Temporal Logics in Computer Science

This comprehensive text provides a modern and technically precise exposition of the fundamental theory and applications of temporal logics in computer science. Part I presents the basics of discrete transition systems, including constructions and behavioural equivalences. Part II examines the most important temporal logics for transition systems and Part III looks at their expressiveness and complexity. Finally, Part IV describes the main computational methods and decision procedures for model checking and model building – based on tableaux, automata and games – and discusses their relationships.

The book contains a wealth of examples and exercises, as well as an extensive annotated bibliography. Thus, the book is not only a solid professional reference for researchers in the field but also a comprehensive graduate textbook that can be used for self-study as well as for teaching courses.

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Temporal Logics in Computer Science
Finite-State Systems

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