1 Introduction

1.1. Natural and Technological Counterflows

Counterflows – flows of a fluid in opposite directions – play important roles in nature and technology. Oppositely directed flows are separated by impermeable surfaces in heat exchangers and similar devices. The surfaces prevent mixing of opposite flows. This book deals with oppositely directed flows with no separating wall. Surprisingly, such flows can be very elongated and survive intense mixing. The intense mixing typical of counterflows is important for chemical technologies, combustion, and other applications. The renowned example of elongated counterflows is the Gulf Stream and the opposite current occurring in the ocean depths. Technological applications of counterflows include hydrocyclones, vortex tubes, and vortex combustors. Counterflows in these devices are wildly turbulent but survive intense mixing; this feature seems paradoxical.

Figure 1 is a schematic of a commercial hydrocyclone. Since 1891, when the first patent on hydrocyclones was granted, hydrocyclones became widely used in solid-liquid and liquid-liquid separation (Schultz et al. 2009). The hydrocyclone shown in Figure 1 is used for separation of oil droplets from water. Oily water tangentially enters the cylindrical container and develops a swirling flow. Rotating water goes to the opposite end near the sidewall. Oil droplets shift to the axis driven by centrifugal buoyancy, accumulate there, go back to the entrance end wall, and leave the container through the central orifice. The cleaned water leaves the container through the remote exhaust. Meridional flow moves in opposite directions near the sidewall and near the axis. This counterflow pattern is crucial for hydrocyclone separators.

A similar flow pattern is typical of vortex tubes. Since the energy separation effect was discovered by Ranque (1933), vortex tubes have been used for spot cooling of cutting edges, sewing needles, electrical cabinets, and other applications. Figure 2 is a schematic of a commercial vortex tube. Pressurized air tangentially enters the cylindrical container and develops a swirling flow. Rotating air moves near the sidewall to the opposite end and is heated up. Part of the air flow leaves the container through the peripheral exhaust. The rest moves back near the axis, is cooled down, and leaves the container through the central exhaust located close to the inlet. Meridional counterflow is a crucial feature of vortex tubes as well.

The transformation of incoming air into hot and cold outflows is counter-intuitive. In addition, the location of hot air at the periphery and of cold air near

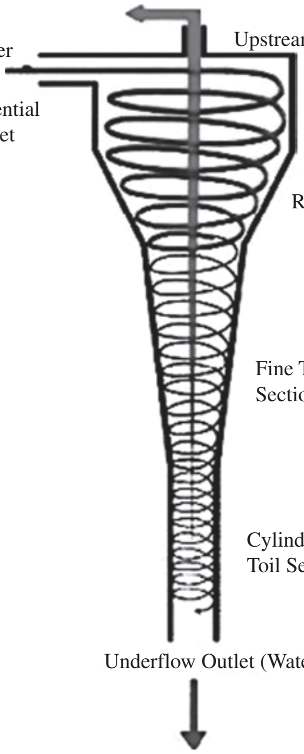


Figure 1. Schematic of a commercial hydrocyclone.

the axis seems contradictory to the thermal stratification typical of centrifugal force. There have been many attempts to explain these paradoxical features of flow in vortex tubes, beginning with Hilsch's paper (1947). Despite rather detailed experimental and theoretical studies, including modern computational fluid dynamics (CFD) simulations (Secchiaroli et al. 2009), a consensus has not yet been achieved regarding the physical mechanism of energy separation in vortex tubes. This book explains the counterflow nature in hydrocyclones, vortex tubes, and other devices, and focuses on the counterflow mechanisms listed in the following sections.

1.2. Physical Mechanisms of Counterflows

1.2.1. Accumulation

A high-speed annular flow converging to a point develops a bipolar jet emerging from that point. The mechanism of bipolar jet generation is inertial and can be explained in terms of a potential motion of an ideal fluid (Batchelor 1967). Figure 3 schematically shows an annular conical flow of a fluid focusing toward point O on the axis of symmetry which is the z-axis in Figure 3. Thin lines represent the axisymmetric surface, SS, separating the flow going forward, i.e., in the positive z-direction in Figure 3, from the flow turning and going back, i.e., in the negative z-direction in Figure 3.

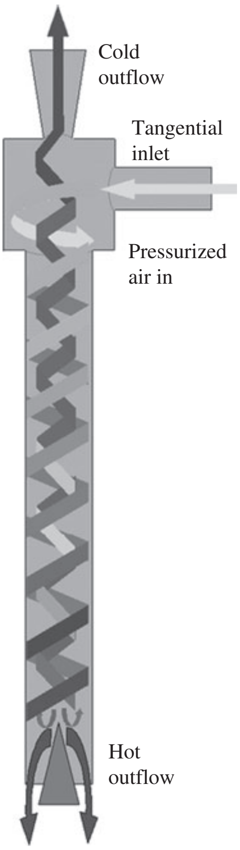


Figure 2. Schematic of a vortex tube.

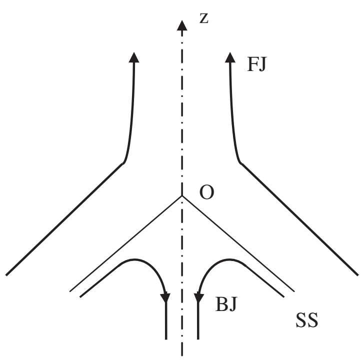


Figure 3. Schematic of a bipolar jet of an inviscid fluid.

The formation of the forward jet, FJ in Figure 3, is obvious since its z-momentum has the same sign as that of the converging flow. In contrast, the development of the backward jet, BJ in Figure 3, is counterintuitive and therefore should be explained. To this end, the Bernoulli equation is helpful, $p_t = p_s + \rho v^2/2 = \text{const}$, where p_t (p_s) is the total (static) pressure, ρ is the fluid density, and v is the velocity magnitude. At the surface of liquid flow, occurring in ambient air, static pressure p_s is equal to atmospheric pressure p_a. For potential flow, p_t is the same constant in the entire liquid flow region. At the stagnation point O where v = 0, static pressure p_s reaches its maximal value, which equals p_t.

Suppose that the backward jet is absent, i.e., no liquid is located below SS in Figure 3. Then pressure is large, close to p_t, above SS and is small, being equal to p_a, below SS in the O vicinity. Since $p_t > p_a$, the pressure difference pushes the fluid down in Figure 3, thus developing the backward jet. The peak of pressure, centered at the stagnation (focusing) point, drives both the forward and backward jets.

The backward jet has an application in artillery shells (Batchelor 1967) where BJ, being driven by explosives, has larger velocity than that of FJ. This feature is utilized to destroy tank armor.

Most spectacular and elongated bipolar jets are observed in space (Pudritz 2000). They originate from stars and galaxy cores whose masses range from 0.01 to 10^9 of the mass of the Sun. Cosmic jets have sizes of 0.1–10^6 parsecs (1 parsec $\approx 3\times10^{13}$ km) and velocities ranging from tens of km/s to relativistic ones.

Most miniature bipolar jets occur in capillary flows. A rapidly growing area of capillary jet and spray applications – fuel atomization, printing, and biotechnology (Bailey 1988) – has attracted the attention of many researchers to flow in liquid meniscus. Surprisingly, this flow is rich with intriguing and practically important features. Since the flow is miniature, that is, the capillary tube diameter is equal to or less than 1 mm, earlier conjectures proposed that the liquid motion in a conical meniscus is slow and unidirectional. Contrary to these expectations, the meridional circulation was discovered in electrospray flows (Hayati et al. 1986a,b).

Circulation development is a nonlinear effect occurring only in a high-speed flow. Estimates and measurements performed for conical meniscus flows revealed that the Reynolds number, *Re*, can exceed 10^3 which is quite sufficient for nonlinear effects. The forward jet (FJ) serves for printing and fuel spraying while the backward jet (BJ) decelerates and turns around, forming the meridional circulation in a conical meniscus. Forcing of the converging conical annular jet can be caused by electric shear stresses (Hayati et al. 1986a,b), gas co-flow (Gañán-Calvo 1998a,b), and other means. Any forcing, resulting in the high-speed converging annular conical jet, can develop meridional circulation.

In micro-fluidic flows, BJ is directed inside the capillary tube. As the shear-stress forcing intensifies the liquid flow, first BJ develops, while FJ can still be blocked by the surface tension. FJ erupts only as the shear-stress forcing exceeds a threshold value.

The jet entrains the ambient liquid in the meniscus. This results in deceleration and divergence of BJ. Figure 4 displays a schematic of a viscous bipolar jet whose streamlines eventually diverge due to the entrainment. As BJ penetrates deep in the needle, it is stopped and reversed by the oppositely directed pipe flow. Thus, the accumulation-induced counterflow develops. The accumulation counterflows are discussed in further detail in Chapter 2.

1.2.2. Swirl Effect

The mechanism of swirl-induced circulation is different and in some sense opposite to the accumulation mechanism: the latter is related to the pressure maximum located at the focal point while the former is related to the pressure minimum, as explained here.

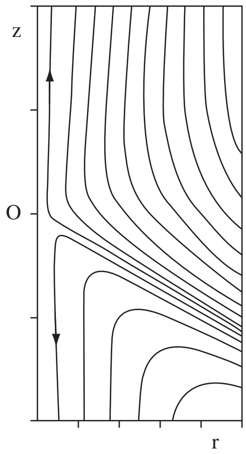


Figure 4. Schematic of a bipolar jet of a viscous fluid.

Swirl reduces pressure near the rotation axis according to the cyclostrophic balance, $\partial p/\partial r = \rho v_s^2/r$, between the centrifugal force and the radial gradient of pressure, where v_s is the swirl velocity. The balance yields that pressure increases with the distance from the axis, r, since $\rho v_s^2/r > 0$.

Now consider how pressure at the axis depends on z in the flow schematically shown in Figure 4. In a high-speed flow, the angular momentum, rv_s, is nearly completely conserved along a streamline. The closer to the axis the streamline approaches, the larger the swirl velocity becomes and the more the pressure drops at the axis, according to cyclostrophic balance.

Near the focal point O in Figure 4, streamlines first converge toward the axis and then diverge from the axis. At the axial coordinate value, where flow convergence changes into flow divergence, a local pressure minimum is developed. As swirl forcing increases, the minimum becomes deeper and the low pressure starts to suck downstream fluid. The suction results in a local flow reversal and swirl-induced circulation, often referred to as vortex breakdown bubble, VBB. Inside VBB, the fluid moves toward the focal point near the axis and moves back at the periphery. This circulation direction is opposite that in the accumulation case. The VBB, emerging from a point, is small in size compared to the ambient flow extent. This swirl-induced counterflow is local.

However, swirl can also cause global meridional circulation, occupying the entire flow region. There are flows where swirl develops both the local and global meridional circulations. An example is the flow in a sealed cylindrical container induced by a rotating end wall (Fig. 5). This flow has been the subject of many experimental and numerical studies of vortex breakdown (VB), starting with Escudier's work (Escudier 1988). The rotating end wall pushes fluid to the sidewall. The fluid moves along the sidewall to the still end, makes a U-turn, and then returns to the rotating end wall along the axis. This is global meridional circulation. As the rotation intensifies, VB occurs near the still end wall; i.e., local circulation develops. Figure 5a shows an experimental observation where the near-axis flow is visualized with the help of a dye (Husain et al. 2003) and Figure 5b shows the numerical simulation of the meridional flow pattern (Herrada & Shtern 2003b).

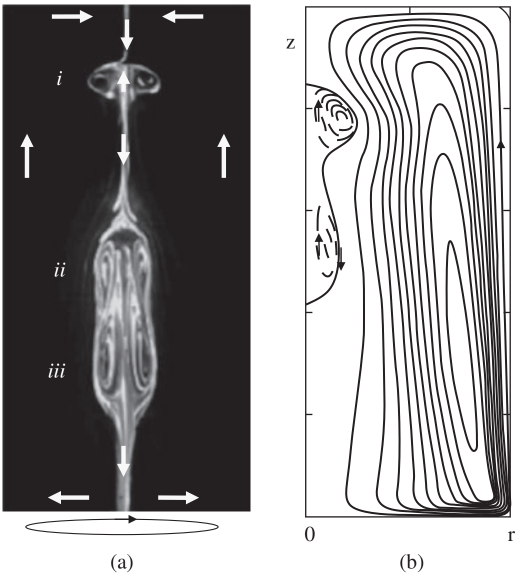


Figure 5. Flow in a sealed cylinder induced by a rotating end wall: (a) experiment and (b) simulation.

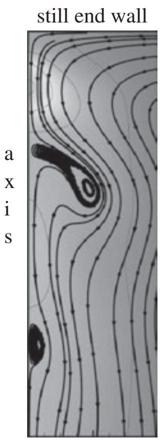


Figure 6. Stream (arrowed curves) and isobaric lines in vortex breakdown region.

VB develops as a result of focused flow convergence near the still end wall driven by the mechanism described previously. Figure 6 shows streamlines (arrowed curves) and isobaric contours in the vortex breakdown region (Shtern et al. 2011a). The convergence of the swirling flow near the still end wall produces the local pressure minimum (pale spot in Fig. 6). The downstream fluid is sucked toward the pressure minimum location developing the first VBB. Downstream of the VBB, the fluid again converges to the axis and the same mechanism is triggered; swirl vorticity accumulates and a new local pressure minimum develops which sucks downstream fluid. This chain-like mechanism can develop a number of VBBs. For example, there are three VBBs, denoted by *i*, *ii*, and *iii*, in Figure 5a.

Chapters 3–7 address effects of swirl in counterflows in greater detail. The appearance of swirl in counterflows via bifurcation is discussed in Chapter 3. Chapter 4 deals with bifurcation of counter-swirl in jetlike and annular flows.

Conical similarity counterflows induced by swirl are covered in Chapter 5. Chapter 6 discusses swirl-induced countercurrents in more general flows: power-law jets, vortex sink with axial flows, and capillary jets. Chapter 7 describes swirl-induced counterflows, vortex breakdown, and double counterflows in cylindrical devices.

1.2.3. Separation

Flow separation from a wall results in the development of a counterflow downstream of the separation line. The mechanism is inertial: separation occurs when a fluid moves against the pressure gradient. Figure 7 illustrates the flow separation near a sharp edge marked by O. In the potential flow of an ideal fluid, schematically shown in Figure 7a, velocity has a singularity at O, becoming infinitely large. According to the Bernoulli equation, pressure unboundedly drops at O.

Therefore, the flow goes against the pressure gradient above O in Figure 7a. In the flow of a viscous fluid, both velocity and pressure are limited; velocity is zero at the entire boundary due to the no-slip, while pressure has a local minimum of a finite depth at O. The downstream fluid is sucked to the pressure minimum location causing flow reversal near the wall above O and the development of counterflow, schematically shown in Figure 7b.

The same separation mechanism works if the edge is not sharp, e.g., in the flow near a cylinder or a sphere. In these flows, velocity (pressure) reaches its maximum (minimum) at the boundary middle and a circulation region develops behind the body.

A similar mechanism works in a diverging channel where velocity (pressure) decreases (increases) downstream. If the divergence angle is sufficiently large, the flow separates from a sidewall and the counterflow develops, as Figure 8 schematically shows. The separation region diminishes the angular extent of through-flow and thus enhances its velocity. This helps through-flow to go against the pressure gradient.

The separation in the flow discussed earlier is a nonlinear phenomenon; it does not occur if the motion is so slow that the inertial effects are small compared with the

Figure 7. Flow schematic near a sharp edge for (a) ideal and (b) viscous fluids.

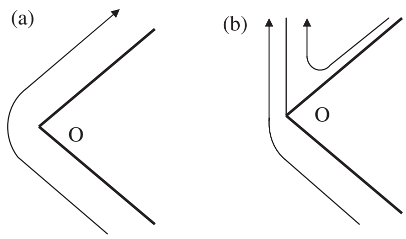
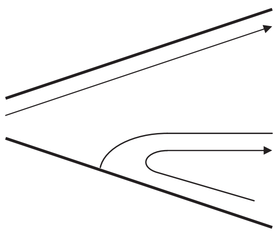


Figure 8. Schematic of a counterflow in a diverging channel.



viscous effects. The separation develops as the Reynolds number, characterizing the flow strength, exceeds a threshold value.

Counterflow development caused by separation in diverging flows is discussed in Chapter 8. Both the separation from a wall and instability-induced internal separation in a planar diverging channel and in the vortex source issuing in a free space are covered there in detail.

1.2.4. Thermal Convection

A well-known driving mechanism of circulatory motion is thermal convection. For example, heating from below produces Bénard cells via the Rayleigh instability (Chandrasekhar 1981). Since this book focuses on elongated counterflows, consider the free convection in a horizontal layer driven by gravity and the horizontal gradient of temperature (Fig. 9a). Near the hot end wall, a fluid is heated, its density decreases, and the buoyancy lifts the fluid and keeps it near the top as the fluid moves to the cold end. Near the cold end wall, the fluid is cooled, its density increases, and the gravity pushes the fluid down and keeps it near the bottom as the fluid goes to the hot end. This results in a circulatory flow schematically depicted by the closed curve with an arrow indicating the motion direction in Figure 9a; g is the gravity acceleration.

A similar flow is thermal convection in a rotating cylindrical container shown in Figure 9b (Shtern et al. 2001). The cylinder rotates around its axis of symmetry, depicted by the dot-dash line in Figure 9b, with angular velocity ω. This generates the centrifugal acceleration, $g_{c.} = \omega^2 r$; r is the distance from the axis. Near the hot end wall, a fluid is heated, its density decreases, and centrifugal buoyancy pushes the fluid from the periphery to the center and keeps it near the axis as the fluid goes to the cold end. Near the cold end wall, the fluid is cooled, its density increases, and centrifugal force pushes the fluid from the center to the periphery and keeps it near the sidewall as the fluid moves to the hot end. This circulation is schematically depicted by the closed curve with the arrow, indicating the meridional motion direction in Figure 9b. This circulatory motion is centrifugal convection.

Comparing Figures 9a and 9b shows that centrifugal and gravitational convections are quite similar. An important difference is that the gravitational convection

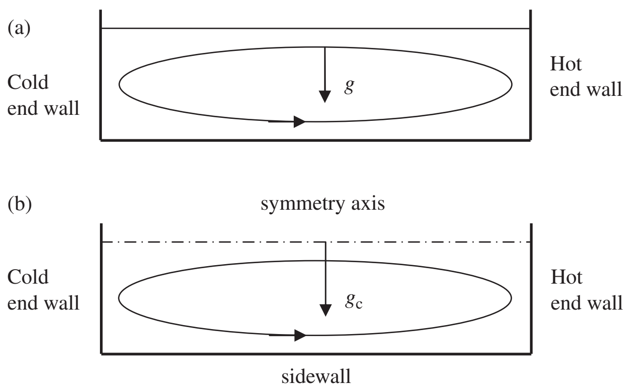


Figure 9. Schematic of (a) gravitational and (b) centrifugal thermal convection.

typically is slow in technological devices. In contrast, centrifugal convection is a high-speed flow, e.g., in vortex tubes, where g_c can be one million times g!

Thermal effects in elongated counterflows are covered in greater detail in Chapters 9–11. Patterns of axisymmetric and three-dimensional distributions of temperature in a number of practical swirling flows are discussed in Chapter 9. Chapter 10 focuses on the onset of thermal convection near a point source of heat and gravity having either conical or Keplerian similarity. These flows developing via bifurcation from the still state mimic bipolar cosmic jets with the help of analytical solutions of the Boussinesq and gas-dynamic equations. Chapter 11 describes a number of thermal-convection flows emerging with no bifurcation. They model counterflows near a volcano, near a hydrothermal vent in the ocean depth, and centrifugal convection of a liquid and a compressible gas in a rotating cylindrical container. The latter flows are modeled by polynomial solutions describing the radial distribution of axial velocity and temperature.

1.3. Counterflow Applications, Control, and Stability

The meridional counterflow is a key feature of vortex separators as mentioned in Section 1.1. Combustion applications involve all kinds of counterflows whose mechanisms are discussed in Section 1.2.

Accumulation counterflows occur in fuel atomizers which disperse a liquid fuel into a spray of droplets. To this end, a high-speed annular conical jet is driven either by surface electric forces or by a gas co-flow. Atomization allows fast evaporation and ignition of the fuel. The stronger the counterflow is, the smaller droplets that are generated, resulting in more efficient combustion.

Swirl-induced circulation is used to stabilize the flame front in combustors. The reversed flow transports the combustion heat back to the fuel nozzle, thus preheating both the fuel and oxidizer and providing stable combustion. Global swirling counterflows are utilized in vortex combustors to efficiently mix air, fuel, and flue gases; this helps reduce harmful emissions. The book also describes double counterflow in a vortex combustor. This flow phenomenon, recently discovered in experiments and explained with the help of numerical simulations, is beneficial for low-emission combustion. The global double counterflow generated by swirl in vortex combustors quickly and efficiently mixes a fuel, an oxidizer, and flue gases, thus providing favorable conditions for combustion at a temperature high enough to burn the carbon monoxide and low enough to prevent nitrogen oxidizing. This significantly reduces harmful emissions.

Centrifugal convection works together with the swirl mechanism and intensifies the global counterflow in vortex combustors. Incoming low-temperature air moves along the sidewall to the end wall where a fuel is injected. The hot flue gases produced by combustion move from the fuel nozzle near the combustor axis to the exhaust. This counterflow is subject to shear-layer instability generating large-scale vortices and small-scale turbulence. The turbulent heat and mass transfer results in efficient preheating of the incoming air and in diluting the flue gases with the air that moderates the exhaust temperature down to a value, e.g., tolerable for turbine blades.

Vortex breakdown (VB) is a local counterflow which occurs in atmospheric swirling jets, like tornadoes, and in a variety of technological flows. For example, VB above a delta wing can cause the loss of aircraft control; VB also helps to collect hazardous emissions in vortex suction devices. Vortex breakdown control is very important for technological applications. Chapter 12 discusses two means of VB control: (a) by adding swirl in the VB region and (b) by a temperature gradient.

One more potential means of VB control is a magnetic field. Chapter 13 examines how magnetic field can appear via bifurcation in accretion, swirling, thermal convection, and electro-vortex conical flows. The latter flows occur in welding and other magneto-hydrodynamic technological processes.

Many counterflows develop via instability. Chapter 14 describes the stability theory of conical flow and explains the origin of disturbances causing bifurcation of swirl, axial symmetry breaking, and hysteretic transitions. In addition, this theory explains strongly nonparallel deceleration instability of swirl-free and swirling jets. The deceleration instability develops at such small values of the Reynolds number that the boundary-layer approximation used in prior studies is invalid.

1.4. Approach

This book does not pretend to encompass all types of counterflows (that seems impossible) and is in a sense an introduction to the topic. Its scope and methods are limited to research in which the author has been closely involved. In particular, this monograph uses applications of analytical studies where it is possible. This includes exact solutions of the Navier-Stokes, Boussinesq, magneto-hydrodynamic, and gas-dynamic equations.

Where exact solutions cannot be obtained, the asymptotic technique is applied for limiting cases such as the Reynolds number and other control parameters tending to infinity. This includes the finding of the boundary-layer and outer solutions as well as their matching to construct uniform approximations valid in the entire flow region. In vicinities of bifurcation points, small-parameter expansions are applied to reveal whether the bifurcation is subcritical or supercritical.

Whenever possible, the governing partial differential equations are reduced to ordinary differential equations with the help of similarity and/or asymptotic approaches. Numerical simulations combined with asymptotic analytical solutions allow us to sometimes cover all ranges of control parameter variations.

The focus on analytical tools is made because the author's view is that analytical results help to better understand the physical mechanism of a flow. For example, a number of analytical solutions discussed in this book become singular at finite values of control parameters. Exploring this paradoxical mathematical feature helps to understand the development of strongly collimated jets such as those observed in cosmic space.

A few physical and numerical experiments discussed in detail here were stimulated and supported by our analytical studies, which indicated a possibility of interesting effects of both fundamental and technological interest.