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Excerpt
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Part I

Theoretical Fundamentals

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1 Games and mechanisms for networked systems: incentives and algorithms

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This chapter presents resource allocation mechanisms which take into account strategic behavior of users in wireless networks. The mechanisms include auctions and pricing schemes which differ in their allocation and communication structures, and model a large class of interference functions and interference-based coupled user utility functions. The convex optimization problems arising in interference coupled multi-carrier systems are addressed using pricing mechanisms. In addition, a regression learning scheme is studied as an alternate way to obtain the user preferences by the mechanism designer. As a main contribution, a three-step process is introduced for designing auction mechanisms by deriving the allocation and pricing functions. In the special case where the parameters of logarithmic user utilities are normalized to sum up to one, the resulting auction mechanism is shown to be budget balanced.

1.1 Introduction

As users play a more active role in strategic resource allocation decisions in wireless networks, the interaction between the individual users and network owners becomes more complex. The wireless users have the opportunity of manipulating the system by misrepresenting their private information for their own benefit. The network owner or designer, in turn, aims to design appropriate incentives and algorithms in order to achieve certain network level goals while eliciting true preferences from users [17].

Mechanisms such as auctions and pricing schemes facilitate designing wireless resource allocation algorithms which can be analyzed within the mathematical framework of strategic (non-cooperative) games. Although the participating players are selfish, these mechanisms ensure that the game outcome is optimal with respect to a global criterion (e.g., maximizing a social welfare function) and strategy-proof, i.e., players have no reason to deceive the designer. The mechanism designer achieves these objectives by introducing specific rules and incentives to the players; in this case by adding resource prices to their utilities. The price of anarchy (PoA), which can be summarized as loss in general efficiency, may result in wireless networks when the users are selfish.

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Network mechanism design aims to mitigate PoA and achieve system level goals such as maximization of aggregate user performance.

In auction-based mechanisms, the mechanism designer explicitly allocates the resources based on bids of the participants in addition to setting prices. Alternatively, pricing mechanisms enforce global objectives only by charging the players for the resources they have utilized and the communication structure between the designer and the players is different from auctions. In either setting, the player preferences represented by utility functions may be coupled or decoupled, i.e., they depend on other players' actions or only on players' own actions, respectively. The methodology and algorithms developed are applied to diverse interference limited network problems such as spectrum management, uplink resource allocation of multi-carrier systems, and energy efficiency.

The celebrated Vickrey–Clarke–Groves (VCG) mechanism [19] is efficient, strategy-proof, and weak budget balanced. A modified VCG mechanism can be obtained for allocation of a divisible resource in which the pricing function is modified for achieving efficiency, strategy-proof, and almost budget balance [13]. However, the VCG mechanism requires considerable communication between the designer and the users especially when the user preferences are of dimension higher than two [26].

There is a rich literature based on the Kelly mechanism [29] in which a Nash implementation is proposed for resource allocation with separable utility functions. These mechanisms suffer from efficiency loss in the presence of price anticipating users. Proportional allocation mechanisms based on the scalar bids by the price anticipating users in which the pricing function is designed appropriately can be used to reach full efficiency [31]. In the area of mechanism design for allocation of divisible resources, many works approximate the infinite dimensional user utility functions with finite-dimensional vectors or scalar quantities. In [27], the players are asked to bid on a scalar parameter of an allowable class of scalar parameterized utility functions which are named as surrogate utility functions. Since resource allocation and payment follow the VCG scheme but are based on scalar parameterized surrogate utility functions, they call it the scalar parameterized VCG mechanism. The outcome is at least one efficient Nash equilibrium (NE) when the marginal utility from actual utility functions of users and the marginal utility from using the declared parameter of surrogate utility functions become equal. However, there can be other multiple Nash equilibria, which are not efficient, due to the approximation by surrogate valuation functions. The mechanisms proposed in [31] are proved to be a special case of this line of work.

In [30], instead of reporting the utility functions, users bid on a two-dimensional signal – the maximum quantity that they want and the per unit price they are willing to pay. In [25] the results in [30] are extended to a multiple divisible good network case. There exist several Nash equilibria out of which one is efficient in the auction game. Reserve prices are used to eliminate some NE which cause arbitrarily large efficiency loss. These mechanisms are easier to implement as compared to the VCG class of mechanisms due to smaller bid-message spaces, but do not have a truthful dominant strategy implementation. The reason behind this is the fact that dominant strategy implementation for divisible goods requires the bid-message space to be rich enough so that each agent can report their utility at each possible real value.

Yates in [42] has proposed an axiomatic approach and defined standard interference functions. General interference functions have been first proposed in [10] and the framework of standard interference functions are proven to be a subset of these functions in [11]. The framework in [10] and a unifying theory in [11] are useful to design cross-layer or physical layer aware resource allocation problems.

In this chapter, the focus is on the log-convex interference functions which are a subset of general interference functions. Log-convexity is a more relaxed assumption than convexity and a useful property that allows one to apply convex optimization techniques to some non-convex problems. In the presence of utility functions based on interference functions the pricing mechanisms get complicated due to the coupling of decision variables. One option is to have pricing functions in which the users are charged for the interference they create to others [21]. There are recent studies on pricing mechanisms with interference coupled utility functions [6]. Here, price discrimination is allowed so that each user is charged a personalized per-unit price for the resource.

There are impossibility results in mechanism design literature due to the information exchange limitation between the players and the designer [32]. Our approach here is to characterize the class of problems for which the global desirable properties are achieved through implementable algorithms. We also follow a scalar strategy approach, however, which allows users to reveal information about their utilities over time by iterative algorithm or learning.

Overview of the chapter

The main theme of the chapter is how to design incentives and algorithms in the case of selfish users having separable or interference coupled utility functions (scalar or higher dimensional) to achieve different properties for network mechanisms.

In the next section, we introduce the system model with the assumptions and the definitions. The general interference functions, which are proven in [10] to have desirable structure for obtaining convex or concave resource allocation problems, are defined next. Then, the utility modeling for interference coupled systems is studied. We characterize the largest class of interference functions and the interference-based utility functions which have convex system optimization. When social welfare maximization (efficiency) is an objective in the game formulation, these results help to analyze the class of interference functions and signal-to-interference-plus-noise ratio (SINR)-based utility functions for which there is a computable unique optimal point.

For systems with a certain class of interference functions and SINR-based utility functions for users, pricing mechanisms are used to obtain certain system level goals. The mechanisms introduced are extension of the Kelly mechanism for interference coupled systems where the prices are functions of Lagrange multipliers and system parameters [14]. An iterative algorithm is used for the implementation. Specifically, we consider uplink resource allocation of *multi-carrier* systems with strategic users having a scalar-parameterized logarithmic utility function. The users decide independently on their power levels without revealing their utility functions, so as to maximize their individual utilities. The utility parameters may indicate the priority of the application, residual queue size, etc. Concurrently, the base station has a social goal such as social welfare (sum of user utility) maximization which may not be achieved due to this strategic behavior of users. We study distributed pricing algorithms in which the users decide on their power levels depending on their utility functions and the prices which are set by the designer. The mechanisms introduced for net utility maximization are modified for operator revenue maximization as well as to obtain energy efficiency in the network.

We introduce next a learning approach to the mechanism design problem. When the users have infinite dimensional utility functions that are unknown to the designer, the designer elicits the information by learning from the bids of the users. We utilize Gaussian process regression learning techniques to infer general player preferences in a mechanism design setting.

The last section of the chapter presents a methodology for designing a class of efficient and strategy-proof scalar mechanisms. The developed approach is applicable to both auction and pricing mechanisms for certain user utility functions with multiplicative scalar parameters, which correspond to user types. A three-step process is presented for designing auction mechanisms by deriving the allocation and pricing functions.

1.2 System model

Consider a mechanism design model where a *designer*, $\mathcal{D}$, influences a set, $\mathcal{A}$, of users who have private utilities (preferences) and compete for limited resources. The designer tries to achieve a global objective such as welfare maximization by making the users reveal their true utilities. For this purpose, the designer imposes certain rules and prices to the users agreeing to participate in the mechanism. However, the designer cannot dictate user actions or modify their private utilities. This setup is applicable to a variety of network resource allocation problems in networking such as flow control, interference management, and spectrum sharing.

In order to analyze such mechanisms, define a N-player strategic game, $\mathcal{G}$, where each user or player $i \in \mathcal{A}$ has a respective scalar *decision variable* x_i such that

$$x = [x_1, \ldots, x_N] \in \mathcal{X} \subset \mathbb{R}^N,$$

where $\mathcal{X}$ is the decision space of all players. The decision variable x_i may represent, depending on the specific problem formulation, the ith player's flow rate, power level, investment or bid in an auction. Due to the inherent coupling between the players, the decisions of players directly affect each other's performance as well as the aggregate allocation of limited resources. For example, the players may share fixed divisible resource C, such that $\sum_i x_i \leq C$.

The *preference* of the ith player is captured by the utility function

$$U_i(x) : \mathcal{X} \to \mathbb{R},$$

which is assumed to be continuous, twice-differentiable, and concave. The designer imposes a price on the actions of players, which is formulated by adding it as a cost term to utility. Hence, the player i has the cost function

$$J_i(x) = c_i(x) - U_i(x), \tag{1.1}$$

and solves the individual optimization problem

$$\min_{x_i} J_i(x). \tag{1.2}$$

The resulting game $\mathcal{G}(\mathcal{A}, x \in \mathcal{X}, U)$ admits a unique solution if certain convexity and compactness conditions are satisfied [1, 4].

It is important to note that we assume here *price anticipating* users, who take into account the effect of their actions on future prices and act accordingly. This is in contrast with *price-taking* users who ignore it, e.g., due to lack of information.

The *Nash equilibrium* (NE) is a widely accepted and useful solution concept in strategic games, where no player has an incentive to deviate from it while others play according to their NE strategies. The NE x^* of the game $\mathcal{G}$ is formally defined as

$$x_i^* := \arg\min_{x_i} J_i(x_i, x_{-i}^*), \forall i,$$

where $x_{-i}^* = [x_1^*, \ldots, x_{i-1}^*, x_{i+1}^*, \ldots, x_N^*]$. The NE is at the same time the intersection point of the players' best responses obtained by solving (1.2) individually.

A stronger concept is *dominant strategy equilibrium* (DSE), which is defined as

$$x_i^D := \arg\min_{x_i} J_i(x_i, x_{-i}),\ \forall x_{-i}\, \forall i,$$

i.e., the players choose the dominant strategy regardless of the actions of others. Hence, DSE is a subset of NE and doesn't require information about the utility or action of other users.

A mechanism $\mathcal{M}$ is a function f which specifies an outcome for every strategy vector $x \in \mathcal{X} \subset \mathbb{R}^N$, of the players. The function f is implemented through allocation and pricing rules.

The *designer objective*, e.g., maximization of aggregate user utilities or social welfare, can be formulated using a smooth objective function

$$V(x, U_i(x), c_i(x)) : \mathcal{X} \to \mathbb{R},$$

where $c_i(x)$ and $U_i(x)$, $i = 1, \ldots, N$ are user-specific pricing terms and player utilities, respectively. Thus, the objective function V characterizes the desirability of an outcome x from the designer's perspective. In some cases when the designer's objective is to satisfy certain minimum performance constraints such as players achieving certain quality-of-service levels, the objective can be characterized by a region (a subset of the game domain $\mathcal{X}$).

The properties of a mechanism and their corresponding game counterparts are summarized in Table 1.1 and in the following definitions.

Table 1.1 Mechanism design objectives.

Mechanism property	Corresponding game property
Efficiency	NE coincides with maximum of objective function
Strategy-proofness	Game admits a truth revealing dominant strategy equilibrium
Budget balance	No net payments at the NE

DEFINITION 1.1 (Efficiency) Efficient mechanisms maximize designer objective, i.e., they solve the problem

$$\max_x V(x, U_i(x), c_i(x)).$$

DEFINITION 1.2 (Strategy-proof) A mechanism is said to be strategy-proof, if and only if, the corresponding game admits a DSE that reveals the true user types (preferences).

DEFINITION 1.3 (Individual rationality (or) voluntary participation (VP)) This property ensures that the utility of all agents should be greater than or equal to the utility they would get by dropping out of the mechanism. The utility that agents get by not participating in the mechanism is usually taken to be zero.

DEFINITION 1.4 (Budget balance) A mechanism is called budget balanced if the net payments add up to zero regardless of user preferences, i.e., $\sum_i c_i(x) = 0$.

1.3 Interference and utility function models

In this section, we study the class of games and mechanism design problems which give unique optimal solutions when the users are in a wireless network coupled by interference. Here we characterize the largest class of interference functions, which allow a convex and concave formulation of the resource allocation problems in interference coupled wireless systems. We give inherent boundaries on the problems which can be characterized as joint convex problems, which could help obtain practically implementable resource allocation strategies. These models for the physical network layer help to characterize the problems for which there exist signal processing algorithms.

Let us define first the SINR of the received signal as

$$\gamma_i(x) = \frac{x_i}{I_i(x)}, \tag{1.3}$$

where $I_i(x)$ denote the interference function.

Yates in [42] proposed standard interference functions using an axiomatic approach. A different class of functions known as general interference functions were proposed in [10] and are defined as follows.

DEFINITION 1.5 (General interference functions) These are interference functions, $I : \mathbb{R}_+^{K+1} \to \mathbb{R}_+$, which satisfy the following properties:

(A1) Conditional positivity: $I(x) > 0$ if $x > 0$

(A2) Scale invariance: $I(\alpha x) = \alpha I(x), \forall \alpha \in \mathbb{R}_+$

(A3) Monotonicity: $I(x) \geq I(\tilde{x})$ if $x \geq \tilde{x}$

(A4) Strict monotonicity: $I(x) > I(\tilde{x})$ if $x \geq \tilde{x}, x_{N+1} \geq \tilde{x}_{N+1}$.

In [11], both the framework in [42] and the framework of general interference functions were compared and it was proved that every standard interference function is a special case of the framework of general interference functions. It means that any problem involving standard interference functions can be reformulated in terms of the framework $A1, A2, A3$. Therefore, the structural results obtained for general interference functions in [8] and [9] can also be applied for standard interference functions.

We focus on the class of *log-convex* interference functions [8], which are a subset of general interference functions. They satisfy $A1$–$A3$ and additionally $I(e^x)$ is log-convex on $\mathbb{R}^{N+1}$. In [9], it was proven that every convex interference function is a log-convex interference function, although the converse is not true.

Most resource allocation problems such as weighted utility maximization are not jointly concave or convex in the "power domain." The aim is to characterize a strictly monotonic increasing and twice continuously differentiable transformation $\psi(s) = x$ which can convexify these resource allocation problems.

The linear interference functions which is a sub-class of log-convex interference function is given by

$$I_i = \sum_{j \neq i} x_j + \sigma,$$

where σ represents the background noise. We begin our analysis with linear interference functions and generalize the results to the case of log-convex interference functions later. In the case of linear interference functions the transformation $x_i = \exp(s_i)$ is the unique transformation which transforms the weighted utility maximization and other commonly occurring optimization problems to jointly convex or concave [7]. We now check whether this exponential transformation works or not when we relax the condition of linear interference functions to other kinds of interference coupling.

The largest class of interference functions, which preserves the concavity of resource allocation strategies of interference coupled wireless systems, is the family of log-convex interference functions [7]. Hence, a certain class of user utility functions is defined as follows.

DEFINITION 1.6 *Conc* is the family of monotonically increasing, differentiable, and concave utility functions. $\mathcal{E}Conc$ is the family of monotonically increasing and differentiable functions U for which $U(\exp\{x\})$ is concave.

Based on the results obtained for linear interference functions and utility functions in the family *Conc*, we consider a subset $\mathcal{E}Conc$ of *Conc*. It was shown that the family of exponential transformation is the unique transformation, such that relevant and frequently encountered problems in interference coupled wireless systems are jointly

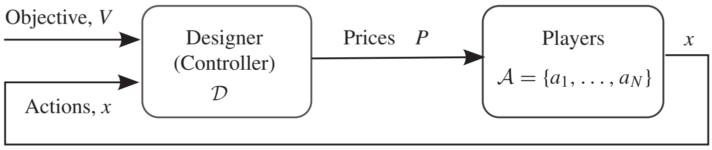

Figure 1.1 A pricing or Pigovian mechanism where the designer charges the user for their direct resource usage.

concave on the s-domain [7]. This is true for linear interference functions and for all utility functions in the class $\mathcal{E}Conc$.

1.4 Pricing mechanisms for multi-carrier wireless systems

In this section, a power allocation problem is considered in multi-carrier systems where the users are allowed to interfere over the different carriers. It is assumed that users are price taking and accept the prices given by the designer as constants which do not depend on their decisions. We study different designer objectives such as net utility maximization, operator revenue maximization, and energy efficiency.

Pricing (Pigovian) mechanisms differ from auction-based ones by the property that the designer does not allocate the resources explicitly, i.e., there is no allocation rule Q or a bidding process. The players obtain resources directly as a result of their actions but are charged for them by the designer observing these actions as depicted in Figure 1.1. This model is suitable for many physical systems where auction mechanisms are not feasible or cause a prohibitively large amount of delay, e.g., due to participating players located in a distributed manner. Example problems from networking include rate control in wired networks, interference management in wireless networks, and power control in optical networks [2, 3, 35, 37].

We consider a multi-carrier system, with narrow band channels, where the transmit power is allocated across multiple orthogonal channels as in orthogonal frequency division multiplexing (OFDM). More than one user is allowed to transmit over a channel to improve the overall capacity of the system and this creates interference between the users. Each user receives a different price for power consumption over different carriers and the prices influence the best user responses.

Due to the inter-cell interference considerations the total power transmitted by the users is to be kept below a threshold in practical multi-cell systems. Another motivation for imposing this threshold is to limit the total power consumption of all the users due to energy considerations. Therefore, the global optimization problem of the designer is constrained with a total power constraint in addition to the individual user power constraints.

Let us consider an uplink multiple access system with spectrum divided into K orthogonal carriers shared among N users. We assume the base station acts as a designer $\mathcal{D}$ who manages the resource sharing among the users. Each user decides on the power