PART I

GENERAL INFORMATION

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Introduction

1.1 What This Book Is About

This is a book on stochastic frontier (SF) analysis, which uses econometric models to estimate production (or cost or profit) frontiers and efficiency relative to those frontiers. Production efficiency relates actual output to the maximum possible, and is defined as the ratio of the actual output to the maximum potential output. More generally, SF analysis can be applied to any problem where the observed outcome deviates from the potential outcome in one direction, that is, the observed outcome is either less or more than the potential outcome. In the context of production efficiency, the potential output, given inputs and technology, is the maximum possible output that defines the frontier and the actual output falls below the frontier due to technical inefficiency. For cost efficiency, the frontier is defined by the potential minimum cost, and the actual cost lies above the minimum frontier owing to inefficiency. Similarly, the profit frontier is defined in terms of the maximum possible profit and profit efficiency is defined as the ratio of actual to maximum possible profit (assuming that they are both positive or negative). Other examples include the observed wage offer being less than the potential maximum; the reported crime rate being less than the true crime because of underreporting; actual investment being less than the potential optimal because of borrowing constraints; and so on. The common denominator in all of these problems is that there is something called the potential maximum or minimum or optimal level, which defines the frontier. This frontier is unobserved. So the question is how to estimate the frontier function so that efficiency can be estimated. Another complicating factor is that the frontier is often viewed as stochastic and the problem is how to estimate efficiency relative to the stochastic frontier when we can estimate only the "deterministic" part of the frontier. This book deals with the issues related to estimating the stochastic frontier econometrically first, and then estimating efficiency relative to the stochastic frontier for each observation.

The best way to understand why this type of analysis is important is to consider the questions that the techniques introduced in this book can answer or, at least, help to answer. The list of questions below is somewhat long but, even then, it is far from exhaustive. Worldwide, efficiency improvement is often regarded as one of the most important goals behind many social and economic policies and reforms. Examples are numerous. For instance, opening up of markets to competition, the removal of trade barriers, and the privatization of state enterprises are all motivated, at least in part, by the potential for efficiency improvements. At a high level, many policies are well understood by economists, but when you consider the details and the specifics of individual industries within the economies, things are less clear.

For instance, how do we measure the improvement in efficiency? Does the efficiency come from the production side – producing more given the same input and technology – or the cost side – costing less to produce the same output? Which one is the appropriate metric? Why do some firms achieve greater efficiency gains than others? What are the determinants of the efficiency gain? Has privatization generally "worked" or is it the opening of the market to competition, rather than privatization per se, that has resulted in efficiency improvements? Has regulation or, for that matter, deregulation been successful? And, at an industry level, are some reforms more successful than others?

Even within a relatively competitive and/or mature industry, there may be public policy questions that could be considered to improve the operation of the market. For example, currently the U.K. government foregoes tax revenues via approved (or tax advantaged) employee share schemes, which are assumed to align employee and employer incentives and thus increase industry productivity and efficiency. But what is the evidence? That is, are companies with such schemes really more productive and efficient than those without such schemes?

Similar questions arise with respect to different forms of corporate ownership and the public-private interfaces within an economy. For instance, when we consider publicly owned corporations, public private partnerships, not-for-profit companies, family owned firms, private companies, or the recent influx of private equity investment, which forms of ownership turn out to be the most effective, and does this depend on the sector? Public-private partnership are frequently used in many parts of the world, but is such an approach really the most cost-effective route in all cases?

At a micro-level, within businesses, there are numerous critical questions that would benefit from the sort of analysis set out in this book. For example, a key strategic question may be whether or not a take-over or merger with a current competitor makes sense. Although there are multiple reasons for considering takeovers, one of the key questions to answer is whether it will result in cost efficiency improvements and/or cost savings through economies of scale and scope. A business may be interested in knowing whether a profit-sharing scheme would help boost employees' incentives and increase production efficiency. For these questions, the measure of efficiency and the effects of efficiency determinants are important.

Examples given here are in the context of production economics, which has traditionally been the main field of research for stochastic frontier analysis. However, recent development in the literature has found wider applications of the analysis in other fields of research in economics and finance. Examples include using the SF model to test the underpricing hypothesis of the initial public offerings and the convergence hypothesis of economic growth. The analysis is also applied to estimate the effects of search cost on observed wage rates, the impact of financing constraints on firms' capital investment, and wage discrimination in the labor market, to name just a few.

1.2 Who Should Read This Book?

The issues raised in the previous section represent some everyday questions that are asked by academics, policy makers, regulators, government advisors, companies, consulting firms, and the like. For them, this book provides practical guidelines to carry out the analysis and help them to answer the questions. Students of industrial organization, government policy, and other fields of economic and financial research will also find the modeling techniques introduced in the book useful.

The increasing demand of the SF analysis from academics and industry is evident from the increasing number of journal articles, conferences, and workshops on the associated topics. There are several journals (e.g., *Journal of Productivity Analysis, Journal of Econometrics, European Journal of Operational Research, Empirical Economics*) that publish efficiency-related papers (or more generally papers that use SF as a tool) on a regular basis. There are several well-established international conferences focusing on the development and applications of efficiency estimation, and they are also held on a regular basis. They include the North American Productivity Workshop, the European Workshop on Efficiency and Productivity Analysis, the Asia-Pacific Productivity Conference, the Helenic Efficiency and Productivity Workshop, and so on.

In terms of applied econometric modeling skills, some familiarity with Stata is assumed, although the reader is taken through the modeling examples step-by-step, so even a non-Stata user should be able to follow the examples.

Throughout the book, we provide Stata codes for estimating systems in both cross-sectional and panel models. We also provide Stata codes for many of the cross-sectional and panel (single equation) models that are not otherwise available. As such, users do not need to do any complex coding for estimating many of the models. The user can also practice running some of the models using the datasets and examples that are used in this book. Because the source codes (the Stata ado-files) are also provided, the more advanced Stata user can tailor the codes for their own models if further extensions are needed.

If the reader is not a Stata user and does not plan to use it, he or she can still benefit from reading the book. It is detailed enough so that one can understand the theory behind the models and follow the discussion of the results from various worked examples.

1.3 The Structure of This Book

Part I: General Information

This section of the book provides the general background material required before examining specific modeling of the subsequent chapters.

- **Chapter 1: Introduction**
  This chapter explains what this book is about, who would find this book of interest, and explains the structure of the rest of the book.
- **Chapter 2: Production, Distance, Cost, and Profit Functions**
  This chapter provides the reader with general background information on the production theory and terminology necessary to understand the remainder of the book. The aim is to provide the reader with a guide to the topics and reference materials for advanced discussions. This chapter is written in such a way that someone familiar with the production theory covered in intermediate microeconomics textbooks would understand the material.

Part II: Single Equation Approach with Cross-Sectional Data

- **Chapter 3: Estimation of Technical Efficiency in Production Frontier Models Using Cross-Sectional Data**

Many of the basic ideas in modeling and applying SF technique are explained in detail in this chapter. Some knowledge of statistics and econometrics is necessary to understand the technical details, although someone without such knowledge can still use, interpret and follow the practical examples. More specifically, this chapter introduces the estimation of a production frontier model as well as inefficiency and efficiency indexes using distribution-free and parametric approaches. For the parametric approach, models with various distributional assumptions including half-normal, truncated-normal, exponential, and so on are discussed and compared.

- **Chapter 4: Estimation of Technical Efficiency in Cost Frontier Models Using Cross-Sectional Data**
  This chapter extends the SF analysis from the production frontier to the cost frontier. It explains the different assumptions used in production and cost functions, and details the differences in the modeling, data requirements and the interpretation of results. Here the focus is on the technical inefficiency and assumes no allocative inefficiency (i.e., all the producers are assumed to be allocatively efficient). It shows how the technical inefficiency in a production frontier model is transmitted to the cost frontier model.
- **Chapter 5: Estimation of Technical Efficiency in Profit Frontier Models Using Cross-Sectional Data**
  This chapter discusses the relationship between production, cost, and profit functions. It also explains how technical inefficiency appears in the different models and explains how to interpret the models.

Part III: System Models with Cross-Sectional Data

- **Chapter 6: Estimation of Technical Efficiency in Cost Frontier Models Using Cost System Models with Cross-Sectional Data**
  This chapter introduces a cost system model that consists of the cost function and the cost share equations, derived from the first-order conditions of the cost minimization problem. It assumes that all the producers are allocatively efficient. The chapter also explains how different covariance structures of the error terms in the system can be used in estimating the model.
- **Chapter 7: Estimation of Technical Efficiency in Profit Frontier Models Using System Models with Cross-Sectional Data**
  This chapter introduces a profit system model that consists of the first-order conditions of profit maximization. An advantage of estimating a profit function using only the first-order conditions is that the profit variable is not directly used in the estimation. Because profit can be negative in real data and hence logarithms cannot be taken, this approach allows us to undertake the estimation using the Cobb-Douglas and/or translog functions without worrying about negative profit.

Part IV: The Primal System Approach

This section of the book examines the primal approach to SF modeling. The terminology "The Primal System Approach" might be confusing to readers because we are explicitly using the first-order conditions of cost minimization and profit maximization, which relate to prices. Here, by primal system approach, we refer to a system approach where the production function is used along with the first-order conditions from either cost minimization or profit maximization. Thus, we are separating the primal system approach from the single equation primal approach which is estimated without using any price information.

- **Chapter 8: Cost Minimization with Technical and Allocative Inefficiency: A Primal Approach**
  This chapter introduces allocative inefficiency and how it may be incorporated in a cost frontier model theoretically. Then it shows the difficulty in empirically estimating such a model. We then present the primal system approach, which estimates both technical and allocative inefficiency. These are introduced into the model via the first-order conditions of cost minimization.
- **Chapter 9: Profit Maximization with Technical and Allocative Inefficiency: A Primal Approach**
  This chapter extends ideas similar to the previous chapter to the case in which producers maximize profit and are allowed to be allocatively inefficient. We call this the primal profit system because we do not use the profit function in this analysis. Instead, we append allocative inefficiency in the first-order condition with respect to output to the cost system discussed in the previous chapter. The problem of using the profit function is that profit has to be positive which is not the case for many applications. The primal approach avoids this problem.

Part V: Single Equation Approach with Panel Data

- **Chapter 10: Single Equation Panel Model**
  This chapter explains the difference between panel data and cross-sectional data, and why the use of panel data may either help or complicate the estimation process. Then it shows how we may avoid such difficulties by adopting a certain modeling strategy. Estimation of some of the more recent formulations that separate time-varying technical inefficiency from fixed firm effects are also considered.
- **Chapter 11: Productivity and Profitability Decomposition**
  This examines how to estimate changes in productivity and profitability over time and decompose these changes into their constituent parts.

Part VI: Looking Ahead

- **Chapter 12: Looking Ahead**
  This chapter briefly sets out some of the topics that we have not covered in the the book.

Appendices

- **Appendix A: Deriving the Likelihood Functions of Single Equation Frontier Models**
  In this appendix, we derive the likelihood functions of the single equation frontier models.
- **Appendix B: Deriving the Efficiency Estimates**
  In this appendix, we derive the inefficiency index and the technical efficiency index.

- **Appendix C: Deriving the Confidence Intervals**
  In this appendix, we derive the confidence intervals for the inefficiency index and the technical efficiency index.
- **Appendix D: Bootstrapping Standard Errors of Marginal Effects on Inefficiency**
  This appendix shows an example of bootstrapping standard errors of variables' marginal effects on inefficiency.
- **Appendix E: Software**
  This appendix explains where to download dataset and Stata .do files used as empirical examples in the book. It also contains instructions on how to download and install the Stata commands written by authors of the book. Detailed explanations on the commands and the syntax are also provided in this appendix.

CAMBRIDGE

Cambridge University Press
978-1-107-02951-4 - A Practitioner's Guide to Stochastic Frontier Analysis Using Stata
Subal C. Kumbhakar, Hung-Jen Wang and Alan P. Horncastle
Excerpt
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2

Production, Distance, Cost, and Profit Functions

2.1 Introduction

In Chapter 1, we introduced a series of questions that the tools discussed in this book are designed to help answer. In this chapter, we provide the reader with the necessary theoretical underpinnings in order to answer these questions and to understand the models that are developed in later chapters. This is important as it is necessary to understand which is the most appropriate tool to use in which circumstance and what the limitations are of the different approaches.

In some of the following sections the text is fairly technical, but these sections are useful as a general reference for the practitioner when modeling specific issues. For example, Section 2.5 on the functional forms of the production function provides the required formulae for some of the key economic issues discussed in many chapters.

The study of the production, cost, and profit functions has a long history and the practical applications of modeling these functions are extensive. In line with the questions introduced in Chapter 1, a summary of the major objectives for studying these functions may include the following:

(i) If a firm were to expand its operations by increasing its inputs by 10 percent, how much would output increase by? How much lower would its unit costs be (and thus how much lower could it reduce its prices by or increase its margins)?
(ii) If a firm were to invest in new IT equipment, how many fewer manual employees would be needed to produce the same level of output? How many more IT personnel would be needed?
(iii) Can we consider the use of certain inputs independently of others?
(iv) Compared to last year, how much more output can be produced for a given level of inputs?
(v) Compared to industry best practice, for a given input level, how much more output can an organization produce compared to its current output level?

From a theoretical perspective, these questions boil down to considering (i) scale economies, (ii) substitutability/complementarity of inputs, (iii) separability of inputs, (iv) technical change, and (v) technical efficiency. Economists are, in general, interested in examining some or all of these economic effects, whereas everyone can benefit from the insights that such studies can shed light on.

In what follows, we first consider the production function. We discuss measurements of the economic effects discussed here and also introduce a number of alternative functional forms that can be used for estimation purposes. We then consider situations in which we have multiple inputs and outputs. We finish the chapter by considering allocative efficiency and expenditure/finance constrained models (in which presence of constraints is manifested in the one-sided error term).

2.2 The Production Function and Technical Efficiency

All production processes represent a transformation of inputs (for example, labor, capital, and raw material) into outputs (which can be either in physical units or services). A production function simply describes this transformation relationship – a "black box" – which converts inputs into outputs. For example, if we consider the simple case of one input and one output, the production function shows the output level that can be produced for a given production technology and a given level of input. We will describe such an output level as the maximum output in the sense that the production technology is used at its full potential. By changing the input level, one can trace the graph of the production function relating the output with various input levels. That is, if we were to plot the maximum possible outputs for different levels of input, the line so produced would represent the firm's production function. Note that it is a technological relationship and does not say whether the input used or the output produced maximizes profit or minimizes cost. Once a particular behavior is assumed, it is possible to determine the optimal level of input and output consistent with profit maximization, cost minimization, or other economic behavior.

In order to examine the economic effects discussed here, we need a more formal definition of a production function. A *production function* is a mathematical representation of the technology that transforms inputs into output(s). If inputs and outputs are treated as two separate categories, the relationship between inputs and outputs can be expressed as $F(\boldsymbol{x}, \boldsymbol{y}) = 0$, where $\boldsymbol{x}$ is a J dimensional non-negative input vector and $\boldsymbol{y}$ is an M dimensional non-negative output vector. This formulation is very general and we will consider a much more restricted formulation, which for a single output case can be expressed as:

$$y = f(x_1, x_2, \ldots, x_J) \equiv f(\boldsymbol{x}), \tag{2.1}$$

where the function $f(\cdot)$ specifies the technology governing the input–output relationship, and is single valued. In this formulation, $f(\boldsymbol{x})$ is the production function, which gives the maximum possible output, for a given $\boldsymbol{x}$. Alternatively, given y and all other inputs, except x_j, this function gives the minimum value of x_j. A well-defined production function should satisfy the following regularity conditions (Chambers [1988], p. 9):

1. $f(\boldsymbol{x})$ is finite, non-negative, real-valued, and single-valued for all non-negative and finite $\boldsymbol{x}$;
2. $f(0) = 0$ meaning that no inputs implies no output;
3. $f(\boldsymbol{x}) \geq f(\boldsymbol{x}')$ for $\boldsymbol{x} \geq \boldsymbol{x}'$ (monotonicity);
4. $f(\boldsymbol{x})$ is continuous and twice-differentiable everywhere;
5. The input requirement set $V(y) = \{\boldsymbol{x} | f(\boldsymbol{x}) \geq y\}$ is a convex set, which implies quasi-concavity of $f(\boldsymbol{x})$;
6. The set $V(y)$ is closed and nonempty for any $y > 0$.