

Cambridge University Press
978-1-107-02913-2 - Superstring Theory: Volume 2: Loop Amplitudes,
Anomalies and Phenomenology: 25th Anniversary Edition
Michael B. Green, John H. Schwarz and Edward Witten
Frontmatter
[More information](#)

Superstring Theory

Volume 2: Loop Amplitudes, Anomalies and Phenomenology 25th Anniversary Edition

Twenty-five years ago, Michel Green, John Schwarz, and Edward Witten wrote two volumes on string theory. Published during a period of rapid progress in this subject, these volumes were highly influential for a generation of students and researchers. Despite the immense progress that has been made in the field since then, the systematic exposition of the foundations of superstring theory presented in these volumes is just as relevant today as when first published.

Volume 2 is concerned with the evaluation of one-loop amplitudes, the study of anomalies, and phenomenology. It examines the low energy effective field theory analysis of anomalies, the emergence of the gauge groups $E_8 \times E_8$ and $SO(32)$, and the four-dimensional physics that arises by compactification of six extra dimensions.

Featuring a new Preface setting the work in context in light of recent advances, this book is invaluable for graduate students and researchers in high energy physics and astrophysics, as well as for mathematicians.

MICHAEL B. GREEN is the Lucasian Professor of Mathematics at the University of Cambridge, JOHN H. SCHWARZ is the Harold Brown Professor of Theoretical Physics at the California Institute of Technology, and EDWARD WITTEN is the Charles Simonyi Professor of Mathematical Physics at the Institute for Advanced Study. Each of them has received numerous honors and awards.

Cambridge University Press

978-1-107-02913-2 - Superstring Theory: Volume 2: Loop Amplitudes,
Anomalies and Phenomenology: 25th Anniversary Edition

Michael B. Green, John H. Schwarz and Edward Witten

Frontmatter

[More information](#)

TO OUR PARENTS

Cambridge University Press

978-1-107-02913-2 - Superstring Theory: Volume 2: Loop Amplitudes,
Anomalies and Phenomenology: 25th Anniversary Edition

Michael B. Green, John H. Schwarz and Edward Witten

Frontmatter

[More information](#)

Superstring Theory
Volume 2: Loop Amplitudes, Anomalies
and Phenomenology
25th Anniversary Edition

Michael B. Green

University of Cambridge

John H. Schwarz

California Institute of Technology

Edward Witten

Institute for Advanced Study, Princeton

Cambridge University Press
978-1-107-02913-2 - Superstring Theory: Volume 2: Loop Amplitudes,
Anomalies and Phenomenology: 25th Anniversary Edition
Michael B. Green, John H. Schwarz and Edward Witten
Frontmatter
[More information](#)

CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org

Information on this title: www.cambridge.org/9781107029132

© M. Green, J. Schwarz and E. Witten 2012

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 1987

First paperback edition 1988

25th Anniversary edition published 2012

A catalogue record for this publication is available from the British Library

ISBN 978-1-107-02913-2 Hardback

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication, and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.

Cambridge University Press

978-1-107-02913-2 - Superstring Theory: Volume 2: Loop Amplitudes,
Anomalies and Phenomenology: 25th Anniversary Edition

Michael B. Green, John H. Schwarz and Edward Witten

Frontmatter

[More information](#)

Contents

	<i>Preface to the 25th Anniversary Edition</i>	xi
8	One-loop diagrams in the bosonic string theory	1
8.1	Open-string one-loop amplitudes	3
	8.1.1 The planar diagrams	7
	8.1.2 The nonorientable diagrams	27
	8.1.3 Nonplanar loop diagrams	32
8.2	Closed-string one-loop amplitudes	37
	8.2.1 The torus	38
	8.2.2 Modular invariance	44
	8.2.3 The integration region	49
	8.2.4 Analysis of divergences	51
	8.2.5 The cosmological constant	54
	8.2.6 Amplitudes with closed-string massless states	57
8.3	Other diagrams for unoriented strings	58
	8.3.1 Higher-order tree diagrams	58
	8.3.2 The real projective plane	63
	8.3.3 Other loop diagrams	66
8.4	Summary	68
8.A	Jacobi Θ functions	69
9	One-loop diagrams in superstring theory	75
9.1	Open-superstring amplitudes	76
	9.1.1 Amplitudes with $M < 4$ massless external states	78
	9.1.2 The planar diagrams	79
	9.1.3 Nonorientable diagrams	85
	9.1.4 Orientable nonplanar diagrams	87
9.2	Type II theories	89
	9.2.1 Finiteness of the torus amplitude	90
	9.2.2 Compactification on a torus	94
	9.2.3 The low-energy limit of one-loop amplitudes	98
9.3	The heterotic string theory	102
	9.3.1 The torus with four external particles	103
	9.3.2 Modular invariance of the $E_8 \times E_8$ and $SO(32)$ theories	108

Cambridge University Press

978-1-107-02913-2 - Superstring Theory: Volume 2: Loop Amplitudes,
Anomalies and Phenomenology: 25th Anniversary Edition

Michael B. Green, John H. Schwarz and Edward Witten

Frontmatter

[More information](#)

vi

Contents

9.4	Calculations in the RNS formalism	112
9.4.1	Modular invariance and the GSO projection	112
9.4.2	The loop calculations	117
9.5	Orbifolds and twisted strings	119
9.5.1	Generalization of the GSO projection	119
9.5.2	Strings on orbifolds	121
9.5.3	Twisted strings in ten dimensions	126
9.5.4	Alternative view of the $SO(16) \times SO(16)$ theory	130
9.6	Summary	133
9.A	Traces of fermionic zero modes	134
9.B	Modular invariance of the functions F_2 and \mathcal{L}	137
10	The gauge anomaly in type I superstring theory	141
10.1	Introduction to anomalies	143
10.1.1	Anomalies in point-particle field theory	143
10.1.2	The gauge anomaly in $D = 10$ super Yang–Mills theory	146
10.1.3	Anomalies in superstring theory	149
10.2	Analysis of hexagon diagrams	152
10.2.1	The planar diagram anomaly	154
10.2.2	The anomaly in the nonorientable diagram	164
10.2.3	Absence of anomalies in nonplanar diagrams	165
10.3	Other one-loop anomalies in superstring theory	167
10.4	Cancellation of divergences for $SO(32)$	168
10.4.1	Dilaton tadpoles and loop divergences	169
10.4.2	Divergence cancellations	171
10.5	Summary	172
10.A	An alternative regulator	173
11	Functional methods in the light-cone gauge	179
11.1	The string path integral	179
11.1.1	The analog model	180
11.1.2	The free string propagator	181
11.1.3	A lattice cutoff	184
11.1.4	The continuum limit	189
11.2	Amplitude calculations	191
11.2.1	Interaction vertices	191
11.2.2	Parametrization of scattering processes	195
11.2.3	Evaluation of the functional integral	197
11.2.4	Amplitudes with external ground states	201
11.3	Open-string tree amplitudes	203
11.3.1	The conformal mapping	204
11.3.2	Evaluation of amplitudes	209

Cambridge University Press

978-1-107-02913-2 - Superstring Theory: Volume 2: Loop Amplitudes,
Anomalies and Phenomenology: 25th Anniversary Edition

Michael B. Green, John H. Schwarz and Edward Witten

Frontmatter

[More information](#)*Contents*

vii

11.4	Open-string trees with excited external states	212
11.4.1	The Green function on an infinite strip	213
11.4.2	Green functions for arbitrary tree amplitudes	214
11.4.3	The amplitude in terms of oscillators	216
11.4.4	The general form of the Neumann coefficients	218
11.4.5	The Neumann coefficients for the cubic open-string vertex	219
11.5	One-loop open-string amplitudes	223
11.5.1	The conformal mapping for the planar loop diagram	223
11.5.2	The Green function	225
11.5.3	The planar one-loop amplitude	226
11.5.4	Other one-loop amplitudes	227
11.6	Closed-string amplitudes	229
11.6.1	Tree amplitudes	229
11.6.2	Closed-string one-loop amplitudes	231
11.7	Superstrings	235
11.7.1	The $SU(4) \times U(1)$ formalism	235
11.7.2	The super-Poincaré generators	239
11.7.3	Supersymmetry algebra in the interacting theory	244
11.7.4	The continuity delta functional	246
11.7.5	Singular operators near the interaction point	248
11.7.6	The interaction terms	251
11.7.7	Tree amplitudes for open superstrings	254
11.8	Summary	255
11.A	The determinant of the Laplacian	256
11.B	The Jacobian for the conformal transformation	265
11.C	Properties of the functions f_m	267
11.D	Properties of the $SU(4)$ Clebsch–Gordan coefficients	269
12	Some differential geometry	271
12.1	Spinors in general relativity	271
12.2	Spin structures on the string world sheet	277
12.3	Topologically nontrivial gauge fields	281
12.3.1	The tangent bundle	281
12.3.2	Gauge fields and vector bundles	283
12.4	Differential forms	287
12.5	Characteristic classes	298
12.5.1	The nonabelian case	300
12.5.2	Characteristic classes of manifolds	302
12.5.3	The Euler characteristic of a Riemann surface	304
13	Low-energy effective action	307
13.1	Minimal supergravity plus super Yang–Mills	308

Cambridge University Press

978-1-107-02913-2 - Superstring Theory: Volume 2: Loop Amplitudes,
Anomalies and Phenomenology: 25th Anniversary Edition

Michael B. Green, John H. Schwarz and Edward Witten

Frontmatter

[More information](#)

viii

Contents

13.1.1	$N = 1$ supergravity in ten and eleven dimensions	310
13.1.2	Type IIB supergravity	314
13.1.3	The coupled supergravity super Yang–Mills system	321
13.2	Scale invariance of the classical theory	326
13.3	Anomaly analysis	330
13.3.1	Structure of field theory anomalies	333
13.3.2	Gravitational anomalies	336
13.3.3	Mixed anomalies	338
13.3.4	The anomalous Feynman diagrams	340
13.3.5	Mathematical characterization of anomalies	343
13.3.6	Other types of anomalies	346
13.4	Explicit formulas for the anomalies	347
13.5	Anomaly cancellations	351
13.5.1	Type I supergravity without matter	352
13.5.2	Type IIB supergravity	352
13.5.3	Allowed gauge groups for $N = 1$ superstring theories	353
13.5.4	The $SO(16) \times SO(16)$ theory	358
14	Compactification of higher dimensions	363
14.1	Wave operators in ten dimensions	363
14.1.1	Massless fields in ten dimensions	363
14.1.2	Zero modes of wave operators	365
14.2	Massless fermions	367
14.2.1	The index of the Dirac operator	369
14.2.2	Incorporation of gauge fields	371
14.2.3	The chiral asymmetry	374
14.2.4	The Rarita–Schwinger operator	375
14.2.5	Outlook	375
14.3	Zero modes of antisymmetric tensor fields	376
14.3.1	Antisymmetric tensor fields	377
14.3.2	Application to axions in $N = 1$ superstring theory	380
14.3.3	The ‘nonzero modes’	386
14.3.4	The exterior derivative and the Dirac operator	387
14.4	Index theorems on the string world sheet	392
14.4.1	The Dirac index	392
14.4.2	The Euler characteristic	394
14.4.3	Zero modes of conformal ghosts	396
14.4.4	Zero modes of superconformal ghosts	397
14.5	Zero modes of nonlinear fields	397
14.6	Models of the fermion quantum numbers	404
14.7	Anomaly cancellation in four dimensions	410
15	Some algebraic geometry	413

Cambridge University Press

978-1-107-02913-2 - Superstring Theory: Volume 2: Loop Amplitudes,
Anomalies and Phenomenology: 25th Anniversary Edition

Michael B. Green, John H. Schwarz and Edward Witten

Frontmatter

[More information](#)*Contents*

ix

15.1	Low-energy supersymmetry	413
	15.1.1 Motivation	413
	15.1.2 Conditions for unbroken supersymmetry	415
	15.1.3 Manifolds of $SU(3)$ holonomy	418
15.2	Complex manifolds	420
	15.2.1 Almost complex structure	421
	15.2.2 The Nijenhuis tensor	421
	15.2.3 Examples of complex manifolds	424
15.3	Kähler manifolds	430
	15.3.1 The Kähler metric	430
	15.3.2 Exterior derivatives	431
	15.3.3 The affine connection and the Riemann tensor	432
	15.3.4 Examples of Kähler manifolds	434
15.4	Ricci-flat Kähler manifolds and $SU(N)$ holonomy	437
	15.4.1 The Calabi–Yau metric	438
	15.4.2 Covariantly constant forms	440
	15.4.3 Some manifolds of $SU(N)$ holonomy	441
15.5	Wave operators on Kähler manifolds	444
	15.5.1 The Dirac operator	444
	15.5.2 Dolbeault cohomology	447
	15.5.3 The Hodge decomposition	449
	15.5.4 Hodge numbers	451
15.6	Yang–Mills equations and holomorphic vector bundles	453
	15.6.1 Holomorphic vector bundles	454
	15.6.2 The Donaldson–Uhlenbeck–Yau equation	456
	15.6.3 Examples	458
15.7	Dolbeault cohomology and some applications	463
	15.7.1 Zero modes of the Dirac operator	465
	15.7.2 Deformations of complex manifolds	467
	15.7.3 Deformations of holomorphic vector bundles	469
15.8	Branched coverings of complex manifolds	471
16	Models of low-energy supersymmetry	475
16.1	A simple Ansatz	475
16.2	The spectrum of massless particles	480
	16.2.1 Zero modes of charged fields	481
	16.2.2 Fluctuations of the gravitational field	483
	16.2.3 The other Bose fields	486
16.3	Symmetry breaking by Wilson lines	486
	16.3.1 Symmetry breaking patterns	488
	16.3.2 A four generation model	491
16.4	Relation to conventional grand unification	495

Cambridge University Press

978-1-107-02913-2 - Superstring Theory: Volume 2: Loop Amplitudes,
Anomalies and Phenomenology: 25th Anniversary Edition

Michael B. Green, John H. Schwarz and Edward Witten

Frontmatter

[More information](#)

x

Contents

16.4.1	Alternative description of symmetry breaking	496
16.4.2	E_6 relations among coupling constants	497
16.4.3	Counting massless particles	500
16.4.4	Fractional electric charges	506
16.4.5	Discussion	510
16.5	Global symmetries	511
16.5.1	CP conservation in superstring models	511
16.5.2	R transformations in superstring models	512
16.5.3	Global symmetries of the toy model	513
16.5.4	Transformation laws of matter fields	518
16.6	Topological formulas for Yukawa couplings	520
16.6.1	A topological formula for the superpotential	522
16.6.2	The kinetic terms	527
16.6.3	A nonrenormalization theorem and its consequences	528
16.6.4	Application to the toy model	532
16.7	Another approach to symmetry breaking	534
16.8	Discussion	541
16.9	Renormalization of coupling constants	544
16.10	Orbifolds and algebraic geometry	548
16.11	Outlook	551
	<i>Bibliography</i>	553
	<i>Index</i>	591

Cambridge University Press

978-1-107-02913-2 - Superstring Theory: Volume 2: Loop Amplitudes,
Anomalies and Phenomenology: 25th Anniversary Edition

Michael B. Green, John H. Schwarz and Edward Witten

Frontmatter

[More information](#)

Preface to the 25th Anniversary Edition

In the twenty-five years since the original publication of these two Volumes, there have been numerous developments in string theory. The curious twists and turns that marked its pre-1987 evolution have continued apace, and current research makes contact with a wide range of areas of mathematics and physics. In the following we will mention briefly some of these developments and then explain why we believe that these volumes are still useful.

Major insights into the non-perturbative structure of string theory followed from the discovery of non-perturbative duality symmetries of super-string theory. This led to the realization that the myriad of apparently distinct superstring theories that arise in ten or fewer dimensions actually are different perturbative approximations to the same underlying theory, which has come to be known as M-theory. Furthermore, M-theory has eleven-dimensional supergravity as another semiclassical limit. The understanding of these interconnections was aided by the simultaneous discovery of the properties of a family of dynamical objects called p -branes, which are extended objects that fill p spatial dimensions, as opposed to the 1 dimension of the string. p -branes can be viewed as solitons that are generalizations of the magnetic monopoles of conventional quantum field theory and the black holes of general relativity. Indeed, these discoveries have stimulated impressive advances in understanding the quantum and thermodynamic properties of large classes of black holes.

An important outcome of these considerations has been striking progress in understanding the nonperturbative structure of the quantum field theories that arise from string theory in various limits. Most notably, it has led to the gauge/gravity correspondence, also known as holographic duality, according to which a quantum theory of gravity in a D -dimensional asymptotically anti-de Sitter spacetime is equivalent to a $(D - 1)$ -dimensional local quantum field theory. The most studied example is an equivalence between four-dimensional maximally supersymmetric $U(N)$ Yang–Mills field theory and type IIB superstring theory in five-dimensional anti-de Sitter space times a five-dimensional sphere with N units of Ramond–Ramond flux.

In 1987, the main focus of this field was the application of string perturbation theory to describe the fundamental forces and particles. This remains a focus, but nowadays nonperturbative methods are used as well. The possibilities for using string theory to construct models of particle

Cambridge University Press

978-1-107-02913-2 - Superstring Theory: Volume 2: Loop Amplitudes,
Anomalies and Phenomenology: 25th Anniversary Edition

Michael B. Green, John H. Schwarz and Edward Witten

Frontmatter

[More information](#)

physics that are at least semi-realistic have proliferated since 1987, and some physicists, in part because of clues coming from cosmological observations, have come to suspect that the “landscape” of string theory possibilities is actually the way the Universe works. In addition, nowadays, string theory is applied to other types of problems in theoretical physics, including the modeling of heavy ion collisions and the theory of quantum critical points relevant to condensed matter physics. From a contemporary point of view, the gauge/gravity correspondence means that string theory and quantum gravity are inevitable parts of the description of strongly coupled quantum systems.

Many of the developments of the last quarter century, which of course are not covered in these two volumes, have been described in more recent books and review articles. These more recent developments are rooted to a large extent in the basic material covered here. Given the immense breadth and volume of recent research, it is extremely difficult for newer books and articles to present all the details of the more basic underlying material. For this reason, we feel that the material contained in these volumes remains of value and we hope it will continue to provide stimulus for further research in this rich field – wherever it eventually may lead.

Michael B. Green
John H. Schwarz
Edward Witten