

Behavioral Rationality and Heterogeneous Expectations in Complex Economic Systems

Recognizing that the economy is a complex system with boundedly rational interacting agents, the book presents a theory of behavioral rationality and heterogeneous expectations in complex economic systems and confronts the nonlinear dynamic models with empirical stylized facts and laboratory experiments. The complexity modeling paradigm has been strongly advocated since the late 1980s by some economists and by multidisciplinary scientists from various fields, such as physics, computer science and biology. More recently the complexity view has also drawn the attention of policy makers, who are faced with complex phenomena, irregular fluctuations and sudden, unpredictable market transitions. The complexity tools – bifurcations, chaos, multiple equilibria – discussed in this book will help students, researchers and policy makers to build more realistic behavioral models with heterogeneous expectations to describe financial market movements and macroeconomic fluctuations, in order to better manage crises in a complex global economy.

CARS HOMMES is Professor of Economic Dynamics at the University of Amsterdam (UvA). After his PhD in Mathematical Economics at the University of Groningen, he founded the Center for Nonlinear Dynamics in Economics and Finance (CeNDEF), an interdisciplinary research group at UvA, pursuing theoretical, experimental and empirical research on complex systems, bounded rationality and behavioral agent-based models in economics and finance.

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Cars Hommes

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“Professor Hommes’ work is a major contribution to the understanding of intertemporal economic fluctuations. In a world in which production and investment behavior is motivated by expectations of the future, the way those expectations are formed becomes of the utmost importance. These expectations lead to dynamic systems, and the author draws on the rich literature developed for the study of mechanical and gravitational phenomena. These lead to the emergence of very complex behavior in markets driven by expectations, especially when different economic agents have different modes of forming expectations from data. The study of this book will have a profound impact on the theoretical and empirical analysis of securities markets and other forms of investment.”

Kenneth J. Arrow, Joan Kenney Professor of Economics and Professor of Operations Research, Emeritus, Stanford University. Winner of the Nobel Prize in Economics 1972

“Cars Hommes has written an excellent book that is a combination of theory, economic modeling and economic experiments. The book is an outgrowth of a course on Nonlinear Economic Dynamics that he has given mostly at the University of Amsterdam for the last 20 years.”

Professor Carl Chiarella, Head of Finance Discipline Group, University of Technology, Sydney

“Henri Poincaré, the great French mathematician and father of non linear dynamic analysis, at the turn of the 20th century chided Walras for his unrealistic assumptions about how individuals make their decisions. He also declared that Bachelier’s random walk hypothesis for financial markets overlooked the tendency of people to act like sheep. Yet Walras and Bachelier are, with reason, regarded as the founders of modern economic and financial theory.

Cars Hommes’ excellent book puts us firmly back on the path that we should have followed had we heeded Poincaré’s warnings and built our economic theory on the foundations that he laid. The book’s careful formal analysis, empirical and experimental evidence provides a solid basis for understanding the volatile evolution of economies. It provides the framework for a better understanding of how economies do actually behave rather than how current economic theory says they should behave. This book could not have come at a more opportune moment.”

Alan Kirman, Professor Emeritus of Economics at Université d’ Aix-Marseille III, France and Director of Studies at Ecole des Hautes Etudes en Sciences Sociales

“Assumptions about the homogeneity of individuals’ expectations have limited economic modeling for some time. In this very complete book, Cars Hommes shows the reader how the world of heterogeneous expectations works in several different contexts. It distinguishes itself by covering theory along with empirical and experimental validation. Researchers interested in getting up to speed in this relatively new area of economics will find this book an excellent overview and tutorial.”

Professor Blake LeBaron, International Business School, Brandeis University

“Without doubt, rational expectations has been a powerful and useful assumption in pushing applied work forward in the last 40 years. But positing that agents have heterogeneous beliefs that deviate from the measure implied by a model opens up new possibilities that promise to allow us to resolve some of our many remaining puzzles about asset prices and quantities. Cars Hommes’ book is a leading example of how productive this approach can be.”

Thomas J. Sargent, W. R. Berkley Professor of Economics and Business, New York University and Senior Fellow, Hoover Institution, Stanford University. Winner of the Nobel Prize in Economics 2011

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Voor Annelies, Thomas & Saar

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Preface

This book has a long history. It grew out of courses on Nonlinear Economic Dynamics (NED), which I have been teaching in the past 20 years at the University of Amsterdam (UvA) and various other places. The NED course has been part of the MSc Econometrics program of the Amsterdam School of Economics, University of Amsterdam since I started at UvA in 1992. I have also taught a condensed version of NED bi-annually between 1996 and 2004 in the Network Algemene en Kwantitatieve Economie (NAKE), a quantitative network of economics PhD courses in the Netherlands. Since 2004 the NED course has been part of the Graduate Program of the Tinbergen Institute, the Graduate school in Economics, Econometrics and Finance in Amsterdam and Rotterdam. More recently, much of the material in this book has been taught at various summerschools and lecture series, in particular the Advanced School on Nonlinear Dynamical Systems in Economics, Udine, Italy, June 2004, the Lecture Series on Heterogeneous Agent Models, Pisa, Italy, June 2006, the Trento Summerschool on Agent-based Finance, Trento, Italy, July 2007 and the International School on Multidisciplinary approaches to Economic and Social Complex Systems, Siena, Italy, June 2010.

I am grateful to many colleagues and friends for inspiration and help over more than two decades. My main PhD thesis advisor at the University of Groningen, Helena Nusse, raised my enthusiasm for chaos and complexity. In Groningen, Floris Takens further deepened my knowledge of nonlinear dynamics and strange attractors, and Ad Pikkemaat taught me the first lessons in mathematical economics. At the University of Amsterdam, this role was taken over by Claus Weddepohl, who was one of the first mathematical economists in the Netherlands and Europe recognizing the importance of nonlinear dynamics and complexity for economics.

I am most grateful to William “Buz” Brock for his inspiration and support over so many years. My visits to the University of Wisconsin, Madison, in the summers of 1994, 1995 and 1997 and our regular discussions thereafter over a coffee or a “spotted cow” either in Amsterdam or Madison, have been extremely stimulating and productive. Our joint work on bounded rationality and heterogeneous expectations in complex economic systems forms the theoretical basis of this book. Buz’s contributions go far

beyond science and his warm friendship has been another reason to keep coming back to Madison.

Since 1998 the Center for Nonlinear Dynamics in Economics and Finance (CeNDEF) provided a most stimulating research environment within the Amsterdam School of Economics at UvA. The CeNDEF group has not only further explored the theory and applications of nonlinear dynamics and complexity in economics, but has also brought these models to the data by testing them with empirical time series data and laboratory experiments with human subjects. At the start of CeNDEF, experimental and empirical work for me were a “jump in the dark” and this book has benefitted enormously from my almost daily discussions and joint work in the past 15 years with CeNDEF researchers, coauthors and friends, particularly with Mikhail Anufriev, Peter Boswijk, Cees Diks, Maurice Koster, Roald Ramer, Joep Sonnemans, Jan Tuinstra and Florian Wagener. I have been fortunate with continuous intellectual challenges from excellent PhD students and postdocs at CeNDEF and would like to thank Tiziana Assenza, Te Bao, Adriana Cornea, Pietro Dindo, Gerwin Griffioen, Peter Heemeijer, Sander van der Hoog, Tatiana Kiseleva, David Kopanyi, Marco van der Leij, Michiel van der Leur, Tomasz Makarewicz, Sebastiano Manzan, Domenico Massaro, Saeed Mohammadian Moghayer, Marius Ochea, Valentyn Panchenko, Raoul Philipse, Daan in’t Veld, Henk van de Velden, Robin de Vilder, Juanxi Wang, Roy van der Weide, Marcin Wolski, Paolo Zeppini, Mei Zhu and Ilija Zovko.

Complexity, bounded rationality and heterogeneity are new and still somewhat controversial topics in economics and my work benefitted greatly from many stimulating discussions, encouragement and joint work with many colleagues and friends: Jasmina Arifovic, Volker Böhm, Giulio Bottazzi, Jean Philip Bouchaud, Bill Branch, Jim Bullard, Serena Brianzoni, Carl Chiarella, Silvano Cincotti, David Colander, Herbert Dawid, Dee Dechert, Paul DeGrauwe, Domenico Delli-Gatti, Roberto Dieci, Giovanni Dosi, Edward Droste, John Duffy, George Evans, Doyne Farmer, Gustav Feichtinger, Mauro Gallegati, Laura Gardini, Andrea Gaunersdorfer, Jacob Goeree, David Goldbaum, Jean-Michel Grandmont, Roger Guesnerie, Tony He, Dirk Helbing, Thorsten Hens, Seppo Honkapohja, Hai Huang, Ken Judd, Alan Kirman, Mordecai Kurz, Yuri Kuznetsov, Laurence Laselle, Blake LeBaron, Axel Leijonhufvud, Marji Lines, Thomas Lux, Rosario Mantegna, Bruce McGough, Alfredo Medio, Paul Ormerod, Damjan Pfajfar, J. Barkley Rosser, Klaus-Reiner Schenk-Hoppé, Andras Simonovits, Gerhard Sorger, Didier Sornette, Shyam Sunder, Leigh Tesfatsion, Fabio Tramontana, Miroslav Verbic, Duo Wang, Frank Westerhoff, Remco Zwinkels and many others.

I hope this book will provide the readers with some of the excitement about nonlinear dynamics and complex systems in economics and finance that I have experienced over the years. The book should not be seen as an in-depth mathematical treatment of nonlinear dynamics, but rather as a collection of the most important and relevant tools to be applied by researchers and policy makers in economics and finance. In the courses I have been teaching about the subject, computer simulations have always played an important role for students as an illustration of the concepts and

richness of nonlinear dynamics. Most of the figures in this book have been generated by the *E&F Chaos* software package, jointly developed at CeNDEF with Cees Diks, Valentyn Panchenko and Roy van der Weide, and freely downloadable at <http://www1.fee.uva.nl/cendef/http://www1.fee.uva.nl/cendef/>.

A special word of thanks goes to Dávid Kopányi, for his assistance in the last year with carefully editing the text and especially producing many illuminating figures and illustrations in the book. Without his help the book would still be unfinished. In addition, I would like to thank Chris Harrison and Phil Good at CUP for their technical support and patience. I gratefully acknowledge financial support for many years of complexity research by the Netherlands Organization for Scientific Research (NWO) and the EU through several FP6 and FP7 EU grants.

Finally and most of all, I thank Annelies, Thomas and Saar for their love and patience over so many years. They are my stable attractors in a complex world.