AN INTRODUCTION TO FLAPPING WING AERODYNAMICS

This is an ideal book for graduate students and researchers interested in the aerodynamics, structural dynamics, and flight dynamics of small birds, bats, and insects, as well as of micro air vehicles (MAVs), which present some of the richest problems intersecting science and engineering. The agility and spectacular flight performance of natural flyers – made possible by their flexible, deformable wing structures as well as outstanding wing, tail, and body coordination – are particularly significant. To design and build MAVs with performance comparable to natural flyers, it is essential to understand natural flyers’ combined flexible structural dynamics and aerodynamics. The primary focus of this book is to address recent developments in flapping wing aerodynamics. This book extends the work presented in *Aerodynamics of Low Reynolds Number Flyers* (Shyy et al. 2008).

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An Introduction to Flapping Wing Aerodynamics

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Hawk preying on an egret, Chi Lu (1477–?), Ming Dynasty, Palace Museum, Beijing
## Contents

*Preface*  
*Preface of the First Edition* (Aerodynamics of Low Reynolds Number Flyers)  
*List of Abbreviations*  
*Nomenclature*

### 1 Introduction

1. Flapping Flight in Nature  
1.1 Geometric Similarity  
1.2 Geometric Similarity  
1.3 Wing Area  
1.4 Wing Loading  
1.5 Aspect Ratio  
1.6 Wing-Beat Frequency  
1.7 Gliding and Soaring  
1.8 Powered Flight: Flapping  
1.9 Power Implication of Flapping Wings  
1.10 Upper and Lower Limits  
1.11 Drag and Power  
1.12 Concluding Remarks

### 2 Rigid Fixed-Wing Aerodynamics

2.1 Laminar Separation and Transition to Turbulence  
2.1.1 Navier-Stokes Equation and the Transition Model  
2.1.2 The $e^N$ Method  
2.1.3 Case Study: SD7003  
2.2 Factors Influencing Low Reynolds Number Aerodynamics  
2.2.1 $Re = 10^3$–$10^4$  
2.2.2 $Re = 10^4$–$10^6$  
2.2.3 Effect of Free-Stream Turbulence

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Contents

2.2.4 Effect of Unsteady Free-Stream 72
2.3 Three-Dimensional Wing Aerodynamics 76
  2.3.1 Unsteady Phenomena at High AoAs 77
  2.3.2 Aspect Ratio and Tip Vortices 78
  2.3.3 Wingtip Effect 83
  2.3.4 Unsteady Tip Vortices 88
2.4 Concluding Remarks 89

3 Rigid Flapping-Wing Aerodynamics ......................... 90
  3.1 Flapping Wing and Body Kinematics 95
  3.2 Governing Equations and Non-Dimensional Parameters 100
    3.2.1 Reynolds Number 100
    3.2.2 Strouhal Number and Reduced Frequency 101
  3.3 Unsteady Aerodynamic Mechanisms in Flapping Wings 103
    3.3.1 Leading-Edge Vortices (LEVs) 106
    3.3.2 Rapid Pitch-Up 111
    3.3.3 Wake Capture 113
    3.3.4 Tip Vortices (TiVs) 114
    3.3.5 Clap-and-Fling Mechanism 116
  3.4 Fluid Physics in $O(10^2$ to $10^3$) Reynolds Number Regime 118
    3.4.1 Effects of Kinematics on Hovering Airfoil Performance 118
    3.4.2 Effects of Wind Gust on Hovering Aerodynamics 129
  3.5 Fluid Physics in $O(10^4$ to $10^5$) Reynolds Number Regime 136
    3.5.1 Flow around a Flat Plate in Shallow and Deep Stall at $Re = 6 \times 10^4$ 137
    3.5.2 Effects of the Reynolds Number 138
    3.5.3 Airfoil Shape Effects: Sane’s Use of Polhamus’s Analogy 139
    3.5.4 2D versus 3D Flat Plate in Shallow Stall 147
  3.6 Approximate Analysis for Non-Stationary Airfoil 149
    3.6.1 Force Prediction for a Pitching and Plunging Airfoil in Forward Flight 149
    3.6.2 Simplified Aerodynamics Models 151
    3.6.3 Some Remarks on Simplified Models 157
    3.6.4 Scaling of the Forces Acting on a Moving Body Immersed in Fluid 162
    3.6.5 Flapping Wing Model versus Rotating Wing Model 165
  3.7 Modeling of Biological Flyers in a Rigid-Wing Framework 166
    3.7.1 Hovering Hawkmoth 166
    3.7.2 Hovering Passerine 170
    3.7.3 Reynolds Number Effects on the LEV and Spanwise Flow: Hawkmoth, Honeybee, Fruit Fly, and Thrips in Hovering Flight 170
  3.8 Concluding Remarks 173

4 Flexible Wing Aerodynamics .............................. 176
  4.1 General Background of Flexible Wing Flyers 176
  4.2 Governing Equations for Wing Structures 185
Contents

4.2.1 Linear Beam Model 186
4.2.2 Linear Membrane Model 187
4.2.3 Hyperelastic Membrane Model 190
4.2.4 Flat Plate and Shell Models 192

4.3 Scaling Parameters for the Flexible Wing Framework 192

4.4 Interactions between Elastic Structural Dynamics and Aerodynamics 195
4.4.1 Fixed Membrane Wing 195
4.4.2 Flapping Flexible Wings 208

4.5 A Scaling Parameter for Force Generation for Flexible Wings 225
4.5.1 Propulsive Force and Non-Dimensional Wingtip Deformation Parameters 226
4.5.2 Scaling and Lift Generation of Hovering Flexible Wing of Insect Size 233
4.5.3 Power Input and Propulsive Efficiency 239
4.5.4 Implications of the Scaling Parameters on the Aerodynamic Performance of Flapping Flexible Wings 243

4.6 Biological Flyers and Flexible Wings 246
4.6.1 Implications of Anisotropic Wing Structure on Hovering Aerodynamic: Hawkmoths 248

4.7 Aerodynamics of Bat Flight 253
4.8 Concluding Remarks 256

5 Future Perspectives ........................................ 259

References 267
Index 293
Preface

This book is about flapping wing aerodynamics. It presents various aspects of the aerodynamics of natural flyers, such as birds, bats, and insects, and of human-engineered micro air vehicles (MAVs) for both rigid and flexible wing structures. This edition focuses on the many recent developments since the publication of our earlier book titled *Aerodynamics of Low Reynolds Number Flyers*. We have substantially expanded Chapter 1 to offer a general and comprehensive introduction to low Reynolds number flight vehicles for both biological flyers and human-made MAVs. In particular, we summarize the scaling laws to relate the aerodynamics and various flight characteristics to a flyer’s size, weight, and speed on the basis of simple geometric and dynamics analyses. In Chapter 2, closely following the previous edition, we discuss the aerodynamics of fixed rigid wings. It considers both two- and three-dimensional airfoils with typically low aspect ratio wings. Both Chapters 3 and 4 have been significantly expanded and updated. Chapter 3 examines the interplay between flapping kinematics and key dimensionless parameters such as the Reynolds number, Strouhal number, and reduced frequency for rigid wings. The various unsteady lift enhancement mechanisms are addressed, including leading-edge vortex, rapid pitch-up and rotational circulation, wake capture, tip vortices, and clap-and-fling. It also discusses both detailed time-dependent and simplified quasi-steady analyses along with experimental observations. Efforts have been made to contrast fixed and flapping wing aerodynamics in the context of geometry and tip, as well as of stall margins. Chapter 3 presents individual and varied objectives in regard to maximizing lift, mitigating drag, and minimizing power associated with flapping wings.

Chapter 4 addresses the role of structural flexibility of low Reynolds number wing aerodynamics. Due to the interplay between structural and fluid dynamics, additional dimensionless parameters appear, resulting in multiple time and length scales. For fixed wings, structural flexibility can further enhance stall margin and flight stability; for flapping wings, passive control can complement and possibly replace active pitching to make the flight more robust and more power efficient. Chapter 4 also discusses the airfoil shape, the time-dependent fluid and structural dynamics, and the spanwise versus chordwise flexibility of a wing. The scaling laws linking lift and power with fluid and structural parameters are of fundamental interest and offer insight into low Reynolds number flight sciences while providing guidelines for
vehicle development. Finally, recent advances and future perspectives are summarized and presented in Chapter 5.

As in the previous edition, we have benefited from collaborations and interactions with many colleagues. In addition to those colleagues named in the previous edition, we would like to acknowledge the generous intellectual and financial support provided by the U.S. Air Force Research Laboratory, in particular the Flight Vehicle Directorate (now Aerospace Systems Directorate) and the Office of Scientific Research.

We feel sure that significant advancements in both scientific and engineering endeavors of flapping wing aerodynamics will continue to be achieved, and we enthusiastically await these new breakthroughs and developments.

Wei Shyy, Hikaru Aono, Chang-kwon Kang, and Hao Liu
Low Reynolds number aerodynamics is important for a number of natural and man-made flyers. Birds, bats, and insects have been of interest to biologists for years, and active study in the aerospace engineering community has been increasing rapidly. Part of the reason is the advent of micro air vehicles (MAVs). With a maximal dimension of 15 cm and nominal flight speeds around 10 m/s, MAVs are capable of performing missions such as environmental monitoring, surveillance, and assessment in hostile environments. In contrast to civilian transport and many military flight vehicles, these small flyers operate in the low Reynolds number regime of $10^5$ or lower. It is well established that the aerodynamic characteristics, such as the lift-to-drag ratio of a flight vehicle, change considerably between the low and high Reynolds number regimes. In particular, flow separation and laminar-turbulent transition can result in substantial change in effective airfoil shape and reduce aerodynamic performance. Since these flyers are lightweight and operate at low speeds, they are sensitive to wind gusts. Furthermore, their wing structures are flexible and tend to deform during flight. Consequently, the aero/fluid and structural dynamics of these flyers are closely linked to each other, making the entire flight vehicle difficult to analyze.

The primary focus of this book is on the aerodynamics associated with fixed and flapping wings. Chapter 1 offers a general introduction to low Reynolds flight vehicles, including both biological flyers and MAVs, followed by a summary of the scaling laws that relate the aerodynamics and flight characteristics to a flyer’s sizing on the basis of simple geometric and dynamics analyses. Chapter 2 examines the aerodynamics of fixed, rigid wings. Both two- and three-dimensional airfoils with typically low aspect ratio wings are considered. Chapter 3 examines structural flexibility within the context of fixed wing aerodynamics. The implications of laminar-turbulent transition, multiple time scales, airfoil shapes, angles-of-attack, stall margin, structural flexibility, and time-dependent fluid and structural dynamics are highlighted.

Unsteady flapping wing aerodynamics is presented in Chapter 4. In particular, the interplay between flapping kinematics and key dimensionless parameters such as the Reynolds number, Strouhal number, and reduced frequency is examined. The various unsteady lift enhancement mechanisms are also addressed, including
Preface of the First Edition

leading-edge vortex, rapid pitch-up and rotational circulation, wake capture, and clap-and-fling.

The materials presented in this book are based on our own research, existing literature, and communications with colleagues. At different stages, we have benefited from collaborations and interactions with colleagues: Drs. Peter Ifju, David Jenkins, Rick Lind, Raphael Haftka, Roberto Albertani, and Bruce Carroll of the University of Florida; Drs. Luis Bernal, Carlos Cesnik, and Peretz Friedmann of the University of Michigan; Drs. Michael Ol, Miguel Visbal, and Gregg Abate, and Mr. Johnny Evers of the Air Force Research Laboratory; Dr. Ismet Gursul of the University of Bath; Dr. Charles Ellington of Cambridge University; Dr. Keiji Kawachi of the University of Tokyo; Mr. Hikaru Aono of Chiba University; Dr. Mao Sun of the Beijing University of Aeronautics and Astronautics. In particular, we have followed the flight vehicle development efforts of Dr. Peter Ifju and his group and enjoyed the synergy between us.

MAV and biological flight is now an active and well-integrated research area, attracting participation from a wide range of talents and specialties. The complementary perspectives of researchers with different training and backgrounds enable us to develop new biological insight, mathematical models, physical interpretation, experimental techniques, and design concepts.

Thinking back to the time we started our own endeavor a little more than ten years ago, substantial progress has taken place, and there is every expectation that significantly more will occur in the foreseeable future. We look forward to it!

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Ann Arbor, Michigan, U.S.A.

Hao Liu
Chiba, Japan

December 31, 2006
# List of Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>2D</td>
<td>two-dimensional</td>
</tr>
<tr>
<td>3D</td>
<td>three-dimensional</td>
</tr>
<tr>
<td>AoA</td>
<td>angle of attack</td>
</tr>
<tr>
<td>DNS</td>
<td>direct numerical simulation</td>
</tr>
<tr>
<td>LES</td>
<td>large-eddy simulation</td>
</tr>
<tr>
<td>LEV</td>
<td>leading-edge vortex</td>
</tr>
<tr>
<td>LSB</td>
<td>laminar separation bubble</td>
</tr>
<tr>
<td>MAV</td>
<td>micro air vehicle</td>
</tr>
<tr>
<td>PIV</td>
<td>particle image velocimetry</td>
</tr>
<tr>
<td>RANS</td>
<td>Reynolds-averaged Navier-Stokes</td>
</tr>
<tr>
<td>TEV</td>
<td>trailing-edge vortex</td>
</tr>
<tr>
<td>TiV</td>
<td>tip vortex</td>
</tr>
<tr>
<td>UAV</td>
<td>unmanned air vehicle</td>
</tr>
</tbody>
</table>
### Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$AR$</td>
<td>aspect ratio</td>
<td>Eq. (1–7)</td>
</tr>
<tr>
<td>$b$</td>
<td>wingspan</td>
<td>Eq. (1–7)</td>
</tr>
<tr>
<td>$c$</td>
<td>chord length</td>
<td>Eq. (1–19)</td>
</tr>
<tr>
<td>$c_3$</td>
<td>unit vector in the direction from the leading edge to the trailing edge</td>
<td>Eq. (4–28)</td>
</tr>
<tr>
<td>$C_D$</td>
<td>drag coefficient</td>
<td>Eq. (2–22)</td>
</tr>
<tr>
<td>$C_{D,F}$</td>
<td>drag coefficient due to skin friction</td>
<td>Eq. (2–22)</td>
</tr>
<tr>
<td>$C_{D,P}$</td>
<td>drag coefficient due to pressure</td>
<td>Eq. (2–22)</td>
</tr>
<tr>
<td>$C_F$</td>
<td>force coefficient</td>
<td>Eq. (3–35)</td>
</tr>
<tr>
<td>$C_L$</td>
<td>lift coefficient</td>
<td>Eq. (1–1)</td>
</tr>
<tr>
<td>$C_T$</td>
<td>tension coefficient, thrust coefficient</td>
<td>Eqs. (3–23) and (4–2)</td>
</tr>
<tr>
<td>$D_{aero}$</td>
<td>aerodynamic drag</td>
<td>Eq. (1–29)</td>
</tr>
<tr>
<td>$D_{ind}$</td>
<td>induced drag</td>
<td>Eq. (1–29)</td>
</tr>
<tr>
<td>$D_{par}$</td>
<td>parasite drag (drag on the body)</td>
<td>Eq. (1–29)</td>
</tr>
<tr>
<td>$D_{pro}$</td>
<td>profile drag</td>
<td>Eq. (1–29)</td>
</tr>
<tr>
<td>$D_w$</td>
<td>drag on a finite wing</td>
<td>Eq. (1–28)</td>
</tr>
<tr>
<td>$e$</td>
<td>span efficiency factor</td>
<td>Eq. (2–22)</td>
</tr>
<tr>
<td>$E$</td>
<td>elastic modulus</td>
<td>Eq. (4–1)</td>
</tr>
<tr>
<td>$f$</td>
<td>flapping (wing-beat) frequency</td>
<td>Eq. (1–12)</td>
</tr>
<tr>
<td>$f_{ext}$</td>
<td>distributed external force per unit</td>
<td>Eq. (4–1)</td>
</tr>
<tr>
<td>$f_n$</td>
<td>natural frequency</td>
<td>Eq. (1–21)</td>
</tr>
<tr>
<td>$g$</td>
<td>gravitational acceleration</td>
<td>Eq. (1–3)</td>
</tr>
<tr>
<td>$h_d$</td>
<td>flapping amplitude</td>
<td>Eq. (3–4)</td>
</tr>
<tr>
<td>$h_t$</td>
<td>thickness of wing, thickness of membrane</td>
<td>Eqs. (4–1) and (4–8)</td>
</tr>
<tr>
<td>$h(t)$</td>
<td>time-dependent flapping displacement</td>
<td>Eq. (3–4)</td>
</tr>
<tr>
<td>$H$</td>
<td>shape factor</td>
<td>Eq. (2–2)</td>
</tr>
<tr>
<td>$I$</td>
<td>moment of inertia</td>
<td>Eq. (1–10)</td>
</tr>
<tr>
<td>$J$</td>
<td>advance ratio</td>
<td>Eq. (3–14)</td>
</tr>
<tr>
<td>$J_T$</td>
<td>torque</td>
<td>Eq. (1–9)</td>
</tr>
<tr>
<td>$k$</td>
<td>reduced frequency, turbulent kinetic energy</td>
<td>Eqs. (1–19) and (2–6)</td>
</tr>
<tr>
<td>$l$</td>
<td>characteristic length</td>
<td>Eq. (1–4)</td>
</tr>
<tr>
<td>$L$</td>
<td>lift, length of membrane after deformation</td>
<td>Eqs. (1–1) and (4–10)</td>
</tr>
</tbody>
</table>
Nomenclature

$L_0$ unstrained membrane length Eq. (4–3)
$L/D$ lift-to-drag ratio or glide ratio Eq. (2–20)
$m$ flyer’s total mass Eq. (1–3)
$\tilde{n}$ amplification factor Eq. (2–12)
$N$ threshold value that triggers turbulent flow in $e^N$ method Eq. (2–17)
$p$ static pressure Eq. (2–5)
$P_{aero}$ total aerodynamic power Eq. (1–30)
$P_{\text{ind}}$ induced power Eq. (1–32)
$P_{\text{inc}}$ inertial power Eq. (1–33)
$P_{\text{par}}$ parasite power Eq. (1–32)
$P_{\text{pro}}$ profile power Eq. (1–32)
$P_{\text{tot}}$ total power required for flight Eq. (1–33)
$q_\infty$ far field dynamic pressure Eq. (4–13)
$R$ wing length Eq. (3–24)
$Re$ Reynolds number
$Re_{f2}$ Reynolds number for 2D flapping motion Eqs. (3–8a) and (3–8b)
$Re_{f3}$ Reynolds number for 3D flapping motion Eq. (3–7)
$Re_T$ turbulent Reynolds number Eq. (2–10)
$Re_\theta$ momentum thickness Reynolds number Eq. (2–12)
$S$ wing area Eq. (1–1)
$S^0$ membrane prestress Eq. (4–8)
$St$ Strouhal number Eq. (3–9)
$t$ time Eq. (2–5)
$T$ wing stroke time scale, thrust Eqs. (1–12) and (1–31)
$u_i$ velocity vector in Cartesian coordinates Eq. (2–4)
$U$ forward flight velocity (free-stream velocity) Eq. (1–1)
$U_f$ flapping velocity Eq. (1–20)
$U_{mp}$ velocity for minimum power (forward flight) Eq. (1–35)
$U_{Mr}$ velocity for maximum range (forward flight) Eq. (1–35)
$U_r$ relative flow velocity Eq. (1–20)
$U_{\text{ref}}$ reference velocity Eq. (1–19)
$w$ vertical velocity in the far wake, transverse deflection Eqs. (3–26) and (4–1)
$w_i$ downwash (induced) velocity Eq. (1–14)
$W$ weight Eq. (1–1)
$W/S$ wing loading Eq. (1–2)
$x_i$ spatial coordinates vector Eq. (2–4)
$\alpha$ angle of attack, feathering angle (pitch angle) of a flapping wing Eqs. (3–3) and (3–5)
$\beta$ stroke plane angle Eq. (3–25)
$\delta^*$ boundary-layer displacement thickness Eq. (2–3)
$\phi$ positional angle of a flapping wing Eq. (3–15)
$\Phi$ stroke angular amplitude Eq. (3–7)
<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>Coefficient of dynamic viscosity</td>
<td>Table 4.2</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>membrane tension, non-dimensional tip, deformation parameter</td>
<td>Eqs. (4–4) and (4–34)</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>circulation</td>
<td>Eq. (2–23)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>phase difference between plunging and pitching motion</td>
<td>Eq. (3–5)</td>
</tr>
<tr>
<td>$\nu$</td>
<td>kinematic viscosity, Poisson’s ratio</td>
<td>Eqs. (2–5) and (4–23)</td>
</tr>
<tr>
<td>$v_{Te}$</td>
<td>effective eddy viscosity</td>
<td>Eq. (2–18)</td>
</tr>
<tr>
<td>$v_T$</td>
<td>turbulent eddy viscosity</td>
<td>Eq. (2–6)</td>
</tr>
<tr>
<td>$\Pi_0$</td>
<td>effective inertia</td>
<td>Eq. (4–32)</td>
</tr>
<tr>
<td>$\Pi_1$</td>
<td>effective stiffness</td>
<td>Eq. (4–15)</td>
</tr>
<tr>
<td>$\Pi_{1,\text{pret}}$</td>
<td>effective pretension</td>
<td>Eq. (4–17)</td>
</tr>
<tr>
<td>$\Pi_2$</td>
<td>effective rotational inertia</td>
<td>Eq. (4–25)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>gliding angle, boundary-layer momentum thickness, elevation angle of a flapping wing</td>
<td>Eqs. (1–17), (2–5), and Eq. (3–2)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>fluid density</td>
<td>Eq. (1–1)</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>structural density</td>
<td>Eq. (4–1)</td>
</tr>
<tr>
<td>$\tau_{ij}$</td>
<td>Reynolds-stress tensor</td>
<td>Eq. (2–6)</td>
</tr>
<tr>
<td>$\omega$</td>
<td>dissipation rate for k-\omega turbulence model</td>
<td>Eq. (2–7)</td>
</tr>
<tr>
<td>$\omega_n$</td>
<td>natural angular frequency of the beam model</td>
<td>Eq. (4–33)</td>
</tr>
<tr>
<td>$\dot{\omega}$</td>
<td>angular acceleration</td>
<td>Eq. (1–11)</td>
</tr>
<tr>
<td>$\eta$</td>
<td>propulsive efficiency for forward flight</td>
<td>Eq. (4–41)</td>
</tr>
<tr>
<td>$\Psi$</td>
<td>the bending angle</td>
<td>Eq. (4–29)</td>
</tr>
<tr>
<td>$()^*$</td>
<td>non-dimensional quantity</td>
<td></td>
</tr>
<tr>
<td>$()$</td>
<td>time-averaged quantity</td>
<td></td>
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</tbody>
</table>