

A CERTAIN UNCERTAINTY: NATURE'S RANDOM WAYS

Based around a series of real-life scenarios, this vivid introduction to statistical reasoning will teach you how to apply powerful statistical, qualitative, and probabilistic tools in a technical context.

From analysis of electricity bills, baseball statistics, and the movement of stock markets, through to the physics of fermions and bosons, and the effects of climate change, each chapter introduces relevant physical, statistical, and mathematical principles step-by-step in an engaging narrative style, helping to develop practical proficiency in the use of probability and statistical reasoning.

With numerous illustrations, which make it easy to focus on the most important information, and full-color figures available online at www.cambridge.org/silverman, this insightful book is perfect for students and researchers of any discipline interested in the interwoven tapestry of probability, statistics, and physics.

MARK P. SILVERMAN is the G. A. Jarvis Professor of Physics at Trinity College, Connecticut. He received his Ph.D. in Chemical Physics from Harvard University, and has since pursued a wide range of experimental and theoretical studies concerning the structure of matter, the behavior of light, and the dynamics of stars and galaxies.

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To Sue, Chris and Jen
(the only certainties in my life)

Books by Mark P. Silverman

- *And Yet It Moves: Strange Systems and Subtle Questions in Physics* (Cambridge University Press, 1993)
- *More Than One Mystery: Explorations in Quantum Interference* (Springer, New York, 1995)
- *Waves and Grains: Reflections on Light and Learning* (Princeton University Press, 1998)
- *Probing the Atom: Interactions of Coupled States, Fast Beams, and Loose Electrons* (Princeton University Press, 2000)
- *A Universe of Atoms, an Atom in the Universe* (Springer, New York, 2002)
- *Quantum Superposition: Counterintuitive Consequences of Coherence, Entanglement and Interference* (Springer, Heidelberg, 2008)

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How is it possible that mathematics, which is indeed a product of human thought independent of all experience, accommodates so well the objects of reality?

Here, in my view, is a short answer: In so far as mathematical statements concern reality, they are not certain, and in so far as they are certain, they do not refer to reality.

—*Albert Einstein*¹

¹ Albert Einstein, from the lecture “Geometrie und Erfahrung” [Geometry and Experience] given in Berlin on 27 January 1921. (Translation from German by M. P. Silverman.)

Preface

An overview – start here

I have heard it said that a preface is the part of a book that is written last, placed first, and never read. Still, I will take my chances; this is, after all, a book about probability and uncertainty. The purpose of this preface is to explain what kind of book this is, why I wrote it, for whom I wrote it, and what I hope the reader will gain by it.

This book is a technical narrative. It is not a textbook (although you can certainly use it that way); there are no end-of-chapter questions or tests, and the level of material does not presuppose the reader to have reached some envisioned state of preparedness. It is not a monograph; it does not survey an entire field of intellectual activity, and there is no list of references apart from a few key sources that aided me in my own work. It is not a popularization; the writing does not sensationalize its subject matter, and explanations may in part be heuristic or analytical, but (I hope) never shallow and hand-waving.

A narrative is a story – albeit in this book one that is meant to instruct as well as amuse. Each chapter, apart from some background material in the beginning, is an account of a scientific investigation I have undertaken – sometimes because the questions at issue are of utmost scientific importance; other times on a whim out of pure curiosity. The various narratives are different, but through each runs a common thread of probability, uncertainty, randomness, and, often enough, serendipity.

Why, you may be thinking, should *my* scientific investigations interest *you*? To this thought, I can give two answers: one brief, the other longer.

The short answer is that I have written six previous books of the same format (narrative descriptions of my researches), which have sold well. Many people who bought (and presumably read) the books found the diversity of subject matter interesting and the expositions clear and informative, to judge from their unsolicited correspondence. It seems reasonable to me, therefore, that a Bayesian forecast of a reader's response to *this* book would employ a favorably biased “prior”.

The longer answer concerns how people learn things. The principal objective of this book, after all, is to share with anyone who reads it part of what I have learned in some 50 years (and still counting) as an experimental and theoretical physicist.

In the course of a long and somewhat unusual scientific career, my researches have taken me into nearly every field of physics. In broad outline, I study the structure of matter, the behavior of light, and the dynamics of stars and galaxies. My investigations of quantum phenomena have employed electron interferometry, radiofrequency and microwave spectroscopy, laser spectroscopy, magnetic resonance, atomic beams, and nuclear spectroscopy. I have examined the reflection, refraction, diffraction, polarization and scattering of light as a classical wave, and the absorption, emission, and correlation of light as a quantum particle (photon). I have reported on the quantum statistics of neutron fluids and Bose–Einstein condensates in exploded, collapsed stars, and the classical statistics of fragments of exploded glass in my laboratory. I have studied the interactions (electromagnetic, nuclear, and gravitational) of real matter on Earth and of dark matter in the cosmos. My interests embraced projects of high scientific significance (such as tests of quantum electrodynamics, of the theory of nuclear decay, of Newtonian gravity and of general relativity) and projects to understand the workings of physically simple, yet surprisingly complicated, physics toys (such as a motor comprising only a AA battery, small cylindrical magnet, and a paper clip; or a passive hollow tube that is fed room temperature air at the center and emits hot air from one end and cold air from the other).

The point of the preceding partial enumeration of research interests is simply this: I was not *trained* to do all the above and more; I had to *teach* myself – and the motivation for learning what I needed to know in each instance derived from the desire to solve a particular problem that interested me. I did not undertake my physics self-instruction out of a desire to absorb abstract principles!

A narrative – a story – humanizes the starkness of physical principles and abstraction of mathematical expressions, and thereby helps provide motivation to learn both. While the personal situations that prompted me to undertake the studies narrated here are unlikely to pertain to you, the reader, I cannot help but believe that the *issues* involved are as relevant to you as they were to me.

Do you travel – and fly in an airplane? Then you may want to read my analysis of the survival of a pilot who fell five miles without a parachute – and how, from that, I developed a protocol for bringing down safely a jumbo jet whose engines all fail.

Do you invest in the stock market to save for retirement? Then you may want to read my statistical analysis of how common stocks behave and what you can expect the market to do for you.

Do you take medications of some kind or have an annual physical exam with a blood test? Then you will be interested in what my statistical analysis reveals about the reliability of the clinical laboratory reports.

Have you ever served on a jury or a committee or some group required to reach a collective judgment? Then you will surely be interested in my theoretical analysis and experimental tests (aided by collaboration with a BBC television show) of the so-called “wisdom-of-crowds” phenomenon.

Do you pay a power company each month for use of electric energy? Are you confident that the meter readings are accurate and that you are being charged correctly? Before answering the second question, perhaps you should read the chapter detailing the statistical analysis of my own electric energy consumption.

Do you enjoy sports, in particular ball games of one kind or another? Then you may be intrigued by my analysis of the ways in which a baseball can move if struck appropriately – or, perhaps of more practical consequence, how I inferred that a certain prominent US ballplayer was probably enhancing his performance with drugs long before the media became aware of it.

Are you concerned about global climate change? Then my statistical study of the climate *under ground* will give you a perspective on what is likely to be the most serious consequence to occur soonest – a consequence that has rarely been given public exposure.

And if you are a scientist yourself – especially a physicist – then you may be utterly astounded, as I was initially, to learn of persistent claims in the peer-reviewed physics literature of processes that, had they actually occurred, would turn nuclear physics (if not, in fact, all laws of physics) upside down. You should therefore find particularly interesting the chapter that describes my experiments and analyses that lay these extraordinary claims to rest.

The foregoing abbreviated descriptions should not disguise the fact that – as mentioned at the outset – this book is a *technical* narrative. The book can be read, I suppose, simply for the stories, skipping over the lines of mathematics. However, if your goal is to develop some proficiency in the use of probability and statistical reasoning, then you will want to follow the analyses carefully. I start the book with basic principles of probability and show every step to the conclusions reached in the detailed explanations of the empirical studies. (Some of the detailed calculations are deferred to appendices.)

A textbook, in which material is laid out in a “linear” progression of topics, may teach statistics more efficiently – but *this* book teaches the application of statistical reasoning *in context* – i.e. the use of principles as they are needed to solve specific problems. This means there will be a certain redundancy – but that is a *good* thing. In many years as a teacher, I have found that an important part of retention and mastery is to encounter the same ideas more than once but in different applications and at increasing levels of sophistication.

Virtually every standard topic of statistical analysis is encountered in this book, as well as a number of topics you are unlikely to find in any textbook. Furthermore, the book is written from the perspective of a “practical physicist”, not a mathematician or statistician – and, where useful, my viewpoint is offered, schooled by some five decades of experimentation and analysis, concerning issues over which confusion or controversy have arisen in the past: for example, issues relating to sample size and uncertainty, use and significance of chi-square tests and *P*-values, the class

boundaries of histograms, the selection of Bayesian priors, the relationship between principles of maximum likelihood and maximum entropy, and others.

As a final point, it should be emphasized that this book is not merely a “statistics book”. Rather, the subject matter at root is statistical *physics*. Every chapter, apart from the first, involves some experimental aspect, whether measured in a laboratory, simulated on a computer, or observed in the world at large. The themes of the narratives concern physical processes from widely different reaches of physics: dynamics of discrete particles, dynamics of fluids, dynamics of heat flow, statistical mechanics of bosons and fermions, creation of non-classical forms of light, transformations of radioactive nuclei, and more. In the process of solving particular problems, there arise – and I will answer – profound questions that are rarely encountered in physics textbooks. Consider thermodynamics, for example. Why is the chemical potential of black-body radiation zero? Is it zero for *all* kinds of photons? Is it zero because the photon is massless? Would a massless neutrino have a zero chemical potential? Read this book and find out.

What background do you need to read this book? Clearly, the more mathematics and physics you know beforehand, the more of the technical details you will be able to understand. An undergraduate physics major should be able to read all of it by the time he or she graduates. In fact, some of the content comes from the physics lectures I give at an undergraduate institution. A person with a knowledge of calculus should be able to read most of it. But anyone with an interest in probability, statistics, and physics should be able to take away something useful and thought-provoking from just the text.

That concludes the short answer, the long answer, and the objectives stated in the first paragraph of the Preface – if you read it.

Note regarding figures: Color figures for this book are available at the Cambridge University Press website www.cambridge.org/silverman.

Mark P. Silverman

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I would like to thank my son Chris for his invaluable help in formatting the text of many of the figures in the book, for designing the beautiful cover of the book, and for his advice on the numerous occasions when my computers or software suddenly refused to co-operate. It is also a pleasure to acknowledge my long-time colleague, Wayne Strange, whose participation in our collaborative efforts to explore the behavior of radioactive nuclei was essential to the successful outcome of that work.

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