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978-1-107-02913-2 - Superstring Theory: Volume 2: Loop Amplitudes,
Anomalies and Phenomenology: 25th Anniversary Edition

Michael B. Green, John H. Schwarz and Edward Witten

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