A Basic Course in Measure and Probability

Originating from the authors' own graduate course at the University of North Carolina, this material has been thoroughly tried and tested over many years, making the book perfect for a two-term course or for self-study. It provides a concise introduction that covers all of the measure theory and probability most useful for statisticians, including Lebesgue integration, limit theorems in probability, martingales, and some theory of stochastic processes. Readers can test their understanding of the material through the 300 exercises provided.

The book is especially useful for graduate students in statistics and related fields of application (biostatistics, econometrics, finance, meteorology, machine learning, etc.) who want to shore up their mathematical foundation. The authors establish common ground for students of varied interests, which will serve as a firm "take-off point" for them as they specialize in areas that exploit mathematical machinery.

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Theory for Applications

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Preface

This work arises from lecture notes for a two semester basic course sequence in Measure and Probability Theory given for first year Statistics graduate students at the University of North Carolina, evolving through many generations of handwritten, typed, mimeographed, and finally LaTeX editions. Their focus is to provide basic course material, tailored to the background of our students, and influenced very much by their reactions and the changing emphases of the years. We see this as one side of an avowed department educational mission to provide solid and diverse basic course training common to all our students, who will later specialize in diverse areas from the very theoretical to the very applied.

The notes originated in the 1960's from a "Halmos style" measure theory course. As may be apparent (to those of sufficient age) the measure theory section has preserved that basic flavor with numerous obvious modernizations (beginning with the early use of the Sierpinski-type classes more suited than monotone class theorems for probabilistic applications), and exposition more tailored to the particular audience. Even the early "Halmos framework" of rings and σ-rings has been retained up to a point since these notions are useful in applications (e.g. point process theory) and their inclusion requires no significant further effort. Integration itself is discussed within the customary σ-field framework so the students have no difficulty in relating to other works.

Strong opinions abound as to how measure theory should be taught, or even if it should be taught: its existence was once described by a Danish statistical colleague as an "unfortunate historical accident" and by a local mathematician as an "unnatural way of approaching integration." In particular he felt that the Caratheodory extension "was not natural" since, as he expressed it "If Caratheodory had not thought of it, I wouldn't have either!"

Perhaps more threatening is the "bottom line" climate in some of today's universities suggesting that training in measure-theoretic probability and statistical theory belongs to the past and should be deemphasized in favor of concentrated computational training for modern project-oriented activity. In this respect we can point with great pride to the many of our graduates making substantial statistical contributions in applications ascribable in (excuse us) "significant measure" to a solid theoretical component in their training. Moreover we ourselves see rather dramatic enrollment increases in our graduate probability courses from students in other disciplines in our own university and beyond, in fields such as financial mathematics with basic probability prerequisite. These (at least local) factors suggest a continuing role for both basic and more advanced course offerings, with the opportunity for innovative selection of special topics to be included.

Our viewpoint regarding presentation, much less single minded than some, is that we would teach (even name) this subject differently according to the particular audience needs. Based on the typical "advanced calculus" and "operational probability" backgrounds of our own students we prefer an essentially non-topological measure theory course followed by one in basic probability theory. For those of a more mathematical bent, the beautiful interplay between measure, topology (and algebra) can be studied at a later stage and is not a substantial part of our standard training mission for first year statistics graduate students. This organization has the incidental advantage that those who do further study have gained an understanding of which arguments (such as the central "σ-ring game") are measure theoretic in nature in contrast to being topological, or algebraic.

Our aim in the first semester is to provide a comprehensive account of general measure and integration theory. This we see as a quite well and naturally defined body of topics, generalizing much of standard real line Lebesgue integration theory to abstract spaces. Indeed a valuable byproduct is that a student may automatically acquire an understanding of real line Lebesgue integration and its relationship to Riemann theory, made visible by a supply of exercises involving real line applications. We find it natural to first treat this body of (general measure) theory, giving advance glimpses from time to time of the probabilistic context. Some authors prefer the immediacy of probabilistic perspective attainable from a primary focus on probability in development *ab initio*, with extensions to general measures being indicated to the degree desired. This is primarily a question of purpose and taste with pros and cons. The only viewpoint we would strongly disagree with is that there exists a uniformly best didactic approach.

In the context of "measure theory" we view σ-finiteness as the "natural norm" for the statement of results, and finite measures as (albeit important) special cases. This, naturally, changes in the second part with primary focus on probability measures and more special resulting theory. In addition to the specialization of general measure theoretic results to yield the basic framework for probability theory there is, of course, an unlimited variety of results which may be explored in the purely probabilistic context and one may argue about which are truly central and a *sine qua non* for a one-semester treatment. There would probably be little disagreement with the topics we have included as being necessary and desirable knowledge, but they certainly cannot be regarded as sufficient for all students. Again our guiding principle has been to provide a course suited as common ground for our students of varied interests and serving as a "take-off point" for them as they specialize in areas ranging from applied statistics to stochastic analysis.

For a course one has to decide whether to emphasize basic ideas, details, or both. We have certainly attempted to strongly highlight the central ideas; if we have erred it is in the direction of including as complete details as possible, feeling that these should be seen at least once by the students. For example, detailed consideration of sets of measure zero, of possibly infinite function values and the specific identification of XxYxZ with (XxY)xZ are not necessarily issues of lasting emphasis in practice but we think it appropriate and desirable to deal with them carefully when introduced in a course. As will be clear, it has not been our intention to produce yet one more comprehensive book on this subject. Rather we have used the facilities of modern word processing as encouragement to give our lecture notes a better organized and repeatedly updated basic course form in the hope that they (and now this volume) will be the more useful to our own students, for whom they are designed, and to others who may share our educational perspectives.

Finally, it is with more than a twinge of sadness that this preface is written in the absence of coauthor Stamatis Cambanis, without whom the lecture notes would not have taken on any really comprehensive form. From the rough (mainly measure - theoretic) notes prepared by MRL in the 1960's, SC and MRL worked together in developing the notes from the mid-1970's as they taught the classes, until Stamatis' untimely death in 1995.

Stamatis Cambanis was a wonderfully sensitive human being and friend, with unmatched concern to give help wherever and whatever the need. He was also The Master Craftsman in all that he did, his character echoing the words of Aristotle: "Είμαστε αυτό που πράττουμε επανειλημμένα. Έτσι, η τελειότητα δεν είναι πράξη αλλά συνήθεια." *(We are what we repeatedly do. Excellence then is not an act but a habit.)*

M.R.L., V.P.

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It is indeed hazardous to list acknowledgements in a work that has been used in developing form for almost half a century, and we apologize in advance for inevitable memory lapses that have caused omissions. It goes without saying that we are grateful to generations of questioning students, often indicating some lack of clarity of exposition in class or in the notes, and leading to needed revisions. Some have studied sections of special interest to them and not infrequently challenged details or phrasing of proofs – again leading to improvements in clarity. In particular Chihoon Lee undertook a quite unsolicited examination of the entire set of notes and pointed out many typographic and other blemishes at that time. Xuan Wang reviewed the entire manuscript in detail. We are especially grateful to Martin Heller who critically reviewed the entire set of book proofs and has prepared a solution set for many of the exercises.

Typing of original versions of the notes was creatively done by Peggy Ravitch and Harrison Williams, who grappled with the early mysteries of LaTeX, pioneered its use in the department, and constantly found imaginative ways to outwit its firm rules. Further residual typing was willingly done by Jiang Chen, James Wilson and Stefanos Kechagias, who also doubled as Greek linguistics advisor. It is a pleasure to record the encouragement and helpful comments of our colleague Amarjit Budhiraja who used the notes as supplementary material for his classes, and the repeated nagging of Climatologist Jerry Davis for publication as a book, as he used the notes as background in his research.

We are especially grateful to the Institute of Mathematical Statistics and the Editors of the IMS Lecture Note Series Anirban DasGupta and the inimitable Susan Murphy for their enthusiasm for production as a volume, and for the conversion of the entire manuscript from older LaTeX and hand corrected pdf files into the new format, through Mattson Publishing Company, the ever patient and gracious Geri Mattson, and the magical group VTeX. In particular we thank IMS Executive Director Elyse Gustafson for her quiet efficiency, willing support and generously provided advice when needed, and Sir David Cox for his ready encouragement as coordinating editor of the new IMS Monograph and Textbook series, in cooperation with Cambridge University Press.

We shall, of course, be most grateful for any brief alert (e.g. to mrl@email.unc.edu or pipiras@email.unc.edu) regarding remaining errors, blemishes or inelegance (which will exist a.s. in spite of years of revision!) as well as general reactions or comments a reader may be willing to share.