Mobile Robotics

*Mathematics, Models, and Methods*

*Mobile Robotics* offers comprehensive coverage of the essentials of the field suitable for both students and practitioners. Adapted from the author's graduate and undergraduate courses, the content of the book reflects current approaches to developing effective mobile robots. Professor Alonzo Kelly adapts principles and techniques from the fields of mathematics, physics, and numerical methods to present a consistent framework in a notation that facilitates learning and highlights relationships between topics. This text was developed specifically to be accessible to senior-level undergraduates in engineering and computer science, and includes supporting exercises to reinforce the lessons of each section. Practitioners will value the author’s perspectives on practical applications of these principles. Complex subjects are reduced to implementable algorithms extracted from real systems wherever possible, to enhance the real-world relevance of the text.

Alonzo Kelly holds undergraduate degrees in aerospace engineering and computer science, and graduate degrees in robotics. Dr. Kelly worked in the aerospace industry for ten years before returning to academia. As a professor at the Robotics institute at Carnegie Mellon University, he teaches mobile robotics at the graduate and undergraduate levels, conducting research in robot simulation, modeling, controls, position estimation, motion planning, and human interfaces.
Mobile Robotics

Mathematics, Models, and Methods

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## Contents

**Preface**  
page xiii

1 Introduction  
1.1 Applications of Mobile Robots  1  
1.2 Types of Mobile Robots  
1.2.1 Automated Guided Vehicles (AGVs)  2  
1.2.2 Service Robots  3  
1.2.3 Cleaning and Lawn Care Robots  4  
1.2.4 Social Robots  4  
1.2.5 Field Robots  5  
1.2.6 Inspection, Reconnaissance, Surveillance, and Exploration Robots  6  
1.3 Mobile Robot Engineering  7  
1.3.1 Mobile Robot Subsystems  7  
1.3.2 Overview of the Text  8  
1.3.3 Fundamentals of Wheeled Mobile Robots  9  
1.3.4 References and Further Reading  11  
1.3.5 Exercise  11  

2 Math Fundamentals  12  
2.1 Conventions and Definitions  12  
2.1.1 Notational Conventions  13  
2.1.2 Embedded Coordinate Frames  17  
2.1.3 References and Further Reading  21  
2.2 Matrices  21  
2.2.1 Matrix Operations  21  
2.2.2 Matrix Functions  24  
2.2.3 Matrix Inversion  25  
2.2.4 Rank-Nullity Theorem  28  
2.2.5 Matrix Algebra  29  
2.2.6 Matrix Calculus  31  
2.2.7 Leibnitz' Rule  39  
2.2.8 References and Further Reading  40  
2.2.9 Exercises  40
## CONTENTS

### 2.3 Fundamentals of Rigid Transforms
- 2.3.1 Definitions
- 2.3.2 Why Homogeneous Transforms
- 2.3.3 Semantics and Interpretations
- 2.3.4 References and Further Reading
- 2.3.5 Exercises

### 2.4 Kinematics of Mechanisms
- 2.4.1 Forward Kinematics
- 2.4.2 Inverse Kinematics
- 2.4.3 Differential Kinematics
- 2.4.4 References and Further Reading
- 2.4.5 Exercises

### 2.5 Orientation and Angular Velocity
- 2.5.1 Orientation in Euler Angle Form
- 2.5.2 Angular Rates and Small Angles
- 2.5.3 Angular Velocity and Orientation Rates in Euler Angle Form
- 2.5.4 Angular Velocity and Orientation Rates in Angle-Axis Form
- 2.5.5 References and Further Reading
- 2.5.6 Exercises

### 2.6 Kinematic Models of Sensors
- 2.6.1 Kinematics of Video Cameras
- 2.6.2 Kinematics of Laser Rangefinders
- 2.6.3 References and Further Reading
- 2.6.4 Exercises

### 2.7 Transform Graphs and Pose Networks
- 2.7.1 Transforms as Relationships
- 2.7.2 Solving Pose Networks
- 2.7.3 Overconstrained Networks
- 2.7.4 Differential Kinematics Applied to Frames in General Position
- 2.7.5 References and Further Reading
- 2.7.6 Exercises

### 2.8 Quaternions
- 2.8.1 Representations and Notation
- 2.8.2 Quaternion Multiplication
- 2.8.3 Other Quaternion Operations
- 2.8.4 Representing 3D Rotations
- 2.8.5 Attitude and Angular Velocity
- 2.8.6 References and Further Reading
- 2.8.7 Exercises

### 3 Numerical Methods

#### 3.1 Linearization and Optimization of Functions of Vectors
- 3.1.1 Linearization
- 3.1.2 Optimization of Objective Functions
- 3.1.3 Constrained Optimization
- 3.1.4 References and Further Reading
- 3.1.5 Exercises

#### 3.2 Systems of Equations
- 3.2.1 Linear Systems
- 3.2.2 Nonlinear Systems
CONTENTS

3.2.3 References and Further Reading 138
3.2.4 Exercises 139
3.3 Nonlinear and Constrained Optimization 140
  3.3.1 Nonlinear Optimization 140
  3.3.2 Constrained Optimization 146
  3.3.3 References and Further Reading 150
  3.3.4 Exercises 150
3.4 Differential Algebraic Systems 151
  3.4.1 Constrained Dynamics 151
  3.4.2 First- and Second-Order Constrained Kinematic Systems 154
  3.4.3 Lagrangian Dynamics 157
  3.4.4 Constraints 162
  3.4.5 References and Further Reading 166
  3.4.6 Exercises 167
3.5 Integration of Differential Equations 168
  3.5.1 Dynamic Models in State Space 168
  3.5.2 Integration of State Space Models 168
  3.5.3 References and Further Reading 172
  3.5.4 Exercises 172
4 Dynamics 173
  4.1 Moving Coordinate Systems 173
    4.1.1 Context of Measurement 174
    4.1.2 Change of Reference Frame 175
    4.1.3 Example: Attitude Stability Margin Estimation 180
    4.1.4 Recursive Transformations of State of Motion 182
    4.1.5 References and Further Reading 186
    4.1.6 Exercises 186
  4.2 Kinematics of Wheeled Mobile Robots 187
    4.2.1 Aspects of Rigid Body Motion 187
    4.2.2 WMR Velocity Kinematics for Fixed Contact Point 191
    4.2.3 Common Steering Configurations 195
    4.2.4 References and Further Reading 200
    4.2.5 Exercises 201
  4.3 Constrained Kinematics and Dynamics 201
    4.3.1 Constraints of Disallowed Direction 202
    4.3.2 Constraints of Rolling Without Slipping 207
    4.3.3 Lagrangian Dynamics 211
    4.3.4 Terrain Contact 217
    4.3.5 Trajectory Estimation and Prediction 220
    4.3.6 References and Further Reading 224
    4.3.7 Exercises 225
  4.4 Aspects of Linear Systems Theory 226
    4.4.1 Linear Time-Invariant Systems 227
    4.4.2 State Space Representation of Linear Dynamical Systems 234
    4.4.3 Nonlinear Dynamical Systems 239
    4.4.4 Perturbative Dynamics of Nonlinear Dynamical Systems 240
    4.4.5 References and Further Reading 244
    4.4.6 Exercises 244
4.5 Predictive Modeling and System Identification 245
  4.5.1 Braking 245
  4.5.2 Turning 247
  4.5.3 Vehicle Rollover 250
  4.5.4 Wheel Slip and Yaw Stability 253
  4.5.5 Parameterization and Linearization of Dynamic Models 256
  4.5.6 System Identification 259
  4.5.7 References and Further Reading 268
  4.5.8 Exercises 269

5 Optimal Estimation 270
  5.1 Random Variables, Processes, and Transformation 270
    5.1.1 Characterizing Uncertainty 270
    5.1.2 Random Variables 272
    5.1.3 Transformation of Uncertainty 279
    5.1.4 Random Processes 289
    5.1.5 References and Further Reading 294
    5.1.6 Exercises 295
  5.2 Covariance Propagation and Optimal Estimation 296
    5.2.1 Variance of Continuous Integration and Averaging Processes 296
    5.2.2 Stochastic Integration 301
    5.2.3 Optimal Estimation 307
    5.2.4 References and Further Reading 315
    5.2.5 Exercises 315
  5.3 State Space Kalman Filters 316
    5.3.1 Introduction 316
    5.3.2 Linear Discrete Time Kalman Filter 319
    5.3.3 Kalman Filters for Nonlinear Systems 321
    5.3.4 Simple Example: 2D Mobile Robot 327
    5.3.5 Pragmatic Information for Kalman Filters 338
    5.3.6 Other Forms of the Kalman Filter 344
    5.3.7 References and Further Reading 344
    5.3.8 Exercises 345
  5.4 Bayesian Estimation 346
    5.4.1 Definitions 346
    5.4.2 Bayes’ Rule 349
    5.4.3 Bayes’ Filters 353
    5.4.4 Bayesian Mapping 358
    5.4.5 Bayesian Localization 365
    5.4.6 References and Further Reading 369
    5.4.7 Exercises 369

6 State Estimation 370
  6.1 Mathematics of Pose Estimation 370
    6.1.1 Pose Fixing versus Dead Reckoning 371
    6.1.2 Pose Fixing 372
    6.1.3 Error Propagation in Triangulation 376
    6.1.4 Real Pose Fixing Systems 384
    6.1.5 Dead Reckoning 385
    6.1.6 Real Dead Reckoning Systems 396
    6.1.7 References and Further Reading 396
    6.1.8 Exercises 397
CONTENTS

6.2 Sensors for State Estimation 398
   6.2.1 Articulation Sensors 398
   6.2.2 Ambient Field Sensors 400
   6.2.3 Inertial Frames of Reference 401
   6.2.4 Inertial Sensors 403
   6.2.5 References and Further Reading 409
   6.2.6 Exercises 410

6.3 Inertial Navigation Systems 410
   6.3.1 Introduction 410
   6.3.2 Mathematics of Inertial Navigation 411
   6.3.3 Errors and Aiding in Inertial Navigation 416
   6.3.4 Example: Simple Odometry-Aided Attitude and Heading Reference System 420
   6.3.5 References and Further Reading 423
   6.3.6 Exercises 424

6.4 Satellite Navigation Systems 425
   6.4.1 Introduction 425
   6.4.2 Implementation 425
   6.4.3 State Measurement 426
   6.4.4 Performance 430
   6.4.5 Modes of Operation 431
   6.4.6 References and Further Reading 433
   6.4.7 Exercises 434

7 Control 435

7.1 Classical Control 435
   7.1.1 Introduction 435
   7.1.2 Virtual Spring-Damper 439
   7.1.3 Feedback Control 441
   7.1.4 Model Referenced and Feedforward Control 447
   7.1.5 References and Further Reading 452
   7.1.6 Exercises 452

7.2 State Space Control 453
   7.2.1 Introduction 453
   7.2.2 State Space Feedback Control 454
   7.2.3 Example: Robot Trajectory Following 458
   7.2.4 Perception Based Control 463
   7.2.5 Steering Trajectory Generation 466
   7.2.6 References and Further Reading 472
   7.2.7 Exercises 472

7.3 Optimal and Model Predictive Control 473
   7.3.1 Calculus of Variations 473
   7.3.2 Optimal Control 476
   7.3.3 Model Predictive Control 482
   7.3.4 Techniques for Solving Optimal Control Problems 484
   7.3.5 Parametric Optimal Control 487
   7.3.6 References and Further Reading 492
   7.3.7 Exercises 492
CONTENTS

7.4 Intelligent Control 493
  7.4.1 Introduction 493
  7.4.2 Evaluation 496
  7.4.3 Representation 499
  7.4.4 Search 507
  7.4.5 References and Further Reading 512
  7.4.6 Exercises 513

8 Perception 514
  8.1 Image Processing Operators and Algorithms 514
    8.1.1 Taxonomy of Computer Vision Algorithms 515
    8.1.2 High-Pass Filtering Operators 517
    8.1.3 Low-Pass Operators 523
    8.1.4 Matching Signals and Images 524
    8.1.5 Feature Detection 526
    8.1.6 Region Processing 529
    8.1.7 References and Further Reading 532
    8.1.8 Exercises 533
  8.2 Physics and Principles of Radiative Sensors 534
    8.2.1 Radiative Sensors 534
    8.2.2 Techniques for Range Sensing 535
    8.2.3 Radiation 539
    8.2.4 Lenses, Filters, and Mirrors 545
    8.2.5 References and Further Reading 550
    8.2.6 Exercises 551
  8.3 Sensors for Perception 551
    8.3.1 Laser Rangefinders 551
    8.3.2 Ultrasonic Rangefinders 555
    8.3.3 Visible Wavelength Cameras 557
    8.3.4 Mid to Far Infrared Wavelength Cameras 560
    8.3.5 Radars 562
    8.3.6 References and Further Reading 564
    8.3.7 Exercises 565
  8.4 Aspects of Geometric and Semantic Computer Vision 565
    8.4.1 Pixel Classification 565
    8.4.2 Computational Stereo Vision 568
    8.4.3 Obstacle Detection 572
    8.4.4 References and Further Reading 576
    8.4.5 Exercises 576

9 Localization and Mapping 579
  9.1 Representation and Issues 580
    9.1.1 Introduction 580
    9.1.2 Representation 580
    9.1.3 Timing and Motion Issues 583
    9.1.4 Related Localization Issues 585
    9.1.5 Structural Aspects 586
    9.1.6 Example: Unmanned Ground Vehicle (UGV) Terrain Mapping 588
    9.1.7 References and Further Reading 592
    9.1.8 Exercises 593
CONTENTS  

9.2 Visual Localization and Motion Estimation 593
  9.2.1 Introduction 593
  9.2.2 Aligning Signals for Localization and Motion Estimation 600
  9.2.3 Matching Features for Localization and Motion Estimation 606
  9.2.4 Searching for the Optimal Pose 612
  9.2.5 References and Further Reading 621
  9.2.6 Exercises 622

9.3 Simultaneous Localization and Mapping 623
  9.3.1 Introduction 623
  9.3.2 Global Consistency in Cyclic Maps 624
  9.3.3 Revisiting 630
  9.3.4 EKF SLAM for Discrete Landmarks 632
  9.3.5 Example: Autosurveying of Laser Reflectors 636
  9.3.6 References and Further Reading 638
  9.3.7 Exercises 639

10 Motion Planning 640
  10.1 Introduction 640
    10.1.1 Introducing Path Planning 641
    10.1.2 Formulation of Path Planning 642
    10.1.3 Obstacle-Free Motion Planning 643
    10.1.4 References and Further Reading 646
    10.1.5 Exercises 646
  10.2 Representation and Search for Global Path Planning 646
    10.2.1 Sequential Motion Planning 647
    10.2.2 Big Ideas in Optimization and Search 653
    10.2.3 Uniform Cost Sequential Planning Algorithms 656
    10.2.4 Weighted Sequential Planning 661
    10.2.5 Representation for Sequential Motion Planning 669
    10.2.6 References and Further Reading 672
    10.2.7 Exercises 672
  10.3 Real-Time Global Motion Planning: Moving in Unknown and Dynamic Environments 673
    10.3.1 Introduction 673
    10.3.2 Depth-Limited Approaches 674
    10.3.3 Anytime Approaches 677
    10.3.4 Plan Repair Approach: D* Algorithm 678
    10.3.5 Hierarchical Planning 686
    10.3.6 References and Further Reading 689
    10.3.7 Exercise 690

Index 691
Preface

Robotics can be a very challenging and very satisfying way to spend your time. A profound moment in the history of most roboticists is the first moment a robot performed a task under the influence of his or her software or electronics. Although a productive pursuit of the study of robotics involves aspects of engineering, mathematics, and physics, its elements do not convey the magic we all feel when interacting with a responsive semi-intelligent device of our own creation.

This book introduces the science and engineering of a particularly interesting class of robots — mobile robots. Although there are many analogs to the field of robot manipulators, mobile robots are sufficiently different to justify their treatment in an entirely separate text. Although the book concentrates on wheeled mobile robots, most of its content is independent of the specific locomotion subsystem used.

The field of mobile robots is changing rapidly. Many specialties are evolving in both the research and the commercial sectors. Any textbook offered in such an evolving field will represent only a snapshot of the field as it was understood at the time of publication. However, the rapid growth of the field, its several decades of history, and its pervasive popular appeal suggest that the time is now right to produce an early text that attempts to codify some of the fundamental ideas in a more accessible manner.

Another indication of timeliness might be the fact that much useful information must be omitted. Many topics, such as perception, are treated only briefly, and others, including legged locomotion, calibration, simulation, human interfaces, and multirobot systems, are omitted completely. The goal of this book is to extract from both the underlying specialties and the depth of mobile robotics research literature a coherent exposition of the concepts, methods, and issues that rise to the forefront in practice, and to represent the core that is unique about this field.

To that end, as much as possible of the material is restricted to two-dimensional wheeled vehicle motion and to structured environments. These assumptions produce a consistent exposition with just enough richness to be relevant and illustrative without overwhelming the reader with details irrelevant to the purpose.

The book follows a logical progression, mimicking the order in which mobile robots are constructed. Each chapter represents a new topic or capability that depends on what came before, and the concepts involved span the fields of numerical methods,
signal processing, estimation and control theory, computer vision, and artificial intelligence in that order.

As of this writing, the Mars Science Laboratory Rover named Curiosity has recently arrived on Mars. It is our third mobile robotic mission to Mars and the legacy of the last (MER) mission is already historic. This book is not for everyone, but for those who are prepared and motivated, if you master the content of the text you will have a very good idea of what is going on inside the brain of a mobile robot, and you will be well prepared to make one of your own.