1. Introduction

1.1 The Early Days of Dual Models

In 1900, in the course of trying to fit to experimental data, Planck wrote down his celebrated formula for black body radiation. It does not usually happen in physics that an experimental curve is directly related to the fundamentals of a theory; normally they are related by a more or less intricate chain of calculations. But black body radiation was a lucky exception to this rule. In fitting to experimental curves, Planck wrote down a formula that directly led, as we all know, to the concept of the quantum.

In the 1960s, one of the mysteries in strong interaction physics was the enormous proliferation of strongly interacting particles or hadrons. Hadronic resonances seemed to exist with rather high spin, the mass squared of the lightest particle of spin J being roughly $m^2 = J/\alpha'$, where $\alpha' \sim 1(\mathrm{GeV})^{-2}$ is a constant that became known as the Regge slope. Such behavior was tested up to about $J = 11/2$, and it seemed conceivable that it might continue indefinitely. One reason that the proliferation of strongly interacting particles was surprising was that the behavior of the weak and electromagnetic interactions was quite different; there are, comparatively speaking, just a few low mass particles known that do not have strong interactions.

The resonances were so numerous that it was not plausible that they were all fundamental. In any case consistent theories of fundamental particles of high spin were not known to exist. Consistent (renormalizable) quantum field theories seemed to be limited to spins zero, one-half, and one, the known examples being abelian gauge theories and scalar and Yukawa theories. That limitation on the possible spins in consistent quantum field theory still seems valid today, though now we would include Yang–Mills theory in the list of consistent theories for spin one. The apparent limitation of consistent quantum field theories to low spin was compatible with the existence of a successful field-theory description of the electromagnetic interactions, in which the basic particles have spin one half and spin one, and was compatible at least with attempts (which

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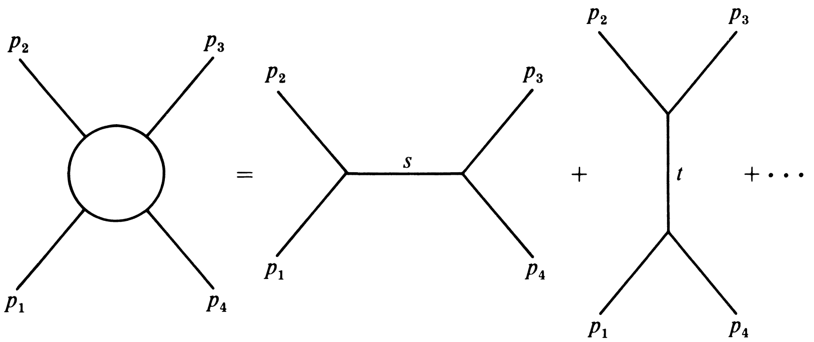

Figure 1.1. An elastic scattering process with incoming particles with momenta p_1, p_2 and outgoing particles with momenta $-p_3, -p_4$ (we adopt the convention that the labels refer to incoming momenta). Both s- and t-channel diagrams are indicated. In field theory the amplitude is constructed as a sum of s-channel and t-channel diagrams.

in time succeeded) at field theories of the weak interactions. But a similar approach to strong interactions did not appear promising.

A related puzzle about strong interactions concerned the high-energy behavior of the scattering amplitudes. Consider an elastic scattering process with incoming spinless particles of momenta p_1, p_2 and outgoing particles of momenta p_3, p_4. We adopt a metric with signature $\{- + + \ldots +\}$, so that the mass squared of a particle is $m^2 = -p^2$. The conventional Mandelstam variables are defined as

$$s = -(p_1 + p_2)^2, \qquad t = -(p_2 + p_3)^2, \qquad u = -(p_1 + p_3)^2. \qquad (1.1.1)$$

They obey the one identity $s+t+u = \sum m_i^2$. We assume that the external states in fig. 1.1 are particles such as pions that transform in the adjoint representation of the flavor group, which for three flavors is $SU(3)$ or $U(3)$. The flavor quantum numbers of the ith external meson are specified by picking a flavor matrix λ_i. We will discuss a term in the scattering amplitude proportional to the group-theory factor $\mathrm{tr}(\lambda_1\lambda_2\lambda_3\lambda_4)$. Since this group-theory factor is invariant under the cyclic permutation $1234 \rightarrow 2341$, Bose statistics require that the corresponding amplitude should be cyclically symmetric under $p_1p_2p_3p_4 \rightarrow p_2p_3p_4p_1$. In terms of Mandelstam variables, this permutation of momenta amounts to $s \leftrightarrow t$, which is the symmetry we will require for the amplitude $A(s,t)$.

In quantum field theory, the leading nontrivial contributions to the scattering amplitude come from the tree diagrams of fig. 1.1. The basic reason that it is difficult to construct sensible quantum field theories of particles of high spin is that tree diagrams with the exchange of high spin particles have bad high-energy behavior. Asymptotically, they exceed unitarity bounds. Consider, for instance, the t-channel diagram. Denote the external particles in fig. 1.1 as ϕ and the exchanged particle as σ. If σ has spin zero fig. 1.1 may involve a simple $\phi^*\phi\sigma$ interaction; the amplitude is then simply $A(s,t) = -g^2/(t-M^2)$ with g being the coupling constant and M the mass of the σ particle. This amplitude vanishes for $t \to \infty$, this being one aspect of the excellent high-energy behavior of the cubic scalar interaction we are discussing.

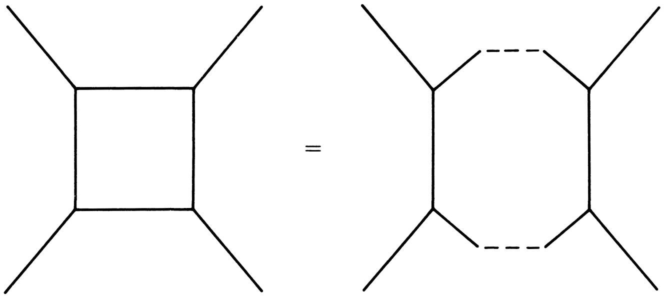

Figure 1.2. A one-loop diagram can be made by sewing together two tree diagrams, as indicated here.

Suppose instead that the sigma particle is a spin J field $\sigma_{\mu_1\mu_2\ldots\mu_J}$. For such a field, the cubic coupling in fig. 1.1 must then be something like $\phi^* \overleftrightarrow{\partial}_{\mu_1} \overleftrightarrow{\partial}_{\mu_2} \ldots \overleftrightarrow{\partial}_{\mu_J} \phi \cdot \sigma^{\mu_1\mu_2\ldots\mu_J}$. In fig. 1.1 there are now $2J$ factors of momenta. If the external particles are scalars then the contribution to the scattering amplitude of the exchange in the t channel of this spin J particle has the form

$$A_J(s,t) = -\frac{g^2(-s)^J}{t-M^2} \tag{1.1.2}$$

at high energies.* The behavior of this amplitude is therefore worse and worse (more and more divergent) for larger and larger J. An objective criterion for what is a 'bad' amplitude is to ask what will happen when we sew together amplitudes like that of (1.1.2) to make loops, as in fig. 1.2.

* This is the behavior of the tree-level scattering amplitude in the asymptotic region of large s, fixed t. The s^J behavior is easily found by contracting the momenta that appear in the interaction vertices in fig. 1.1. The exact formula (for moderate s) is more complicated, involving a Legendre polynomial $P_J(\cos\theta_t)$ (θ_t is the center-of-mass scattering angle in the t channel). We prefer to write only the high-energy behavior, which is transparent and adequate for our purposes.

The one-loop integrand in n dimensions is roughly $\int d^n p\, A^2/(p^2)^2$, with A being the tree amplitude of (1.1.2). In four dimensions such a loop diagram is convergent for $J < 1$, has a potentially renormalizable logarithmic divergence for $J = 1$, and has a nasty unrenormalizable divergence for $J > 1$.

There are strongly interacting particles of various mass and spin that might be exchanged in the t channel, so we must think of a t-channel amplitude of the general form

$$A(s,t) = -\sum_J \frac{g_J^2(-s)^J}{t - M_J^2}, \tag{1.1.3}$$

where now we allow for the possibility that the couplings g_J and masses M_J of the exchanged particles may depend on J (and perhaps on other quantum numbers that we do not indicate). Of course, one might take the point of view that the strong interactions are so strong that a Born-like approximation as in (1.1.3) is hopeless. But let us be optimists and see how well we can do. What is the high-energy behavior of the sum in (1.1.3)? If this is a *finite* sum, the high-energy behavior is simply determined by the hadron of largest J that contributes in (1.1.3). This is very different from what is observed in nature; the actual high-energy behavior of hadron scattering amplitudes is much softer than the behavior of any individual term in (1.1.3). (In fact, Regge asymptotic behavior of the type described in §1.1.2. is a reasonable approximation to experiment.) On the other hand, it is not reasonable to think of (1.1.3) as a finite sum. There certainly does not seem to be any such thing as a 'hadron of highest spin'. With (1.1.3) viewed as an infinite sum, it is certainly conceivable that the whole sum might have a high-energy behavior better than the behavior of any individual term in the series, just as the function e^{-x} is smaller for $x \to \infty$ than any individual term in its power series expansion $e^{-x} = \sum_{n=0}^{\infty} (-x)^n/n!$

Regarding (1.1.3) as an infinite sum has another consequence. In a physical process such as the elastic scattering of pions, we expect the t-channel poles that appear in (1.1.3), but we also expect s-channel resonances or in other words poles in the amplitude at certain values of s. In fact, the cyclic symmetry that we discussed earlier requires that the coefficient of $\mathrm{tr}(\lambda_1\lambda_2\lambda_3\lambda_4)$ in the scattering amplitude have both s- and t-channel poles or neither. A *finite* sum (1.1.3) defines an amplitude $A(s,t)$ that has no s-channel poles; for fixed t, (1.1.3) manifestly defines an entire function of s, as long as there are only a finite number of terms in the sum. It is precisely for this reason that the perturbative expansion of ordinary quantum field theories satisfies crossing symmetry by including both s- and t-channel diagrams. In the case of an infinite sum, things are different. Though each term in (1.1.3) is an entire function of s, the infinite sum might diverge at some finite values of s, giving poles in the s channel. Thus, once we accept the fact that (1.1.3) is essentially an *infinite* series, it is no longer obvious that s-channel terms must be included separately; they may be already implicit in (1.1.3).

Similar remarks could be made if we took as our starting point resonant scattering or in other words contributions to scattering amplitudes with s-channel poles. We would then construct an amplitude analogous to (1.1.3) but with s-channel poles rather than t-channel poles:

$$A'(s,t) = -\sum_J \frac{g_J^2(-t)^J}{s - M_J^2}. \tag{1.1.4}$$

Symmetry under cyclic permutation of the external momenta requires that the same masses and couplings appear in (1.1.4) as in (1.1.3). Studying (1.1.4) we would again observe that a finite sum of the type in (1.1.4) inevitably has a high-energy behavior much worse than the observed behavior of hadrons, but this is not inevitably true for an infinite sum of this type. Furthermore, a finite sum (1.1.4) would certainly define (for fixed s) an entire function of t, but this might not be true for an infinite sum.

Pursuing these thoughts still further, one might imagine that if the couplings g_J and masses M_J are cunningly chosen, then the s-channel and t-channel amplitudes $A(s,t)$ and $A'(s,t)$ might be equal. In this case, the entire amplitude could be written as a sum over only s-channel poles, as in (1.1.4), *or* as a sum over only t-channel poles, as in (1.1.3). This would be a sharp contrast to the field-theory situation in which one ordinarily needs a sum over *both s- and t*-channel poles.

Equality of the s- and t-channel amplitudes was advocated around 1968 by Dolen, Horn and Schmid, who argued, on the basis of an approximate evaluation of (1.1.3) and (1.1.4) (carried out with the help of experimental data), that the equality $A(s,t) = A'(s,t)$ was indeed approximately obeyed for small values of s and t. This was called the 'duality' hypothesis, the hypothesis that s- and t-channel diagrams give alternative or 'dual' descriptions of the same physics. Is duality an approximation or a principle? At first sight it looks well nigh impossible to choose the resonance masses and couplings to obey exactly the duality relation $A(s,t) = A'(s,t)$. However, a way of doing this was found by Veneziano in 1968. Veneziano simply postulated a formula for the scattering amplitude, namely

$$A(s,t) = \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))}. \tag{1.1.5}$$

Here Γ is the Euler gamma function,

$$\Gamma(u) = \int_0^\infty t^{u-1}e^{-t}dt, \tag{1.1.6}$$

and $\alpha(s)$ is the 'Regge trajectory', for which Veneziano postulated the linear form $\alpha(s) = \alpha(0) + \alpha' s$; α' and $\alpha(0)$ are known in Regge-pole theory as the Regge slope and the intercept, respectively.

1.1.1 The Veneziano Amplitude and Duality

It is not evident at first sight that the Veneziano amplitude obeys duality, but we will now show that it does. First of all, we need to know something about the gamma function. This function obeys the identity

$$\Gamma(u+1) = u\Gamma(u). \tag{1.1.7}$$

This is proved, starting from (1.1.6), by simple integration by parts:

$$\Gamma(u+1) = -\int_0^\infty t^u \frac{d}{dt}e^{-t}dt = u\int_0^\infty t^{u-1}e^{-t}dt = u\Gamma(u). \tag{1.1.8}$$

It is evident from (1.1.6) that $\Gamma(1) = 1$. If u is a positive integer, then repeated use of (1.1.7) implies that

$$\Gamma(u) = (u-1)!. \tag{1.1.9}$$

The integral representation of the Γ function in (1.1.6) is valid as long as the real part of u is positive, and shows that Γ has no singularities in this part of the complex u plane. The recursion relation (1.1.7) can be used to extend the domain of definition of Γ and determine its singularities. Writing (1.1.7) in the form

$$\Gamma(u) = \frac{\Gamma(u+1)}{u} \tag{1.1.10}$$

gives a definition of the gamma function for Re$u > -1$, since the right hand side of (1.1.10) has already been defined in that region. Equation

(1.1.10) also shows that Γ has a simple pole at $u = 0$ with residue 1. This process can be generalized; repeated use of (1.1.7) gives

$$\Gamma(u) = \frac{\Gamma(u+n)}{u(u+1)\ldots(u+n-1)} \tag{1.1.11}$$

for any positive integer n. The right-hand side of (1.1.11) is uniquely defined by the integral representation (1.1.6) as long as $\operatorname{Re} u > -n$, so we obtain a unique analytic continuation of the gamma function in this region. Since n is arbitrary, the gamma function actually has a unique analytic continuation throughout the whole complex u plane. From (1.1.11) we can see that the only singularities of Γ are simple poles at $u = 0, -1, -2, \ldots$. The behavior for u near $-n$ (n a non-negative integer) can be read off from (1.1.11) and is

$$\Gamma(u) \sim \frac{1}{u+n}\frac{(-1)^n}{n!}. \tag{1.1.12}$$

Now we wish to discuss the analytic behavior of the function

$$B(u,v) = \frac{\Gamma(u)\Gamma(v)}{\Gamma(u+v)}, \tag{1.1.13}$$

which is called the Euler beta function. It is related to the Veneziano amplitude by $A(s,t) = B(-\alpha(s), -\alpha(t))$. Evidently, (1.1.13) has a simple pole when u or v is a non-positive integer. There are no double poles in (1.1.13), since while $\Gamma(u)$ and $\Gamma(v)$ may simultaneously have poles, when this occurs the denominator in (1.1.13) has a pole at the same time. This is an important point, because simple poles are the only singularities allowed in tree amplitudes in relativistic quantum mechanics. The behavior of $B(u,v)$ for $v \sim -n$ (n being a non-negative integer) is evidently

$$B(u,v) \sim \frac{1}{v+n}\frac{(-1)^n}{n!}(u-1)(u-2)\ldots(u-n). \tag{1.1.14}$$

Here we are using (1.1.7) to write the residue of the pole at $v = -n$ as a polynomial in u; this is again an important step since the residue of a pole in relativistic quantum mechanics must be a polynomial. As a function of v for fixed u, $B(u,v)$ has only the singularities indicated in (1.1.14). We claim now that (for $\operatorname{Re} u > 0$ so that the following infinite sum converges) we can write

$$B(u,v) = \sum_{n=0}^{\infty} \frac{1}{v+n}\frac{(-1)^n}{n!}(u-1)(u-2)\ldots(u-n). \tag{1.1.15}$$

The idea here is that the sum on the right of (1.1.15) reproduces all of the singularities of the beta function, so could differ from it only by an entire function of v, that is, a function without singularities in the complex v plane. Such a function could not vanish for large $|v|$. As the sum on the right-hand side of (1.1.15) vanishes for positive u and large $|v|$ (away from the real axis), and we will presently see that $B(u,v)$ has the same property, they must be equal.

We can immediately express (1.1.15) as a formula for the Veneziano amplitude:

$$A(s,t) = -\sum_{n=0}^{\infty} \frac{(\alpha(s)+1)(\alpha(s)+2)\ldots(\alpha(s)+n)}{n!} \frac{1}{\alpha(t)-n}. \qquad (1.1.16)$$

While the Veneziano amplitude was defined originally to manifestly obey $A(s,t) = A(t,s)$, this symmetry is not at all apparent in (1.1.16). Because of the underlying symmetry, we can immediately write down the alternative expansion

$$A(s,t) = -\sum_{n=0}^{\infty} \frac{(\alpha(t)+1)(\alpha(t)+2)\ldots(\alpha(t)+n)}{n!} \frac{1}{\alpha(s)-n}. \qquad (1.1.17)$$

Now, with the simple choice of 'Regge trajectory', $\alpha(t) = \alpha' t + \alpha(0)$, the singularities of (1.1.16) are simple poles corresponding, as in (1.1.3), to t-channel exchange of particles of mass $M^2 = (n - \alpha(0))/\alpha'$, $n = 0,1,2,\ldots$. The residue of the pole at $\alpha(t) = n$ is (with the linear choice of Regge trajectory) an nth order polynomial in s, corresponding, in view of (1.1.3), to the fact that the particles of mass $(n - \alpha(0))/\alpha'$ have spin at most n. The smallest possible mass of a particle of spin J is thus $(J - \alpha(0))/\alpha'$, and this is why α' is called the 'Regge slope'; the particles of mass $M^2 = (J - \alpha(0))/\alpha'$ are said to lie on the 'leading Regge trajectory'. We are interested in the case $\alpha' > 0$, since these particles would otherwise be all or almost all tachyons.

The equality of (1.1.16) and (1.1.17) is the seemingly impossible property of 'duality': the *same* amplitude can be written as a sum of s-channel poles as in (1.1.17) *or* as a sum of t-channel poles as in (1.1.16).

One thing that is *not* obvious in either (1.1.17) or (1.1.16) is the sign of the residues of the s-channel and t-channel poles. In (1.1.2) for exchange of a spin J particle, the coefficient of $-(-1)^J/(t - M^2)$, which is called the residue of the pole, must be positive (since g^2 must be positive). More generally, the residues of poles must be positive in a relativistic quantum theory, for unitarity and absence of ghosts. We are thus led to the question of whether the residues in (1.1.16) and (1.1.17) are positive, something that is far from obvious. Much early work on dual models was concerned with this question, culminating eventually in the 'no-ghost theorem', which asserts that ghosts (or negative residues) are absent if certain rather surprising restrictions are placed on the value of $\alpha(0)$ and the dimension of space-time. In particular, it turned out that the dimension of space-time should be 26, and the constant $\alpha(0)$ in the Regge trajectory $\alpha(s) = \alpha' s + \alpha(0)$ should be 1. (The no-ghost theorem suggests but by itself does not uniquely determine those values.) We shall return to these matters in the next chapter.

Next, we would like to work out an interesting integral representation for the Veneziano amplitude. Consider the function

$$C(u,v) = \int_0^1 dx\, x^{u-1}(1-x)^{v-1}. \tag{1.1.18}$$

It obeys

$$\begin{aligned} C(u-1,v+1) &= \int_0^1 dx\, x^{u-2}(1-x)^v = \frac{1}{u-1}\int_0^1 dx\,(\frac{d}{dx}x^{u-1})(1-x)^v \\ &= \frac{v}{u-1}\int_0^1 dx\, x^{u-1}(1-x)^{v-1} = \frac{v}{u-1}C(u,v). \end{aligned} \tag{1.1.19}$$

where we have integrated by parts. The beta function obeys the same identity $B(u-1,v+1) = \frac{v}{u-1}B(u,v)$ by virtue of (1.1.7). C also obeys

$$\begin{aligned} C(u+1,v) &= \int_0^1 dx\, x^u(1-x)^{v-1} \\ &= \int_0^1 dx\, x^{u-1}(1-x)^{v-1} - \int_0^1 dx\, x^{u-1}(1-x)^v \\ &= C(u,v) - C(u,v+1). \end{aligned} \tag{1.1.20}$$

The analogous beta function identity $B(u+1,v)+B(u,v+1) = B(u,v)$ is likewise a consequence of (1.1.7). These recursion relations together with the similar asymptotic behavior of the functions $B(u,v)$ and $C(u,v)$ and the fact that they are equal at $u = v = 1$ imply that in fact $B(u,v) = C(u,v)$. Therefore, we obtain an integral representation for the Veneziano amplitude:

$$A(s,t) = \int_0^1 x^{-\alpha(s)-1}(1-x)^{-\alpha(t)-1}dx. \tag{1.1.21}$$

This integral representation is quite important, since it is in this form that the Veneziano amplitude usually appears in most approaches to calculating string scattering amplitudes.

1.1.2 High-Energy Behavior of the Veneziano Model

Our next task is to understand the asymptotic behavior of the Veneziano amplitude for high energy. We consider first the Regge region of large s, fixed t. The physical region for elastic scattering is positive s, negative t or vice versa. Large s, fixed t corresponds to small angle scattering at high energy; it was the phenomenology of this region that gave birth to Regge-pole theory and eventually to dual models.

To explore the asymptotic behavior of the Veneziano amplitude, we first need to know the asymptotic behavior of the gamma function. The behavior of $\Gamma(u)$ for large u can be easily extracted from the integral representation

$$\Gamma(u) = \int_0^\infty dt\, t^{u-1}e^{-t}. \tag{1.1.22}$$

For large u, the integral is dominated by the region $t \approx u-1$. A saddle point evaluation in this region gives Stirling's formula

$$\Gamma(u) \sim \sqrt{2\pi}u^{u-1/2}e^{-u}. \tag{1.1.23}$$

Although our derivation assumed positive u, Stirling's formula is actually valid for large u throughout the u plane as long as one keeps away from the negative u axis, where $\Gamma(u)$ has poles. From (1.1.23), we see that the Veneziano amplitude

$$A(s,t) = \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))} \tag{1.1.24}$$

has in the region of large s, fixed t the asymptotic behavior

$$A(s,t) \sim \Gamma(-\alpha(t))(-\alpha(s))^{\alpha(t)}. \tag{1.1.25}$$

For a linear Regge trajectory, $\alpha(s) \sim \alpha' s$, the asymptotic behavior for