CAMBRIDGE
Cambridge University Press & Assessment
978-1-107-01880-8 — Applied Optimization Methods for Wireless Networks
Y. Thomas Hou , Yi Shi , Hanif D. Sherali
Excerpt
More Information

CHAPTER 1 Introduction

The present moment is the only moment available to us, and it is the door to all moments.

Thich Nhat Hanh

1.1 Book overview

The goal of this book is to offer a course to graduate students by demonstrating an important set of mathematical tools (with a focus on optimization techniques) and to show how they can be used to address some challenging problems in wireless networks.

Book organization This book consists of four parts.

- Part I, consisting of three chapters, is devoted to optimization and designing algorithms that can offer optimal solutions.
- Part II, consisting of five chapters, is devoted to techniques that can offer provably near-optimal solutions.
- Part III, consisting of two chapters, is devoted to some highly effective heuristics.
- Part IV, consisting of only one chapter, is devoted to some miscellaneous topics in the broader context of wireless network optimizations. This part will be expanded in a future edition.

Structure of each chapter Each chapter starts with a brief overview of a particular optimization technique and subsequently is followed by a comprehensive coverage of a case study in wireless networking. The goal of giving a pointer to the underlying theory at the beginning of each chapter is to offer a direction to students on what they should explore further in formal course work or textbooks in these areas. These pointers are not meant to be comprehensive tutorials, each of which could constitute a book on its own. The first section of each chapter is not meant to be a replacement of formal courses in operations research (OR) or computer science (CS) and certainly does not offer a short cut for students in their formal education in these subject areas.

Instead, the focus of each chapter is on how to apply the technique under discussion to solve a challenging problem in wireless networking. Each chapter is self-contained and shows all the details involved in problem formulation, reformulation, and customization of optimization techniques in order to devise a final solution. Our guiding principles in the choice of a case study in each chapter are the following:

- To reflect state-of-the-art wireless network research rather than discuss some classic but outdated problems that are no longer of current research interest. In this spirit, the problems that we chose are mainly in the context of multi-hop wireless networks, with the underlying wireless technologies being cognitive radio (CR), multiple-input multiple-output (MIMO), and cooperative communications (CC), among others.
- To offer a reasonable level of difficulty or challenge in each case study rather than a simple problem that the students would hardly encounter in their research. This is in contrast to simple and diverse examples that are typically presented in standard optimization or algorithmic textbooks. In this book, we want to show readers how the method introduced in each chapter can be applied to solve a hard wireless networking problem, which they are likely to encounter in research. For this purpose, each chapter is dominated by a case study in terms of length coverage. We believe this approach will help readers better appreciate the underlying method and to gain a better understanding on its application in practice.
- To offer only essential background on the underlying wireless communication technology that is needed in formulating the problem rather than a comprehensive overview of the technology. This is because the main goal of this book is to learn various optimization techniques and apply each one to solve a wireless networking problem as a case study rather than offering a comprehensive tutorial on various wireless communication technologies. Therefore, we decided to minimize the coverage on wireless technologies and offer references that the readers can study further on their own.
- To help readers develop strong problem formulation and reformulation skills. This can only be taught with examples with sufficient sophistication and complexity. We believe such formulation/reformulation skills are important for research and thus want to teach the readers such skills in detail in each case study.

Key characteristics of the book

- Presents a collection of useful optimization techniques (one technique in each chapter), with an emphasis on how each technique is put into action to solve challenging wireless networking problems.

- Combines techniques from both operations research (OR) and computer science (CS) disciplines, with a strong focus on solving optimization problems in wireless networks.
- Shows various tricks and step-by-step details on how to develop optimization models and reformulations, particularly in the context of cross-layer optimization problems involving flow routing (network layer), scheduling (link layer), and power control (physical layer).
- Discusses case studies that focus on multi-hop wireless networks (e.g., ad hoc and sensor networks) and incorporates a number of advanced physical layer technologies such as MIMO, cognitive radio (CR), and cooperative communications (CC).
- Contains problem sets at the end of each chapter. PowerPoint slides for each chapter are available to both the students and instructors. A solutions manual is available to the instructors.

1.2 Book outline

This book has four parts. Part I of this book, consisting of Chapters 2 to 4, is devoted to optimization and designing algorithms that can offer optimal solutions.

- Chapter 2 reviews linear programming (LP) and shows how it can be employed to solve certain problems in wireless networks. Although the LP methodology itself is rather basic and straightforward, special care is still needed to ensure that it is used correctly, as we demonstrate in the case study in this chapter. The case study is rather interesting as it shows that even in LP-based problem formulations, deep insights can be gained once we dig deep into it. In particular, the case study considers lexicographic max-min (LMM) rate allocation and node lifetime problems in a wireless sensor network (WSN). We introduce the *parametric analysis* (PA) technique, which is very useful in its own right. The concept of LMM is also important, and can be employed as a fairness criterion for other problems in wireless networking research. Through the case study in this chapter, the readers will gain a rather deep understanding of LP and its applications in wireless networks.
- Chapter 3 reviews convex programming, which is a popular and powerful tool for studying nonlinear optimization problems. Once a problem is shown to be a convex program, then there are standard solution techniques and we may even directly apply a solver to obtain an optimal solution. For many cross-layer convex optimization problems, the research community is more interested in exploring a solution in its dual domain. There are two reasons for this approach. First, many cross-layer problems, once properly formulated in the dual domain, can be decomposed into subproblems, each of which may be decoupled from the other layers. Such a

layering-based decomposition offers better insights and interpretations for the underlying problems. Second, once a problem is decomposed in its dual domain, the solution may be implemented in a distributed fashion, which is a highly desirable feature for networking researchers. In the case study of this chapter, we study a cross-layer optimization problem for a multi-hop MIMO network, which involves variables at the transport, network, link, and physical layers. We show that the problem can be formulated as a convex program. By studying the problem in its dual domain, we show that the dual problem can be decomposed into two subproblems: one subproblem solely involving variables at the transport and network layers and the other problem involving variables at the link and physical layers. We describe how the dual problem can be solved by a cutting-plane method and how the solution to the primal problem can be recovered from the solution to the dual problem.

- Chapter 4 illustrates how an optimization problem can be solved by clever algorithmic techniques from CS. For certain problems, general optimization methods from OR may not always be the best approach. In fact, a formulation following OR's optimization approach may lead to a solution with nonpolynomial-time complexity. But a solution with nonpolynomial-time complexity does not mean that the problem is not in P. In fact, we may well develop a different algorithm to solve the problem with polynomial-time complexity. This is what we illustrate in this chapter, where we develop a polynomial-time algorithm. In the case study, we consider a relay node assignment problem in cooperative communications (CC). Our objective is to assign a set of available relay nodes to different source–destination pairs so as to maximize the minimum data rate among all the pairs. Following the OR optimization approach, we show that the problem can be formulated as a mixed-integer linear programming (MILP) problem, which is NP-hard in general. But this does not mean that the problem is NP-hard. Instead, by following a CS algorithm design approach, we develop a polynomial-time exact algorithm for this problem. A novel idea in this algorithm is a linear marking mechanism, which is able to achieve polynomial-time complexity at each iteration. We give a formal proof of the optimality of the algorithm.

Part II of this book, consisting of Chapters 5 to 9, is devoted to techniques that can offer provably near-optimal solutions.

- Chapter 5 presents the branch-and-bound framework and shows how it can be applied to solve discrete and combinatorial optimization problems. Such problems are typically considered most difficult in nonconvex optimization and the branch-and-bound framework offers a general-purpose and effective approach. The effectiveness of branch-and-bound resides in the careful design of each component of its framework, such as computation of a lower bound, local search of an upper bound, and selection of partitioning variables (in the case of a minimization problem). It should be noted that

Cambridge University Press & Assessment
978-1-107-01880-8 — Applied Optimization Methods for Wireless Networks
Y. Thomas Hou , Yi Shi , Hanif D. Sherali
Excerpt
More Information

the worst-case complexity of a branch-and-bound-based method remains exponential, although a judicious design of each component could achieve reasonable computational times in practice. In the case study, we consider a per-node power control problem for a multi-hop CRN. This problem has a large design space that involves a tight coupling relationship among power control, scheduling, and flow routing, which is typical for a cross-layer optimization problem. We develop a mathematical model and a problem formulation, which is a mixed-integer nonlinear programming (MINLP) problem. We show how to apply the branch-and-bound framework to design a solution procedure.

- Chapter 6 presents the Reformulation-Linearization Technique (RLT) for deriving tight linear relaxations for any monomial. Simply put, RLT can be applied to any polynomial term of the form $\prod_{i=1}^{n}(x_i)^{c_i}$ in variables x_i, where the c_i-exponents are constant integers. Given such generality, RLT is a powerful tool in deriving tight linear relaxations. In the case study, we consider a throughput maximization problem in a multi-hop CRN under the signal-to-interference-and-noise-ratio (SINR) model. We develop a mathematical formulation for joint optimization of power control, scheduling, and flow routing. We present a solution procedure based on the branch-and-bound framework and apply RLT to derive tight linear relaxations for a product of variables. In this case study, we also learn how to identify the core optimization space for the underlying problem and how to exploit different physical interpretations of the core variables in developing a solution.
- Chapter 7 presents a linear approximation algorithm, which is a powerful method to tackle certain nonlinear optimization problems. We show how such an approach could be employed to solve a nonlinear programming (NLP) problem in a wireless sensor network (WSN). In addition to the linear approximation technique, the problem in the case study is interesting on its own, and shows how the so-called wireless energy transfer technology can be employed to address network lifetime problems in a WSN.
- Chapter 8 shows how to design a polynomial-time approximation algorithm to provide an $(1-\varepsilon)$-optimal solution to a nonconvex optimization problem. The case study focuses on a classic base station placement problem in a WSN. The design of the $(1-\varepsilon)$-optimal approximation algorithm is based on several clever techniques such as discretization of cost parameters (and distances), partitioning of the search space into a finite number of subareas, and representation of subareas with fictitious points (with tight bounds on costs). These three techniques can be exploited to develop approximation algorithms for other problems. We prove that the approximation algorithm is $(1-\varepsilon)$-optimal.
- Chapter 9 is a sequel to Chapter 8. Again, our interest is on the design of a $(1-\varepsilon)$-optimal approximation algorithm for a mobile base station problem. But the problem is much harder than that in the last chapter. By allowing the base station to be mobile, both the location of the base station and the

multi-hop flow routing in the network are time-dependent. To address this problem, we show that as far as the network lifetime objective is concerned, we can *transform* the time-dependent problem to a location (space)-dependent problem. In particular, we show that flow routing only depends on the base station location, regardless of *when* the base station visits this location. Further, the specific time instances for the base station to visit a location are not important, as long as the total sojourn time for the base station to be present at this location is the same. This result allows us to focus on solving a location-dependent problem. Based on the above result, we further show that to obtain a $(1-\varepsilon)$-optimal solution to the location-dependent problem, we only need to consider a finite set of points within the smallest enclosing disk for the mobile base station's location. Here, we follow the same approach as that in Chapter 8, i.e., discretization of energy cost through a geometric sequence, division of a disk into a finite number of subareas, and representation of each subarea with a fictitious cost point (FCP). Then we can find the optimal sojourn time for the base station to stay at each FCP (as well as the corresponding flow routing solution) so that the overall network lifetime (i.e., sum of the sojourn times) is maximized via a single LP problem. We prove that the proposed solution can guarantee that the achieved network lifetime is at least $(1-\varepsilon)$ of the maximum (unknown) network lifetime. This chapter offers some excellent examples on how to transform a problem from time domain to space domain and how to prove results through construction.

Part III, consisting of Chapters 10 and 11, is devoted to some highly effective heuristics.

- Chapter 10 presents an effective approach to address a class of mixed-integer optimization problems. The technique, called sequential fixing (SF), is designed to iteratively determine (fix) binary integer variables. It is a heuristic procedure and has polynomial-time complexity. Its performance is typically measured by comparing its solution value to some performance bound, e.g., a lower bound for a minimization problem, or an upper bound for a maximization problem. Based on our own experience, we find that this SF technique is very efficient and can offer highly competitive solutions. As a case study, we study an optimization problem in a multi-hop CRN. Since the problem formulation is an MINLP model, we develop a lower bound to estimate the optimal objective value. Subsequently, we present an SF algorithm for this optimization problem. Numerical examples show that the solutions produced by this SF algorithm can offer objective values that are very close to the computed lower bounds, thus confirming their near-optimality.
- Chapter 11 presents metaheuristic methods, which are an important class of heuristic methods and have been applied to solve some very complex problems in wireless networks. In this chapter, we give a review of some well-

known metaheuristic methods (e.g., basic local search, simulated annealing (SA), tabu search (TS), and genetic algorithms (GA)). In the case study, we focus on developing a GA-based method to solve a multi-path routing problem for MD video. We find that a GA-based solution is eminently suitable to address this particular problem, which involves complex objective functions and exponential solution space. By exploiting the survival-of-the-fittest principle, a GA-based solution is able to evolve to a population of better solutions after each iteration and eventually offers a near-optimal solution.

Part IV, currently consisting of only Chapter 12, is devoted to some miscellaneous topics in the broader context of wireless network optimizations. This part will be further expanded to include other topics in a future book edition.

- Chapter 12 presents an asymptotic capacity analysis for wireless ad hoc networks. Such an analysis addresses an achievable per-node throughput when the number of nodes goes to infinity. We focus on so-called random networks, where each node is randomly deployed and each node has a randomly chosen destination node. In this asymptotic capacity analysis, the results are derived in the form of $\Omega(\cdot)$, $O(\cdot)$, and $\Theta(\cdot)$ and the underlying analysis is very different from what we do in the other chapters, which focus on optimization problems for *finite-sized* networks. We show that the asymptotic capacity analysis heavily depends on the underlying interference model. In this chapter, we consider three interference models (i.e., the protocol model, the physical model, and the generalized physical model) and show how to develop asymptotic capacity bounds for each model.

1.3 How to use this book

This book is written as a textbook and is mainly aimed at graduate students (particularly students in electrical and computer engineering) pursuing advanced research and study in wireless networks. The book could be adopted for a second or third graduate course in networking. The prerequisites for this course are a graduate course in networking.

At Virginia Tech, we have been this book in an Electrical and Computer Engineering (ECE) and Computer Science (CS) cross-listed course titled "ECE/CS 6570: Advanced Foundations of Networking." The prerequisite for this course is a graduate course in networking. We cover Chapters 2, 3, 4, 5, 8, 10, and 11 in a one semester course and the response from the students has been overwhelmingly positive! For each chapter, we have prepared PowerPoint slides, which are available to both the students and instructors. We have also prepared a solutions manual for all end-of-chapter problems, which is only available to the instructors. To access these materials, please visit www.cambridge.org/hou.

In addition to its primary role as a graduate textbook, researchers in academia who are active in conducting research in wireless networks will find this book a very useful reference to expand their toolboxes in problem solving. Further, researchers and engineers in industry and government laboratories who perform active research in wireless networks will also find this book to be a useful reference.

PART I

Methods for Optimal Solutions

CHAPTER

2 Linear programming and applications

The real voyage of discovery consists not in seeking new landscapes but in having new eyes.

Marcel Proust

2.1 Review of key results in linear programming

Linear programming (LP) is a problem consisting of a linear objective function and a set of linear constraints (equations or inequalities) with real variables. Such a problem aims to maximize or minimize a specific objective function by systematically choosing the values of the real variables within an allowed set (i.e., solution space). In an LP problem, both the objective function to be optimized and all the constraints restricting the variables are linear.

A general form of an LP problem is as follows:

$$\begin{aligned}
\text{Maximize} \quad & c_1x_1 + c_2x_2 + \ldots + c_nx_n \\
\text{subject to} \quad & a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \le b_1 \\
& a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \ge b_2 \\
& a_{31}x_1 + a_{32}x_2 + \ldots + a_{3n}x_n = b_3 \\
& \vdots \\
& a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = b_m \\
& x_j \ge 0 \quad (1 \le j \le n).
\end{aligned}$$

The function $c_1x_1 + c_2x_2 + \ldots + c_nx_n$ is the objective function to be maximized, where $c_1, c_2, \ldots, c_n$ are constant coefficients, and $x_1, x_2, \ldots, x_n$ are the so-called *decision variables* to be determined. The equality (or inequality) $\sum_{j=1}^{n} a_{ij}x_j =$ (or $\le$ or $\ge$) b_i is the ith constraint, where a_{ij} is a constant