An Introduction to Celestial Mechanics

This accessible text on classical celestial mechanics—the principles governing the motions of bodies in the solar system—provides a clear and concise treatment of virtually all the major features of solar system dynamics. Building on advanced topics in classical mechanics, such as rigid body rotation, Lagrangian mechanics, and orbital perturbation theory, this text has been written for well-prepared undergraduates and beginning graduate students in astronomy, physics, mathematics, and related fields. Specific topics covered include Keplerian orbits; the perihelion precession of the planets; tidal interactions among the Earth, Moon, and Sun; the Roche radius; the stability of Lagrange points in the three-body problem; and lunar motion. More than 100 exercises allow students to gauge their understanding; a solutions manual is available to instructors. Suitable for a first course in celestial mechanics, this text is the ideal bridge to higher-level treatments.

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Preface

The aim of this book is to bridge the considerable gap that exists between standard undergraduate mechanics texts, which rarely cover topics in celestial mechanics more advanced than two-body orbit theory, and graduate-level celestial mechanics texts, such as the well-known books by Moulton (1914), Brouwer and Clemence (1961), Danby (1992), Murray and Dermott (1999), and Roy (2005). The material presented here is intended to be intelligible to an advanced undergraduate or beginning graduate student with a firm grasp of multivariate integral and differential calculus, linear algebra, vector algebra, and vector calculus.

The book starts with a discussion of the fundamental concepts of Newtonian mechanics, as these are also the fundamental concepts of celestial mechanics. A number of more advanced topics in Newtonian mechanics that are needed to investigate the motions of celestial bodies (e.g., gravitational potential theory, motion in rotating reference frames, Lagrangian mechanics, Eulerian rigid body rotation theory) are also described in detail in the text. However, any discussion of the application of Hamiltonian mechanics, Hamilton-Jacobi theory, canonical variables, and action-angle variables to problems in celestial mechanics is left to more advanced texts (see, for instance, Goldstein, Poole, and Safko 2001).

Celestial mechanics (a term coined by Laplace in 1799) is the branch of astronomy that is concerned with the motions of celestial objects—in particular, the objects that make up the solar system-under the influence of gravity. The aim of celestial mechanics is to reconcile these motions with the predictions of Newtonian mechanics. Modern analytic celestial mechanics started in 1687 with the publication of the Principia by Isaac Newton (1643-1727) and was subsequently developed into a mature science by celebrated scientists such as Euler (1707-1783), Clairaut (1713-1765), D'Alembert (1717–1783), Lagrange (1736–1813), Laplace (1749–1827), and Gauss (1777–1855). This book is largely devoted to the study of the "classical" problems of celestial mechanics that were investigated by these scientists. These problems include the figure of the Earth; tidal interactions among the Earth, Moon, and Sun; the free and forced precession and nutation of the Earth; the three-body problem; the secular evolution of the solar system; the orbit of the Moon; and the axial rotation of the Moon. However, any discussion of the highly complex problems that arise in modern celestial mechanics, such as the mutual gravitational interaction between the various satellites of Jupiter and Saturn, the formation of the Kirkwood gaps, the dynamics of planetary rings, and the ultimate stability of the solar system, is again left to more advanced texts (see, in particular, Murray and Dermott 1999).

A number of topics, closely related to classical celestial mechanics, are not discussed in this book for the sake of brevity. The first of these is positional astronomy—the

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branch of astronomy that is concerned with finding the positions of celestial objects in the Earth's sky at a particular instance in time. Interested readers are directed to Smart (1977). The second excluded topic is the development of numerical methods for the solution of problems in celestial mechanics. Interested readers are directed to Danby (1992). The third (mostly) excluded topic is astrodynamics: the application of Newtonian dynamics to the design and analysis of orbits for artificial satellites and space probes. Interested readers are directed to Bate, Mueller, and White (1977). The final excluded topic is the determination of the orbits of celestial objects from observational data. Interested readers are again directed to Danby (1992).