Dynamic Models for Volatility and Heavy Tails

The volatility of financial returns changes over time and, for the last thirty years, Generalized Autoregressive Conditional Heteroscedasticity (GARCH) models have provided the principal means of analyzing, modelling and monitoring such changes. Taking into account that financial returns typically exhibit heavy tails – that is, extreme values can occur from time to time – Andrew C. Harvey's new book shows how a small but radical change in the way GARCH models are formulated leads to a resolution of many of the theoretical problems inherent in the statistical theory. The approach can also be applied to other aspects of volatility, such as those arising from data on the range of returns and the time between trades. Furthermore, the more general class of Dynamic Conditional Score models extends to robust modelling of outliers in the levels of time series and to the treatment of time-varying relationships. As such, there are applications not only to financial data but also to macroeconomic time series and to time series in other disciplines. The statistical theory draws on basic principles of maximum likelihood estimation and, by doing so, leads to an elegant and unified treatment of nonlinear time-series modelling. The practical value of the proposed models is illustrated by fitting them to real data sets.

Andrew C. Harvey is Professor of Econometrics at the University of Cambridge and a Fellow of Corpus Christi College. He is a Fellow of the Econometric Society and of the British Academy. He has published more than 100 articles in journals and edited volumes and is the author of three books, *The Econometric Analysis of Time Series*, *Time Series Models* and *Forecasting and Structural Time Series Models and the Kalman Filter* (Cambridge University Press, 1989). He is one of the developers of the STAMP computer package.

Cambridge University Press
978-1-107-03472-3 - Dynamic Models for Volatility and Heavy Tails: With Applications to Financial and Economic Time Series
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CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town,
Singapore, São Paulo, Delhi, Mexico City

Cambridge University Press
32 Avenue of the Americas, New York, NY 10013-2473, USA

www.cambridge.org
Information on this title: www.cambridge.org/9781107630024

First published 2013

Printed in the United States of America

A catalog record for this publication is available from the British Library.

Library of Congress Cataloging in Publication Data

Harvey, A. C. (Andrew C.)
Dynamic models for volatility and heavy tails : with applications to financial and economic time series / Andrew C. Harvey.
p. cm. – (Econometric society monographs)
Includes bibliographical references and index.
ISBN 978-1-107-03472-3 (hbk.) – ISBN 978-1-107-63002-4 (pbk.)
1. Econometrics. 2. Finance – Mathematical models. 3. Time-series analysis. I. Title.
HB139.H369 2013
330.01′5195–dc23 2012036508

ISBN 978-1-107-03472-3 Hardback
ISBN 978-1-107-63002-4 Paperback

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Preface

This book sets out a class of nonlinear time series models designed to extract a dynamic signal from noisy observations. The signal may be the level of a series or it may be a measure of scale. Changing scale is of considerable importance in financial time series where volatility clustering is an established stylized fact. Generalized autoregressive conditional heteroscedasticity (GARCH) models are widely used to extract the current variance of a series. However, using variance (or rather, standard deviation) as a measure of scale may not be appropriate for non-Gaussian (conditional) distributions. This is of some importance, because another established feature of financial returns is that they are characterized by heavy tails.

The dynamic equations in GARCH models are filters. Just as the filters for linear Gaussian location models are linear combinations of past observations, so GARCH filters, because of their Gaussian origins, are usually linear combinations of past squared observations. The models described here replace the observations or their squares by the score of the conditional distribution. Furthermore, when modelling scale, an exponential link function is employed, as in exponential GARCH (EGARCH), thereby ensuring that the filtered scale remains positive. The unifying feature of the models in the proposed class is that the asymptotic distribution of the maximum likelihood estimators is established by a single theorem that delivers an explicit analytic expression for the asymptotic covariance matrix of the estimators. Furthermore, the conditions under which the asymptotics go through are relatively straightforward to verify. There is no such general theory for GARCH models: analytic expressions for the asymptotic covariance matrix of the maximum likelihood estimators cannot be found even in the most basic cases, and for some models, most notably EGARCH, there is no asymptotic theory except for very special cases that are never used in practice.

Other properties of the proposed models may be found. These include analytic expressions for moments, autocorrelation functions and multistep forecasts. The properties, particularly for the volatility models, which employ an exponential link function, are more general than is usually the case. For example, expressions for unconditional moments, autocorrelations and the conditional moments of multistep predictive distributions can be obtained for absolute values of the observations raised to any power.

The generality of the approach is further illustrated by consideration of dynamic models for non-negative variables. Such models have been used for modelling duration, range and realized volatility in finance. Again, the use of an exponential link function combined with a dynamic equation driven by the conditional score gives a range of analytic results similar to those obtained with the new class of EGARCH models.

Estimating a dynamic level embedded in noise is explicitly an exercise in signal extraction. A general treatment of Gaussian models is based on the state space form and the Kalman filter. When the noise comes from a heavy-tailed distribution, such as Student's t, the filter proposed here can be regarded as an approximation to a filter for the signal plus noise model that can only be obtained by computer simulation techniques, as in Durbin and Koopman (2012). However, its properties are obtained by treating it as a model in its own right. Such a model is said to be observation-driven, as opposed to the unobserved components model, which is parameter-driven. Turning to scale, GARCH models are not usually seen as vehicles for signal extraction, but this is precisely what they are. That this is the case becomes clearer if they are viewed as observation-driven approximations to parameter-driven stochastic volatility models. Indeed, this was part of the original motivation for the formulation of EGARCH models. The development of the class of observation-driven models in this book acknowledges the link with parameter-driven models, and in doing so, it takes a step towards a unified theory of nonlinear time series models.

The book assumes that the reader is familiar with the basic ideas and technicalities of time series. The mathematics is not too demanding given a good understanding of statistical concepts such as conditional distributions and maximum likelihood estimation. Hence it should be accessible to graduate students in the more technical areas of economics and finance, as well as to statisticians. Sections marked with an asterisk (*) are more technical and/or tangential to the main argument and can be skipped without loss of continuity.

The idea of using the score to drive the dynamics in non-Gaussian models is not new, but up to now has had no firm theoretical foundation. The research for this book began in 2008 with a working paper I wrote with a student, Tirthankar Chakravarty, on EGARCH models. At the same time, Siem-Jan Koopman and his co-workers were independently developing a range of score-driven models.[1] They also produced a working paper in 2008. Because Siem-Jan and I have co-authored many papers on unobserved component models, it is perhaps not too surprising that we hit on the same idea, albeit by different routes. One of the difficulties we faced was that the models lacked a convincing asymptotic

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[1] Rather than the term dynamic conditional score (DCS) models, which I use here, Creal, Koopman and Lucas (2011) prefer the name generalized autoregressive score (GAS). However, despite the attraction of the acronym, the term 'autoregressive' seems to me to convey a more limited dynamic structure than is actually the case.

theory for maximum likelihood estimation. Fortunately, a six-month visit to Carlos III University in Madrid in 2010 provided me with the inspiration to develop the necessary theory. I'm grateful to the Bank of Santander for its support under the Carlos III program for Chairs of Excellence. Further work was done when I was a visiting Fernand Braudel Fellow at the European University Institute in Florence towards the end of 2011. It was there, in the garden of the Villa San Paolo, that Anders Rahbek gave me a memorable tutorial on the finer points of advanced asymptotic theory for time series. I'm grateful to Anders and to all the other colleagues who have provided comments and support during the work on the project. These include Philipp Andres, Tirthankar Chakravarty, Frank Diebold, Rob Engle, Gloria Gonzalez-Rivera, Peter Hansen, Stan Hurn, Ryoko Ito, Siem-Jan Koopman, Alessandra Luati, Mark Salmon, Steve Satchell, Richard Smith, Genaro Sucarrat, Abderrahim Taamou, Stephen Thiele and Paolo Zaffaroni. Universities at which the ideas were presented include Oxford, Warwick, Queensland, Monash, Hanover, EUI, Carlos III, Alicante, New York, Columbia and Pennsylvania. Special thanks go to Esther Ruiz at Carlos III and Mardi Dungey at the University of Tasmania, where I spent three weeks in December 2010. Finally I'd like to thank Rosa Matzkin and two anonymous readers for their helpful and constructive comments, and Peihang Lu for editorial assistance.

Acronyms and Abbreviations

ACD	autoregressive conditional duration
ACF	autocorrelation function
AIC	Akaike information criterion
APARCH	asymmetric power ARCH
ARCH	autoregressive conditional heterocedasticity
ARIMA	autoregressive integrated moving average
BIC	Bayesian information criterion
CAViaR	conditional autoregressive value at risk by regression quantiles
CDF	cumulative distribution function
CPI	consumer price index
CV	coefficient of variation
DCC	dynamic conditional correlation
DCS	dynamic conditional score
EGARCH	exponential GARCH
ES	expected shortfall
EWMA	exponentially weighted moving average
GARCH	generalised autoregressive conditional heteroscedasticity
GED	general error distribution
GG	generalised gamma
IF	innovations form
IGARCH	integrated GARCH
IID	independent and identically distributed
IRW	integrated random walk
KF	Kalman filter
LIE	law of iterated expectations
LM	Lagrange multiplier
LR	likelihood ratio
MA	moving average
MCMC	Markov chain Monte Carlo
MD	martingale difference
MEM	multiplicative error models
MGF	moment generating function

ML	maximum likelihood
MMSE	minimum mean square error (estimate)
MSE	mean square error
NID	normally and independently distributed
PDF	probability distribution function
PIT	probability-integral transform
QARMA	quasi-ARMA
QML	quasi-maximum likelihood
QQ	quantile-quantile
RMSE	root mean square error
SD	standard deviation
SE	standard error
SRE	stochastic recurrence equation
SSF	state space form
STM	structural time series model
SV	stochastic volatility
UC	unobserved components
VaR	value at risk
WN	white noise