STRONG-COUPLING THEORY OF HIGH-TEMPERATURE SUPERCONDUCTIVITY

High-temperature superconductivity has transformed the landscape of solid state science, leading to the discovery of new classes of materials, states of matter, and concepts. However, despite being over a quarter of a century since its discovery, there is still no single accepted theory to explain its origin. This book presents one approach, the strong-coupling or bipolaron theory, which proposes that high-temperature superconductivity originates from competing Coulomb and electron–phonon interactions. The author provides a thorough overview of the theory, describing numerous experimental observations, and giving detailed mathematical derivations of key theoretical findings at an accessible level. Applications of the theory to existing high-temperature superconductors are discussed, as well as possibilities of liquid superconductors and higher critical temperatures. Alternative theories are also examined to provide a balanced and informative perspective. This monograph will appeal to advanced researchers and academics in the fields of condensed matter physics and quantum field theories. A selection of colour figures is available online at www.cambridge.org/alexandrov.

ALEXANDRE S. ALEXANDROV was Professor of Theoretical Physics at Loughborough University, UK and Visiting Professor at UNICAMP, Campinas, Brasil. His research focused on high-temperature superconductivity, polarons, molecular electronics, colossal magnetoresistance, and charged Bose liquids.
Alexandre Alexandrov was born on 30 July 1946. He graduated from high school in Novgorod, Russia and was admitted to the Moscow Engineering Physics Institute (MEPhI), from where he graduated in 1970 cum laude. He was awarded his Ph.D. for a thesis on quantum transport in semiconductors in 1973. He then became interested in the theory of superconductivity, where he is known for developing a bipolaron theory. His first highly cited work on bipolarons in superconductors was conducted with colleagues in Grenoble in 1980 during his year-long stay in France. As noted by Bednorz and Muller in their Nobel address, these ideas inspired their successful search for high-Tc superconductivity in cuprates.

Towards the end of the Soviet Union, Alexandrov moved as a guest scientist to RWTH, Aachen, Germany, hosted by Herbert Cappelman, where he published a few seminal papers on bipolaronic superconductivity and on the Kohn–Luttinger mechanism of superconductivity. In 1992 he moved to Cambridge, UK, as Mott Bye-Fellow at Gonville and Caius College, where he started a fruitful collaboration with Sir Nevill Mott and other scientists in the Interdisciplinary Research Centre for Superconductivity, at the Cavendish Laboratory. In 1995 he moved to the Loughborough University of Technology as Professor and Chair of theoretical physics and later served there as Head of the Physics Department.

Alexandre Alexandrov’s main focus was on superconductivity, especially on mechanisms of high-Tc superconductivity, still a controversial topic. He was always a strong proponent of the decisive role of strong electron–phonon coupling (EPC) and the bipolaronic mechanism of high-Tc. He constructed a number of semi-phenomenological and microscopic models of bipolarons in HTSC that allowed explanation of various data that are difficult to interpret, using strongly correlated models without a strong EPC, e.g. an unconventional oxygen isotope effect, normal state pseudogap, specific heat features, etc. His deep critical analysis of many popular models of superconductivity and strongly correlated systems has generated a lot of interest and very lively discussion in the community.

He died on 15 August 2012, leaving a wife and son.

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Preface

The 1986 epoch-making discovery [1] by J. Georg Bednorz and K. Alex Müller of superconductivity in cuprates at incredibly high temperatures by the accepted standards is now recognised as one of the greatest scientific revolutions of the twentieth century. This discovery inspired a multitude of researchers worldwide to synthesise materials that are superconducting at temperatures more than six times higher than the earlier ones. High-temperature superconductivity has completely transformed the landscape of solid state science; it has led to the discovery of new classes of materials, of new states of matter, and of new concepts.

Strikingly, after more than 25 years of extensive experimental and theoretical efforts there is still little consensus on the origin of high-temperature superconductivity. The only consensus there is is that charge carriers are bound into pairs with an integer spin. Pairing of two fermionic particles has been evidenced in cuprate superconductors from the quantization of magnetic flux in units of the flux quantum [2]. Soon after that discovery the late Sir Nevill Mott answering his own question: ‘Is there an explanation?’ [Nature 327 (1987) 185] expressed the view that the Bose–Einstein condensation (BEC) of small bipolarons, predicted by us in 1981, could be the one. Several authors then contemplated BEC of real-space tightly-bound electron pairs, but with a purely electronic mechanism of pairing rather than with an electron–phonon interaction (EPI). However a number of other researchers criticised the bipolaron (or any real-space pairing) scenario as incompatible with some observed angle-resolved photoemission spectra (ARPES), with effective masses of carriers, and an unconventional symmetry of the superconducting order parameter in cuprates. Since then the controversial issue of whether the electron–phonon interaction is crucial for high-temperature superconductivity or whether it is weak and inessential has been one of the most challenging problems of contemporary condensed-matter physics. Unconventional symmetries of the order parameter allowed some researchers to maintain that a purely repulsive interaction between electrons provides superconductivity without phonons in
a number of high-temperature superconductors. However recent semi-analytical [3] and numerical (Monte Carlo) [4, 5] studies have shed strong doubts on the possibility of high-temperature superconductivity caused by repulsive interactions. Also a growing number of experimental and theoretical studies suggest that the true origin of high-temperature superconductivity should be found in a proper combination of the finite-range Coulomb repulsion with a significant finite-range EPI [6, 7].

This book is conceived as a fairly basic introduction to one of the modern theories of high-temperature superconductivity, the bipolaron theory, which extends the canonical BCS theory to strong electron–phonon coupling. A long time ago F. London suggested that the remarkable superfluid properties of 4He were intimately linked to the Bose–Einstein ‘condensation’ of the entire assembly of Bose particles [8]. The crucial demonstration that superfluidity was linked to the Bose particles and the Bose–Einstein condensation came after experiments on liquid 3He, whose atoms are fermions, which failed to show the characteristic superfluid transition within a reasonably wide temperature interval around the critical temperature for the onset of superfluidity in 4He. In sharp contrast, 3He becomes a superfluid only below a very low temperature of some 0.0026 K. Here we have a superfluid formed from pairs of 3He fermions below this temperature. The three orders-of-magnitude difference between the critical superfluidity temperatures of 4He and 3He kindles the view that the Bose–Einstein condensation might represent the ‘smoking gun’ of high-temperature superconductivity. Nowadays it seems rather natural that the first proposal for high-temperature superconductivity, made by Ogg Jr. in 1946, was the pairing of individual electrons [9]. If two electrons are chemically coupled together the resulting combination is a boson with integer total spin. Thus an ensemble of such two-electron entities can, in principle, be condensed into the Bose–Einstein superconducting condensate. This idea was further developed as a natural explanation of superconductivity by Schafroth [10], and Butler and Blatt [11] in 1955.

A breakthrough in the microscopic understanding of conventional superconductors arrived in 1957 when Bardeen, Cooper and Schrieffer proposed that two electrons in a superconductor were indeed correlated in real space, but on a very large (practically macroscopic) coherence length of about 10 000 times the average inter-electron spacing [12]. The BCS theory was derived from an early demonstration by Fröhlich [13] that conduction electrons in states near the Fermi energy attract each other on account of their weak interaction with vibrating ions of a crystal lattice. Cooper then showed that any two electrons were paired in momentum space due to their quantum interaction (i.e. the Pauli exclusion principle) with all other electrons in the Fermi sea. These Cooper pairs strongly overlap in real space, in sharp contrast with the model of non-overlapping real-space pairs discussed earlier by
Ogg Jr. and Schafroth, Butler and Blatt. A self-consistent derivation of the BCS master equation for the superconducting order parameter was proposed by Eliashberg in 1959 [14] based on Migdal’s theory of weakly coupled electrons and phonons [15]. In 1959 using the Green’s function formalism Gor’kov [16] reconciled the microscopic BCS theory with the phenomenological theory of superconductivity proposed earlier by Ginzburg and Landau (1950) [17] and extended by Abrikosov to type II superconductors [18]. This reconciliation was a real triumph of the BCS theory since the Ginzburg–Landau–Abrikosov phenomenology accounted remarkably well for the electromagnetic properties of conventional superconductors near the transition temperature.

Nowadays it has become clear that the Ogg–Schafroth and BCS descriptions are actually two opposite extremes of the electron–phonon interaction. Extending the BCS theory to embrace the strong interaction between electrons and ion vibrations, a Bose liquid of tightly bound electron pairs surrounded by the lattice deformation (i.e. of small bipolarons) was predicted by Alexandrov and Ranninger in 1981 [19, 20] with a further prediction in 1983 that high-temperature superconductivity should exist in the crossover region, of the electron–phonon interaction strength, from the BCS-like polaronic to bipolaronic superconductivity [21]. Many high-temperature superconductors are highly polarisable ionic lattices where the Fröhlich EPI [13] creates an effective attraction of carriers virtually equal to their long-range Coulomb repulsion. In ionic lattices like cuprate superconductors both interactions are quite strong (of the order of 1 eV) compared with rather low Fermi energies of doped carriers because of the poor screening by non- or near-adiabatic carriers. In these conditions the standard BCS theory is inapplicable.

Here the general multi-polaron theory is described in detail with both interactions being strong. It is demonstrated that the true origin of high-temperature superconductivity is found in a proper combination of the strong Coulomb repulsion with the equally strong finite-range Fröhlich EPI, and that the theory is fully compatible with key experimental observations in a great number of high-temperature superconductors. While in most analytical and numerical models of high-temperature superconductivity, proposed so far [22], both interactions have been introduced as input parameters not directly related to the material, this book presents the analytical multi-polaron theory of high-temperature superconductivity in highly polarisable ionic lattices with the generic unscreened Coulomb and Fröhlich interactions avoiding any ad hoc assumptions of their range and relative magnitude.

The impetus to write this book has come from my students on the course ‘High-temperature superconductivity’ and from my colleagues and participants of many international meetings on the subject, in particular those held in recent years: Materials and Mechanisms of Superconductivity (M2S-HTSC- V1), Houston, USA, February 2000; Major Trends in Superconductivity in the New Millennium,
Preface


The author has benefited from illuminating discussions with many colleagues. My long-standing collaboration with Alexander Andreev, Alexander Bratkovsky, Jozef Devreese, Peter Edwards, Jim Hague, Yurii Firsov, Viktor Kabanov, Pavel Kornilovitch, Dragan Mihailovic, John Samson and Peter Zhao, and the support during the writing of the book by the European Union Framework Programme 7 (NMP3-SL-2011-263104- HINTS), the UNICAMP visiting professorship programme, by ROBOCON 2009-12 and Loughborough University (UK) are greatly appreciated. I am grateful to Elena Sladkovskaia and Maxim Alexandrov for their understanding and careful reading of the manuscript.

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