Geometric Methods in Signal and Image Analysis

This comprehensive guide offers a new approach for developing and implementing robust computational methodologies that uncover the key geometric and topological information from signals and images.

With the help of detailed real-world examples and applications, readers will learn how to solve complex signal and image processing problems in fields ranging from remote sensing to medical imaging, bioinformatics, robotics, security, and defense. With an emphasis on intuitive and application-driven arguments, this text not only covers a range of methods in use today, but also introduces promising new developments for the future, bringing the reader up-to-date with the state-of-the-art in signal and image analysis.

Covering basic principles as well as advanced concepts and applications, and with examples and homework exercises, this is an invaluable resource for graduate students, researchers, and industry practitioners in a range of fields including signal and image processing, biomedical engineering, and computer graphics.

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A. Ben Hamza

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Preface

This book is about the use of modern geometric methods for signal and image analysis. It provides a comprehensive coverage of the subject from the basic principles to state-of-the-art concepts and applications. The objective is to give the reader a sound understanding of the major theoretical concepts and computational approaches for applying geometric techniques and methodologies in solving various problems that arise naturally in signal and image processing, computer graphics, computer-aided design, bioinformatics, and other disciplines. The emphasis throughout is on intuitive and application-driven arguments. All methods are illustrated by well-chosen examples and applications, and are selected from core areas of modern geometric and topological computing. Furthermore, the purpose is for the reader to become aware of some recent developments in this fast-growing field.

Audience

The book is intended as a comprehensive and concise reference for geometric and topological methods in signal and image processing. The topics covered in this book are essential for research in numerical geometry and computational algebraic topology, and desirable for students, researchers, and practitioners pursuing research in signal and image processing, computer vision, computer graphics, computer-aided design, and other related fields.

The content grew from notes developed for graduate and undergraduate courses in signal processing, image processing, and computer graphics given at North Carolina State University and Concordia University, primarily targeted at electrical engineering, computer science, and software engineering students.

Chapter organization and topics covered

This book abandons the classical definition-theorem-proof model, and instead heavily relies on effective computational techniques with concrete applications to image analysis, computer vision, geometry processing, and computer graphics. The pitfalls of including all the technical details at the expense of foregone physical intuition of

Preface

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many heavily mathematical texts are largely avoided. The first chapter presents a brief motivation behind geometric methods and their various applications in imaging and computer graphics. Chapters 2 and 3 lay the foundations for our coverage of geometry and topology, and are essential to the rest of the book. The remaining three chapters are, however, almost completely independent of each other. All chapters include lots of examples sprinkled throughout the book to keep the reader actively involved in the process of learning and discovering. In addition, each chapter concludes with applications to signal and image analysis, and/or geometry processing of three-dimensional shapes.

Building up gently from a very low level, Chapters 2 and 3 carefully introduce the reader to the basics of group theory and vector spaces. Motivated by the concept of symmetry, we introduce the theory of groups. We define the notion of a group, and describe in detail the essential group-theoretical concepts through illustrative examples. We then present the formal definition of a vector space, along with some of its basic properties. We also provide an introduction to the theory of linear operators between vector spaces, and examine their key properties. In particular, we show that each linear operator can be represented by a matrix, which plays an important role in the algebra of vector spaces. This allows us to work directly with matrices in lieu of linear operators. We also discuss the eigenvalue decomposition of linear operators and their associated matrices. Motivated by the basic properties of the dot product and its key role in expressing the concepts of length, angle, and area in the Euclidean geometry, we introduce the fundamental metric function of the inner product and discuss how metric concepts of geometry may be applied to general or abstract vector spaces. In addition, we discuss two eigenvalue-based methods - singular value decomposition and principal component analysis - which are commonly used to reduce high-dimensional data into fewer dimensions while retaining important information.

Chapter 4 introduces the local theory of curves and surfaces, focusing primarily on their local properties. By local properties, we mean the properties that are defined in a neighborhood of a point on the curve or surface. In the local theory of curves, we cover the concepts of the tangent, speed, length, arc-length, curvature, and torsion of parametrized curves, and Frenet–Serret apparatus, and we also describe the fundamental theorem of curves. In the local surface theory, we introduce the essential notions of tangent plane, vector fields, Gauss map, orientability, first and second fundamental forms, and surface curvatures.

Motivated by Chapter 4, we introduce in Chapter 5 the basic concepts of manifold theory. For the purpose of visualizing manifolds intuitively, the reader should basically keep in mind the familiar example of a surface in the three-dimensional Euclidean space, which is the space we live and move around in. We describe in particular smooth manifolds and smooth functions on manifolds, vector fields, pushforward and pullback, the Whitney embedding theorem, connections on manifolds, and quotient topology. We also discuss the basic elements of Riemannian geometry, including a Riemannian metric on a Riemannian manifold, the Laplace–Beltrami operator, and isometry between manifolds. In addition, we introduce the concepts of a Morse function and topological Reeb graph.

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Chapter 6 introduces the reader to topological data analysis (TDA), also referred to as computational topology, as an algebraic alternative to graph-based analysis. This approach is particularly important and powerful when geometry of the data is either absent or unaccounted for. One case in point is the analysis of a sensor network. The nodes represent randomly deployed sensors, whose precise coordinates are unknown. These sensors typically have a wireless communication ability within some range, thereby establishing connectivity with other sensors within range. The topology of such a constructed flag graph can be systematically investigated to provide insight into the network functionality and failures. In its raw form, such analysis may be viewed as generically addressing a point cloud as a sampled data from a manifold. Other important results which are developed include higher-order Laplacians, which represent natural generalizations of the so-called graph Laplacian. They are also a very important and popular topic of research.

Prerequisites

We assume some familiarity with the basic concepts of linear algebra and calculus, albeit the book is largely self-contained. On its own, the present book may be used as a textbook for a single semester graduate course in geometric methods for signal and image analysis. Furthermore, a one semester advanced undergraduate course could also be partly based on Chapters 2, 3, and 4.

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