Index

Entries for technical terms and for theorems give the principal location(s) where you will find them explained. Entries for names link to not-merely-bibliographical occurrences.

\[ \Delta_0 \text{ wff}, 74 \]
\[ \Pi_1 \text{ reflection}, 265 \]
\[ \Pi_1 \text{-sound}, 81, 265 \]
\[ \Pi_1 \text{ wff}, 76 \]
\[ \Pi_1 \text{-complete}, 79 \]
\[ \Pi_1 \text{-equivalent wff}, 79 \]
\[ \Pi_2 \text{ wff}, 79, 88 \]
\[ \Sigma_0 \text{ wff}, 79 \]
\[ \Sigma_1 \text{-complete}, 79 \]
\[ \Sigma_1 \text{ wff}, 76 \]
\[ \Sigma_1 \text{-consistent}, 186 \]
\[ \Sigma_1 \text{ wff}, 79 \]
\[ \beta \text{-function}, 114 \]
\[ \varepsilon_0, 243 \]
\[ \lambda \text{-calculus}, 341 \]
\[ \lambda \text{-definable}, 341 \]
\[ \mu \text{-recursive}, 287 \]
\[ \mu, \text{ see minimization operator} \]
\[ \omega, 156, 223 \]
\[ \omega \text{-consistent}, 157 \]
\[ \omega \text{-incomplete}, 121, 156 \]
\[ \omega \text{-rule}, 30 \]

\[ \# \text{, xiii} \]
\[ \overline{x}, 100 \]
\[ \bar{x}, 117 \]
\[ \omega, 188 \]
\[ \vdash, 234 \]
\[ \square, 245 \]
\[ \ast, 145 \]
\[ \sim, 141 \]
\[ \models, 32 \]
\[ \models, 33 \]
\[ \models, 208 \]
\[ \not\models, 69 \]

absurdity constant, 234
ACAs, 217

acceptable g.n. scheme, 138
Ackermann, Wilhelm, 288, 303, 339, 340
Ackermann–Péter function, 290
admissible interpretation, 208, 212
Albert of Saxony, 257
algorithm, 14, 339
analytic, 2
ancestral, 229

arithmetic

Baby, see BA
basic, 1
language of basic, 1, 37
neat, 210
Peano
First–order, see PA
Second–order, see PA_2
Robinson, see Q
True, 193, 217

arithmetically sound, 163
arithmetization of syntax, 55, 136–141
atomic wff, 38

axiom, 2, 29
Axiom of Infinity, 133, 273
axiomatizable set of wffs, 47
axiomatized theory, 26, 27, 31, 161, 301
effectively, 31
p.r., 161
recursively, 301

BA, 62
Baby Arithmetic, see BA
basic arithmetic, 1
Benacerraf, Paul, 281
Bernays, Paul, 246, 277
bijective function, 9
Black, Robert, 365
Blass, Andreas, 231
bounded minimization, 108
bounded quantifiers, 73

canonically capture, 129

Cantor, Georg, 12

Cantor’s Theorem, 12
capture


canonically, 129

function, 44, 119

function, strongly, 123

function, weakly, 120

property, 43

Carnap’s Equivalence, 180

Carnap, Rudolf, 180
categorical theory, 213

characteristic function, 9

Chinese Remainder Theorem, 116

Church, Alonzo, 303, 341–343, 347, 350

Church’s Theorem, 302

Church’s Thesis, 294, 342

interpretive vs. labour-saving, 296

Church–Turing Thesis, 338

‘class’ vs. ‘set’, 96
closed wff, 28
codomain, 8
complete logic, 33

complete theory, 32, 33

vs. complete logic, 33, 216

Completeness Theorem, 33

composition of functions, 98, 101

Comprehension Schema, 210

Con, 234

concatenation function, 145

conservative, 122, 217

real, 275

weakly, 275

consistency, 32

sentence, 6, 234, 238

sentence, provable, 263

unprovability of, 6, 236–238

consistency-minded proof, 264

contentual, 273

correctly decides, 32

Craig, William, 192

Craig’s Re-axiomatization Theorem, 192

Curry, Haskell B., 257

Curry’s paradox, 257

dataspace, 359

decidable

property, 18

set, 19

theory, 32
decision problem, 301

Dedekind, Richard, 90, 98, 214
definition by cases, 108
definition by primitive recursion, 98
derivability conditions, 246

deriving vs. proving, 3

Detlefsen, Michael, 278
diagonal argument, 12, 51
diagonal function, 13, 106
diagonalization, 13, 143

Diagonalization Lemma, 180

Jeroslow’s, 249
diagonalize out, 107, 294, 342

‘do until’ loop, 106, 285
domain, 8
dTuring-computable, 321
e.e., see effectively enumerable
effectively axiomatizable, 47
effectively axiomatized theory, 31
effectively computable, 15, 294
effectively decidable, 18, 19
effectively enumerable, 20

Basic Theorem about e.e. sets, 24

Entscheidungsproblem, 303

eumerable set, 10
equinumerous, 132

Erdős, Paul, 48

express

function, 43

property, 41

extend a theory, 81, 164, 194

extension

of function, 42, 102

of predicate, 206

of property, 19, 103

extensionality, 101

Feferman, Solomon, 264, 265, 269

Fermat’s Last Theorem, 80, 102, 172

First Incompleteness Theorem, 3, 165–166

first-order

Peano Arithmetic, see PA

384
Index

vs. second-order arithmetic, 217
vs. second-order logic, 37, 204–208
Fitch, Frederic, 68
fixed point, 181
Fixed Point Theorem, 181
‘for’ loop, 104
formal deduction, 25
formal language, 25
interpreted, 27
formal theory, 27, 31, 161, 301
Formalized First Theorem, 234–235
Formalized Second Theorem, 249
formally undecidable, 134
Frege, Gottlob, 3, 25, 95, 96, 103, 130, 131, 133, 272
Friedman, Harvey, 225, 227
function
μ-recursive, 287
Ackermann–Péter function, 290
bijective, 9
characteristic, 9
codomain, 8
diagonal, 13, 106
domain, 8
initial, 100
injective, 8
one-one correspondence, 9
one-to-one, 8
onto, 8
partial, 8, 286, 314
partial computable, 336
primitive recursive, 99, 101
range, 8
regular, 286
state-of-play, 326, 329
surjective, 8
total, 8, 15
Turing-computable, 314
g.n., see Gödel number
Gdl, 153
Gentzen, Gerhard, 68, 221, 241
Gödel, Kurt
quotations, 134, 166, 180, 200, 218, 235, 237, 263, 268, 274, 282
reported views, 169, 283
Gödel number, 136–139
super, 139
Gödel sentence, 153, 158–160, 182, 249
canonical, 158
showing to be true, 171–173, 269–271
Gödel’s Completeness Theorem, 33
Gödel’s Incompleteness Theorems, see First, Second Incompleteness Theorem
Gödel-Rosser Theorem, 189
Goldbach type (of wff), 78, 155
Goldbach’s conjecture, 1, 78, 80, 172, 353
Goodstein, Rueben L., 222
Goodstein sequence, 222
Goodstein’s Theorem, 223
Grundgesetze der Arithmetik, 95, 130
halt standardly, 314
halting problem, 330
Henkin, Leon, 255
Herbrand, Jacques, 342
hereditary property, 133, 215, 230
Hilbert, David, 30, 246, 273, 277, 303, 339, 340
Hilbert-style system, 68
Hilbert’s Programme, 7, 267, 272–279
Hume’s Principle, 132
i-quadruple, 312
iΔ0, 84
iff, 1
incompleteness, 4, 48, 163, 220
incompleteness theorems
semantic vs. syntactic, 167–171
stated, 48, 52, 154, 158, 163, 165, 187, 189, 190, 192, 236, 248, 251, 261, 304, 308, 333, 336
indenumerable, 12
induction
as rule of inference, 60
Axiom, 61, 208
course of values, 58
Schema, 60, 83
strong, 58
transfinite, 224
inexhaustibility, 175
initial function, 100
injective function, 8
interpretation, 29

385
admissible, 208
Isaacson, Daniel, 225
Isaacson’s Thesis, 225
isomorphic interpretations, 213
Jeroslow, R. G., 249
Kant, Immanuel, 272
Kirby, Laurence, 221
Kirby-Paris Theorem, 223
Kleene, Stephen, 99, 288, 334, 341, 342, 349, 351
Kleene’s Normal Form theorem, 334
Kolmogorov, A. N., 359
Koskensilta, Aatu, 231
Kreisel, Georg, 169, 208, 224, 255, 269, 355
Kripke, Saul A., 257, 355
KU-algorithm, 361
KU-computable, 362

$L_A$, 37
$L_{2A}$, 206
Lakatos, Imre, 350
Leibniz, G. W., 51
Leibniz’s Law, 62
length
of computation, 334
of proofs, 201
Liar paradox, 4, 198, 257
Löb, Martin H., 246, 255
Löb’s Theorem, 255
logicism, 3, 5, 130–134, 175, 273
Loglsh, 205
LOOP program, 105
Löwenheim, Leopold, 340
Löwenheim-Skolem Theorem, 213, 357
Lucas, John, 280, 283

Mac Lane, Saunders, 209
Martin, R. M., 229
Miša, Peter, 183
minimization
bounded, 108
operator, 107, 286
regular, 286
unbounded, 287

model, 69, 213
slim, 214
Mostowski, Andrzej, 159, 171, 237
Myhill, John, 229

$N$, 9
natural numbers, 1
neat second-order arithmetic, 210
negation-complete, 2, 32
Nelson, Edward, 91
nice theory, 165
nice’ theory, 304
nice+ theory, 250
nice* theory, 261
numeral, 38
1-consistency, 186
one-one correspondence, 9
open wff, 40
order-adequate, 73
ordinal numbers, 223, 224

$P$, Gödel’s type theory, 166, 235
p.r. adequate theory, 124
p.r. axiomatized, 161
p.r. function, see primitive recursive function
PA, 92
PA2, 211
PA*, 230
Parikh, Rohit, 88
Paris, Jeff, 221
Paris-Harrington theorem, 225
partial function, 8, 286, 314
Peano, Giuseppe, 90
Penrose, Roger, 283
Péter, Róza, 99, 289, 293
Potter, Michael, 269
Presburger, Mojžesz, 93
Presburger Arithmetic, 93
Prf, 177
Primitive Recursive Arithmetic, 124
primitive recursive function, 99, 101
primitive recursive property, 107
Principia Mathematica, 130–134, 166, 273
proof system, 30
Prov, 177
<table>
<thead>
<tr>
<th>Term</th>
<th>Page(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>provability predicate</td>
<td>177–178</td>
</tr>
<tr>
<td>Rosser, 188</td>
<td></td>
</tr>
<tr>
<td>Putnam, Hilary, 280</td>
<td></td>
</tr>
<tr>
<td>Q, 67</td>
<td></td>
</tr>
<tr>
<td>Quine, W. V., 173</td>
<td></td>
</tr>
<tr>
<td>r.e., see recursively enumerable</td>
<td></td>
</tr>
<tr>
<td>Ramsey, Frank, 211</td>
<td></td>
</tr>
<tr>
<td>range, 8</td>
<td></td>
</tr>
<tr>
<td>real vs. ideal, 273</td>
<td></td>
</tr>
<tr>
<td>real-conservative, 275</td>
<td></td>
</tr>
<tr>
<td>real-consistent, 275</td>
<td></td>
</tr>
<tr>
<td>real-sound, 275</td>
<td></td>
</tr>
<tr>
<td>recursive, 98</td>
<td></td>
</tr>
<tr>
<td>recursively adequate theory, 298</td>
<td></td>
</tr>
<tr>
<td>recursively axiomatized theory, 301</td>
<td></td>
</tr>
<tr>
<td>recursively decidable</td>
<td></td>
</tr>
<tr>
<td>property, 300</td>
<td></td>
</tr>
<tr>
<td>set, 300</td>
<td></td>
</tr>
<tr>
<td>theory, 301</td>
<td></td>
</tr>
<tr>
<td>recursively enumerable, 302</td>
<td></td>
</tr>
<tr>
<td>recursively solvable, 301</td>
<td></td>
</tr>
<tr>
<td>reflection schema, 265, 266</td>
<td></td>
</tr>
<tr>
<td>regular function, 286</td>
<td></td>
</tr>
<tr>
<td>regular minimization, 286</td>
<td></td>
</tr>
<tr>
<td>Robinson Arithmetic, see Q</td>
<td></td>
</tr>
<tr>
<td>Robinson, Julia, 196</td>
<td></td>
</tr>
<tr>
<td>Rosser, Barkley, 185, 341</td>
<td></td>
</tr>
<tr>
<td>Russell, Bertrand, 95, 96, 130, 131, 134, 211, 235, 273</td>
<td></td>
</tr>
<tr>
<td>Russell’s paradox, 96, 131</td>
<td></td>
</tr>
<tr>
<td>schema, pl. schemata, 63</td>
<td></td>
</tr>
<tr>
<td>Second Incompleteness Theorem, 6–7, 236, 249, 261</td>
<td></td>
</tr>
<tr>
<td>second-order</td>
<td></td>
</tr>
<tr>
<td>entailment, 208</td>
<td></td>
</tr>
<tr>
<td>Peano Arithmetic, see PA₂</td>
<td></td>
</tr>
<tr>
<td>self-halting problem, 329</td>
<td></td>
</tr>
<tr>
<td>sentence, 28</td>
<td></td>
</tr>
<tr>
<td>set theory, 5, 131, 134, 194, 227, 273, 277, 280</td>
<td></td>
</tr>
<tr>
<td>ZF, 135</td>
<td></td>
</tr>
<tr>
<td>ZFC, 273</td>
<td></td>
</tr>
<tr>
<td>Shapiro, Stewart, 283</td>
<td></td>
</tr>
<tr>
<td>Skolem, Thoralf, 93, 99</td>
<td></td>
</tr>
<tr>
<td>slim model, 214</td>
<td></td>
</tr>
<tr>
<td>Smiley, Timothy, 50</td>
<td></td>
</tr>
<tr>
<td>Solovay, Robert, 218</td>
<td></td>
</tr>
<tr>
<td>sound theory, 3, 32</td>
<td></td>
</tr>
<tr>
<td>arithmetically, 163</td>
<td></td>
</tr>
<tr>
<td>speed-up, 201–203, 218</td>
<td></td>
</tr>
<tr>
<td>squeezing argument, 355</td>
<td></td>
</tr>
<tr>
<td>state of Turing machine, 320</td>
<td></td>
</tr>
<tr>
<td>state-of-play function, 326, 329</td>
<td></td>
</tr>
<tr>
<td>successor function, 38</td>
<td></td>
</tr>
<tr>
<td>sufficiently expressive, 46</td>
<td></td>
</tr>
<tr>
<td>sufficiently strong, 50</td>
<td></td>
</tr>
<tr>
<td>super Gödel number, 139</td>
<td></td>
</tr>
<tr>
<td>surjective function, 8</td>
<td></td>
</tr>
<tr>
<td>syntax, 28</td>
<td></td>
</tr>
<tr>
<td>arithmetization of, 55, 136–141</td>
<td></td>
</tr>
<tr>
<td>T_A, 193</td>
<td></td>
</tr>
<tr>
<td>Tarski, Alfred, 196, 199, 200</td>
<td></td>
</tr>
<tr>
<td>Tarski’s Theorem, 197–199</td>
<td></td>
</tr>
<tr>
<td>term, 28, 38</td>
<td></td>
</tr>
<tr>
<td>closed, 38</td>
<td></td>
</tr>
<tr>
<td>theorem, 32</td>
<td></td>
</tr>
<tr>
<td>theory</td>
<td></td>
</tr>
<tr>
<td>axiomatized, 27, 31, 161</td>
<td></td>
</tr>
<tr>
<td>categorical, 213</td>
<td></td>
</tr>
<tr>
<td>complete, 2, 32</td>
<td></td>
</tr>
<tr>
<td>consistent, 32</td>
<td></td>
</tr>
<tr>
<td>effectively axiomatized, 31, 301</td>
<td></td>
</tr>
<tr>
<td>effectively decidable, 32</td>
<td></td>
</tr>
<tr>
<td>extend, 81, 164, 194</td>
<td></td>
</tr>
<tr>
<td>formal, 27, 31, 161, 301</td>
<td></td>
</tr>
<tr>
<td>good, 52</td>
<td></td>
</tr>
<tr>
<td>nice, 165</td>
<td></td>
</tr>
<tr>
<td>nice', 301, 304</td>
<td></td>
</tr>
<tr>
<td>nice*, 250</td>
<td></td>
</tr>
<tr>
<td>nice*, 261</td>
<td></td>
</tr>
<tr>
<td>of rational fields, 195</td>
<td></td>
</tr>
<tr>
<td>of real closed fields, 195</td>
<td></td>
</tr>
<tr>
<td>p.r. adequate, 124</td>
<td></td>
</tr>
<tr>
<td>recursively adequate, 298</td>
<td></td>
</tr>
<tr>
<td>recursively axiomatized, 301</td>
<td></td>
</tr>
<tr>
<td>recursively decidable, 301</td>
<td></td>
</tr>
<tr>
<td>sound, 3, 32</td>
<td></td>
</tr>
<tr>
<td>total function, 8, 15</td>
<td></td>
</tr>
<tr>
<td>transfinite induction, 224</td>
<td></td>
</tr>
<tr>
<td>True Basic Arithmetic, 193, 309</td>
<td></td>
</tr>
<tr>
<td>Turing, Alan, 16, 17, 303, 310, 342, 343, 347</td>
<td></td>
</tr>
</tbody>
</table>
Index

Turing machine, 16, 310–320
  halting problem, 330
  self-halting problem, 329
  state of, 320
Turing program, 313
dextral, 321
Turing’s Thesis, 17, 314, 338
Turing-computable function, 314
unbounded search, 106, 285
universal closure, 84
Uspenskii, V. A., 359
von Neumann, John, 200

Waismann, Friedrich, 349
Wang, Hao, 283
weakly-conservative, 275
wff, 14
  atomic, 38
  closed, 28
  Goldbach type, 78, 155
  open, 40
Whitehead, Alfred N., 131, 134, 273
Wiles, Andrew, 102, 172
Zach, Richard, 31, 269
ZFC, ZFC set theory, 135, 273