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978-1-107-03472-3 - Dynamic Models for Volatility and Heavy Tails: With Applications to  
Financial and Economic Time Series

Andrew C. Harvey

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