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PART I

Truth and indexicality

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CHAPTER I

Semantical indices

When we say

(1) It is raining in Waitarere

we are speaking in ordinary English. But our concern in this book is not with how to speak English per se. Still less is it how to teach the meaning of English sentences to anyone who does not understand English. We are simply using English as an example of a language in which sentences are being used which can illustrate the world–time parallel, and we choose English simply because readers of this book will already understand it. We are though concerned with how to analyse the truth or falsity of ordinary English sentences like (1). And we say that (1) is true at a time t iff[1] there is rain in Waitarere at t. In other words

(2) 'It is raining in Waitarere' is true at a time t iff it is raining in Waitarere at t,

i.e.,

(3) (1) is true at a time t iff it is raining in Waitarere at t.

Since what we want is an analysis of (1) we are treating (1) as a sentence of our *object language*. (3) is part of our *metalanguage*. One can present a grammar of Italian in English. Italian is the object language and English is the metalanguage. One can also present a grammar of English in English.[2]

[1] 'Iff' is a standard abbreviation in logic for 'if and only if'.

[2] If the grammar of English is stated in English it would of course be of no use in teaching English to someone who knows no English, but from that it does not follow that it would be incorrect. Readers unfamiliar with the language/metalanguage distinction might think that a sentence like (3) is contentless, or analytically true, but that is not so. If 'is raining' had meant what 'is sunny' means then, without any change in Waitarere's weather, it would change the truth value of (3). (Compare the riddle 'If you call a tail a leg, how many legs does a donkey have?' The answer is " 'Four'; calling a tail a leg does not make it one.")

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The metalanguage in (3) is a version of English which makes reference to truth at a time. In this chapter we are using as examples of our object language sentences of English, like (1). In later chapters we will introduce predicate tense and modal languages as the formalised object languages in which we will discuss the world–time parallel.

Although (3) has also been written in English there is good reason to think of the metalanguage as 'tenseless' in the sense that, where α is a sentence of the object language, a phrase like 'α is true at *t*' should be understood in such a way that it does not even make sense to speak of *when* it is so that α is true at *t*. The metalanguage, as in (3), speaks of a sentence as being true or false *at a time*, where the object language has a sentence, (1), which has no mention of moments of time. For any time, (1) is true at that time iff

(4) It is raining at *t*

is (absolutely) true when *t* is assigned the time in question, and that will be so iff it is raining in Waitarere at the time in question. Even though (4) expresses a temporal fact, since it will be true for some values of *t* and false for others, (4), unlike (1), is understood to be tenseless in that it makes no sense to ask *when* it is true.[3] In that respect it is like

(5) $2 + 2 = 4$

which is simply true, and where there is nothing in it which could be described as tense. (4) is true simpliciter if it is raining at the time assigned to *t* and false simpliciter if it is not. A genuinely tenseless language is one which does not have the resources to speak of *when* its sentences are true. One could read (4) as '*t* is a moment of rain (at Waitarere)', but we have to bear in mind that *any English* explanation of the meaning of (4) is an explanation in a language which is inescapably tensed. What this means is that in saying that (4) is tenseless we are thinking of it as like (5) in that you can't sensibly ask *when* it is true.[4] For the same reason (3) is also tenseless, even though it speaks about a sentence (1) which is of course tensed, since our object language, English, is a tensed language.

At this point we should establish some of the logical terminology which we shall have occasion to make use of throughout this book. Begin with ∼.

[3] Wahlberg 2010 appears to suggest that this cannot be the end of the matter, since it does not address issues like what it would be for something to be so at a time.

[4] An example closer to our concerns of a genuinely tenseless metalanguage would be a language of first-order predicate logic (see pp. 51–55 and p. 216).

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Where α is a sentence ∼α means 'it is not so that α'. We can specify the meaning of an operator like ∼ by the principle:

(6) ∼α is true if α is false and false if α is true.

Other operators can be treated in a similar fashion:

(7) α ∧ β means 'it is so that both α and β' – so that α ∧ β is true if both α and β are true, and false if either one of them is false.
(8) α ∨ β means 'either α or β', in the sense that α ∨ β is true iff at least one of α or β is true.
(9) α ⊃ β means 'if α then β' in the sense that α ⊃ β is true iff α is false or β is true.
(10) α ≡ β means 'if α then β and if β then α' in the sense that α ≡ β is true iff α and β have the same truth value, i.e., iff they are both true or both false. ≡ represents 'iff'.

When tense is involved (6)–(10) need refinement since we cannot speak simply of truth and falsity, but we have to understand it to be truth or falsity at a time. So that (6) has to be stated as

(11) For any time t, ∼α is true at t if α is false at t, and false at t, if α is true at t.

Tense logicians use the letters F to mean 'it will one day be the case that', and P to mean 'it was once the case that'. With this notation we can express the sentence

(12) It once rained in Waitarere

as

(13) P(It is raining in Waitarere).

In saying this we make no claims about the syntax of English or any other natural language. Indeed that had better be so, since this is supposed to be a *logical* investigation of the world–time parallel, not a grammatical discussion about natural language. What our logical language does is provide a framework in which a sentence like (13) can be understood as built up from a simpler sentence in just the way that a formula of logic or mathematics can be built up out of simpler components. This reflects the view that we understand sentences of the languages we speak not by learning them one at a time, but by building them up from the words in them together with

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their grammatical structure. The languages of formal logic are too simple to provide a realistic model of natural language, but they are sufficient for studying the world–time parallel. Notice already a divergence between (13) and how we would express the past truth of (1) in English. If in English we were to say

(14) It was the case that it is raining at Waitarere

the present tense of 'is' is in tension with the prefix 'it was the case'. It would be more natural to say

(15) It was the case that it *was* raining in Waitarere

but our aim is to explain how the truth of (13) depends on the truth of (1) at this or that time.[5] For that purpose we write $t < t'$ to mean that time t is earlier than time t'. We can then say that (13) is true at a time t iff there is some t' such that $t' < t$ and (1) is true at t'. This is clearly independent of the meaning of the particular sentence in question, and we may state this as a schematic principle, where α is any sentence:

(16) $P\alpha$ is true at t iff there is some t' such that $t' < t$ and α is true at t'.

In this logic P is a sentential operator. We could put this by saying that in this logic *tense* is a feature of whole sentences. It is important to stress this since in later chapters we shall be discussing work which concerns whether verbs like 'is' are tensed or tenseless, and what is crucial to learn at this stage is that from the point of view of logic the tensed/tenseless distinction is one which can only be drawn at the level of whole sentences. This comes out more clearly in a sentence like

(17) A child prodigy studied at this school.

In (17) the obvious implication is that the person was a prodigy *at the time of studying*. So that whatever the tense is doing it must be captured in a

[5] This reflects A. N. Prior's view that the present is semantically basic. (See for instance Prior 1957, p. 10, and Prior 1968a (p. 171 in Prior 2003).) In a language like English it is more likely that P operates on a pre-tense form – something like ***shout Bugsy*** where ***shout*** is the infinitival form. Despite the simplicity of classical tense logic, natural language, as would be expected, needs refinements. Here is a more dramatic case of the phenomenon illustrated by (15). Assume that ***yesterday*** α is true at a time t iff, where t' is a moment in the day preceding the day in which t is a moment, α itself is true at t'. If 'Bugsy shouted yesterday' had the form ***yesterday P shout Bugsy*** it would be true at t iff Bugsy shouts at some t'' which precedes some t' where t' is on the day preceding the day in which t occurs – obviously the wrong result.

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manner which takes you back to a time when a person who was then a prodigy was then studying. (17) would be formalised as

(18) P(A child prodigy studies at this school).

What (18) makes clear is that the 'pastness' is captured only by P, and that not only is the *studying* something which happened in the past, which is what the surface English grammar might suggest, but also the being a prodigy. And indeed (18) gets the truth conditions right, since it is true at a time t iff there is some $t' < t$, such that

(19) A child prodigy studies at this school

is true at t'. There are of course refinements to do with what is called 'aspect', by which a distinction is made between 'studies' and 'is studying', and the semantics for the English tense system is far more complex than our little logical language allows. But within its limits it will suffice to illustrate the world–time parallel. We shall take up the appropriate representation of (19) in Chapter 5 when we introduce tense and modal predicate logics.

To express

(20) It will one day rain in Waitarere

we use the operator F to mean 'it will one day be the case that', and express (20) as

(21) F(It is raining in Waitarere).

We can say that for any sentence α

(22) $F\alpha$ is true at t iff there is some t' such that $t < t'$ and α is true at t'.

Then (21) is true at t iff there is some t' such that $t < t'$ and (1) is true at t'. In (20) the verb 'rain' is in the infinitival form, whereas in (21) the embedded (1) is in the present tense. 'Will' in English is what is called a 'modal' verb.[6] Modal verbs like ***will***, ***would***, ***can***, ***could***, ***may***, ***might*** and the like are marked by three features. (i) They do not have a separately marked third-person singular. (ii) They are negated by a *not* which follows the verb rather than by 'does not'. (iii) They take a 'naked infinitive'. Thus we say 'Bugsy might sneeze', not 'Bugsy mights to sneeze,' and we say

[6] Perhaps a reason for this difference between past and future is the view that *in the case of the future* actuality and possibility may be hard to tell apart. In any case examples with ***will*** in them will correspond with examples with other modal words.

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'Bugsy might not sneeze', rather than 'Bugsy doesn't might to sneeze'.[7] This already indicates the strength of the parallel, but at present all we need note are the similarities between past and future. A sentence like

(23) One day a child prodigy will study at this school

can be formalised by analogy with (17) as

(24) F(A child prodigy studies at this school).

Using P and F it is easy to define an operator H in such a way that $H\alpha$ means that α always *has* been so, and an operator G so that $G\alpha$ means that α is always *going* to be true. G and H are straightforwardly related to F and P:

$$G\alpha \equiv \sim F \sim \alpha$$
$$H\alpha \equiv \sim P \sim \alpha.$$[8]

To say that α is always going to be true is to say that it is not so that it will one day be false, and to say that α has always been true is to say that it is not so that it was once false. One can also introduce operators ***always*** and ***sometimes***, where ***always*** α means that α is and always has been and always will be so, and ***sometimes*** α means that α was either once so, or is now so, or will one day be so. ***Always*** α and ***sometimes*** α do not change their truth values with the passage of time. In place of *sometimes* we shall use $\otimes$, and in place of *always* we shall use $\oplus$. If time is linear we may define these operators in terms of F, P, G and H:

$$\otimes\alpha =_{\text{df}} (F\alpha \vee \alpha \vee P\alpha)$$
$$\oplus\alpha =_{\text{df}} (G\alpha \wedge \alpha \wedge H\alpha).$$

What we have so far may be called an *indexical* treatment of tense, in the sense that the semantics of tensed sentences is given by specifying whether they are true or false at a time. The time is what can be called a *semantical index*, or an *index of truth*. This is a neutral term because the question of what indices are needed is a somewhat open-ended one. By an index of truth we merely mean something with respect to which a sentence is determined for truth or falsity. Calling something an index of truth says

[7] An interesting case here is the verb ***need***, which can be either a modal or an 'ordinary' verb. Thus we can say either 'Tallulah need not be here tomorrow', or 'Tallulah doesn't need to be here tomorrow.'

[8] Alternatively we can take G and H as primitive and define F and P, which is how we shall proceed in Chapter 5.

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nothing about its intrinsic nature. In fact, keeping questions of the nature of the indices separate from their role as indices of truth enables us to study the logical parallel. (Other authors use the term 'indexical' differently from us, and we shall note these divergent uses from time to time.)

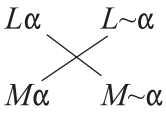
The indexical treatment can easily be extended to modality. If we use the symbol M for 'might' we can formulate

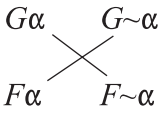
(25) A child prodigy might study at this school

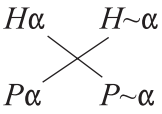
as

(26) M(A child prodigy studies at this school).

Modal notions involve possibility. Correlative with possibility is *necessity*, and we write $L\alpha$ to mean that it is necessary that α. Among truths, some are necessarily true, they couldn't have been otherwise. Others, while no less actually true, might easily have been false. And among falsehoods some, though false, might have been true – they are possible – while others are not possible, they could not have been true. The relations between necessity and possibility can be characterised by a *square of opposition*. The impossible is what *has to be false*, i.e. 'not possibly' means the same as 'necessarily not'. Similarly 'possibly not' means the same as 'not necessarily'. There are also similar squares in the temporal case:







The way to interpret these squares is this. In each case the top two formulae cannot both be true, while the bottom two formulae cannot both be false. Finally, each formula is equivalent to the negation of the diagonally opposite formula. As the definitions of G and H on p. 8 make clear, if α is always going to be true then it is not so that $\sim\alpha$ will ever be true, and anlogously with H – if α has always been so, then $\sim\alpha$ was never so, and so on.

Necessary truths include the facts of mathematics and logic. Among those actually true but not true by necessity are such facts as that someone is talking to you, or that the number 2 bus runs to Miramar. To account for modality we speak of sentences as true or false in different possible worlds. Thus

(27) The number 2 bus goes to Miramar

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is true in world *w* iff the number 2 bus does indeed go to Miramar in *w*. Since it doesn't *have* to do so there will be worlds in which it does not, and since it *could* do so there will be worlds in which it does. In the case of modality an extra complexity appears which does not seem to arise in the temporal case. That is the phenomenon of different senses of possibility. If we ask

(28) Can people fly to the moon?

one answer is yes, because it has been done. Another answer is no, because the equipment is not ready, even though it could have been ready. Take a sentence like

(29) Auckland might be the capital of New Zealand.

If someone asserts (29) it probably indicates ignorance. If they know their New Zealand geography they would know that Auckland isn't the capital. In order to express possibilities which are known not to be realised, English combines modal words with the past tense. So that the possibility that the decision to transfer to Wellington in 1865 might have gone the other way could be expressed as

(30) Auckland might have been the capital of New Zealand.

In ordinary English these sentences can be expressed by various combinations of modal words and tense auxiliaries like ***may***, ***might***, ***might have***, and so on. Yet what they all seem to have in common is that possibilities are *contextually constrained*. In (29) it is our knowledge which rules out certain possibilities; in (28) perhaps it is technological facts; in other cases it might be economic constraints. More basic will be the constraints of the laws of nature in the physical universe, and so on. The truths of mathematics and logic are a limiting case of senses of possibility. For there is *no* sense of 'possible' in which $2 + 3 = 4$, though of course the *symbol* '4' might have stood for the number 5. While we may not be able to jump more than 3 metres high, or travel faster than light, or disappear at will, these facts are because of the way the world is. The 'worlds' in which these things are so, although possible worlds, are not possible for us because of various constraints, and we will speak of *logical* possibility or *logical* necessity when we mean something which could or must be true without any constraints whatsoever.[9] A sentence α is said to be *necessary* in a world *w* provided α

[9] Some philosophers contrast logical with metaphysical necessity. We make no such distinction, and discuss this on p. 55.

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