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978-1-107-02040-5 - A Basic Course in Measure and Probability: Theory for Applications

Ross Leadbetter, Stamatis Cambanis and Vlasos Pipiras

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