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978-1-107-03115-9 - Mobile Robotics: Mathematics, Models, and Methods

Alonzo Kelly

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Mobile Robotics

Mathematics, Models, and Methods

Mobile Robotics offers comprehensive coverage of the essentials of the field suitable for both students and practitioners. Adapted from the author's graduate and undergraduate courses, the content of the book reflects current approaches to developing effective mobile robots. Professor Alonzo Kelly adapts principles and techniques from the fields of mathematics, physics, and numerical methods to present a consistent framework in a notation that facilitates learning and highlights relationships between topics. This text was developed specifically to be accessible to senior-level undergraduates in engineering and computer science, and includes supporting exercises to reinforce the lessons of each section. Practitioners will value the author's perspectives on practical applications of these principles. Complex subjects are reduced to implementable algorithms extracted from real systems wherever possible, to enhance the real-world relevance of the text.

Alonzo Kelly holds undergraduate degrees in aerospace engineering and computer science, and graduate degrees in robotics. Dr. Kelly worked in the aerospace industry for ten years before returning to academia. As a professor at the Robotics institute at Carnegie Mellon University, he teaches mobile robotics at the graduate and undergraduate levels, conducting research in robot simulation, modeling, controls, position estimation, motion planning, and human interfaces.

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Preface

Robotics can be a very challenging and very satisfying way to spend your time. A profound moment in the history of most roboticists is the first moment a robot performed a task under the influence of his or her software or electronics. Although a productive pursuit of the study of robotics involves aspects of engineering, mathematics, and physics, its elements do not convey the magic we all feel when interacting with a responsive semi-intelligent device of our own creation.

This book introduces the science and engineering of a particularly interesting class of robots – mobile robots. Although there are many analogs to the field of robot manipulators, mobile robots are sufficiently different to justify their treatment in an entirely separate text. Although the book concentrates on wheeled mobile robots, most of its content is independent of the specific locomotion subsystem used.

The field of mobile robots is changing rapidly. Many specialties are evolving in both the research and the commercial sectors. Any textbook offered in such an evolving field will represent only a snapshot of the field as it was understood at the time of publication. However, the rapid growth of the field, its several decades of history, and its pervasive popular appeal suggest that the time is now right to produce an early text that attempts to codify some of the fundamental ideas in a more accessible manner.

Another indication of timeliness might be the fact that much useful information must be omitted. Many topics, such as perception, are treated only briefly, and others, including legged locomotion, calibration, simulation, human interfaces, and multi-robot systems, are omitted completely. The goal of this book is to extract from both the underlying specialties and the depth of mobile robotics research literature a coherent exposition of the concepts, methods, and issues that rise to the forefront in practice, and to represent the core that is unique about this field.

To that end, as much as possible of the material is restricted to two-dimensional wheeled vehicle motion and to structured environments. These assumptions produce a consistent exposition with just enough richness to be relevant and illustrative without overwhelming the reader with details irrelevant to the purpose.

The book follows a logical progression, mimicking the order in which mobile robots are constructed. Each chapter represents a new topic or capability that depends on what came before, and the concepts involved span the fields of numerical methods,

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PREFACE

signal processing, estimation and control theory, computer vision, and artificial intelligence in that order.

As of this writing, the Mars Science Laboratory Rover named *Curiosity* has recently arrived on Mars. It is our third mobile robotic mission to Mars and the legacy of the last (MER) mission is already historic. This book is not for everyone, but for those who are prepared and motivated, if you master the content of the text you will have a very good idea of what is going on inside the brain of a mobile robot, and you will be well prepared to make one of your own.