INTRODUCTION TO PLASMA PHYSICS
With Space, Laboratory and Astrophysical Applications

Introducing the basic principles of plasma physics and their applications to space, laboratory, and astrophysical plasmas, this new edition provides updated material throughout. Topics covered include single-particle motions, kinetic theory, magnetohydrodynamics, small-amplitude waves in hot and cold plasmas, and collisional effects. New additions include the ponderomotive force, tearing instabilities in resistive plasmas, the magnetorotational instability in accretion disks, charged particle acceleration by shocks, and a more in-depth look at nonlinear phenomena. A broad range of applications are also explored: planetary magnetospheres and radiation belts, the confinement and stability of plasmas in fusion devices, the propagation of discontinuities and shock waves in the solar wind, and analysis of various types of plasma waves and instabilities that can occur in planetary magnetospheres and laboratory plasma devices. With step-by-step derivations and self-contained introductions to mathematical methods, this book is ideal as an advanced undergraduate to graduate-level textbook, or as a reference for researchers.

DONALD A. GURNETT is a pioneer in the study of waves in space plasmas, and has been active in teaching plasma physics and conducting experimental space physics research for over fifty years. He is currently the James A. Van Allen/R. J. Carver Professor of Physics at the University of Iowa and has received numerous awards for both his teaching and research. In 1994 he received the Iowa Board of Regents Award for Faculty Excellence, and in 1998 was elected a member of the National Academy of Sciences.

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INTRODUCTION TO PLASMA PHYSICS

With Space, Laboratory and Astrophysical Applications

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Dedicated to Marie, my loving wife for over fifty years, who has enthusiastically supported my various endeavors, especially this book.

Don Gurnett

And, to Melissa, without whose love and support this book and so much else would not be possible.

Amitava
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Preface

This textbook is intended for a full year introductory course in plasma physics at the senior undergraduate or first-year graduate level. It is based on lecture notes from courses taught by the authors for more than three decades at the University of Iowa, Columbia University, University of New Hampshire, and Princeton University. During these years, plasma physics has grown increasingly interdisciplinary, and there is a growing realization that diverse applications in laboratory, space, and astrophysical plasmas can be viewed from a common perspective. Since the students who take a course in plasma physics often have a wide range of interests, typically involving some combination of laboratory, space, and astrophysical plasmas, a special effort has been made to discuss applications from these areas of research. The emphasis of the book is on physical principles, less so on mathematical sophistication. An effort has been made to show all relevant steps in the derivations, and to match the level of presentation to the knowledge of students at the advanced undergraduate and early graduate level. The main requirements for students taking this course are that they have taken an advanced undergraduate course in electricity and magnetism and that they are knowledgeable about using the basic principles of vector calculus, i.e., gradient, divergence, and curl, and the various identities involving these vector operators. Although extensive use is made of complex variables, no special background is required in this subject beyond what is covered in an advanced calculus course. Relatively advanced mathematical concepts that are not typically covered in an undergraduate sequence, such as Fourier transforms, Laplace transforms, the Cauchy integral theorem, and the residue theorem, are discussed in sufficient detail that no additional preparation is required. Although this approach has undoubtedly added to the length of the book, we believe that the material covered provides an effective and self-contained textbook for teaching plasma physics. MKS units are used throughout. Problem solutions are available to instructors at www.cambridge.org/9781107027374

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Preface

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