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978-1-107-01688-0 — Predictive Control for Linear and Hybrid Systems
Francesco Borrelli , Alberto Bemporad , Manfred Morari
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Predictive Control

for Linear and Hybrid Systems

Model Predictive Control (MPC), the dominant advanced control approach in industry over the past 25 years, is presented comprehensively in this unique book. With a simple, unified approach, and with attention to real-time implementation, it covers predictive control theory including the stability, feasibility, and robustness of MPC controllers. The theory of explicit MPC, where the nonlinear optimal feedback controller can be calculated efficiently, is presented in the context of linear systems with linear constraints, switched linear systems, and, more generally, linear hybrid systems. Drawing upon years of practical experience and using numerous examples and illustrative applications, the authors discuss:

- The techniques required to design predictive control laws, including algorithms for polyhedral manipulations, mathematical, and multiparametric programming.
- How to validate the theoretical properties and to implement predictive control policies.

The most important algorithms feature in an accompanying free online MATLAB toolbox, which allows easy access to sample solutions. Predictive Control for Linear and Hybrid Systems is an ideal reference for graduate, postgraduate and advanced control practitioners interested in theory and/or implementation aspects of predictive control.

Francesco Borrelli is a chaired professor at the Department of Mechanical Engineering of the University of California, Berkeley. Since 2004 he has served as a consultant for major international corporations in the area of real-time predictive control. He was the founder and CTO of BrightBox Technologies Inc., and is the co-director of the Hyundai Center of Excellence in Integrated Vehicle Safety Systems and Control at UC Berkeley. His research interests include constrained optimal control, model predictive control and its application to advanced automotive control, robotics, and energy-efficient building operation.

Alberto Bemporad is a professor and former director of the IMT School for Advanced Studies Lucca. He has published numerous papers on model predictive control and its application in multiple domains. He has been a consultant for major automotive companies and cofounder of ODYS S.r.l., a company specialized in advanced control and optimization software for industrial production. He is the author or co-author of various MATLAB toolboxes for model predictive control design, including the Model Predictive Control Toolbox and the Hybrid Toolbox.

Manfred Morari was a professor and head of the Department of Information Technology and Electrical Engineering at ETH Zurich. During the last three decades he has shaped many of the developments and applications of model predictive control through his academic research and interactions with companies from a wide range of sectors. The analysis techniques and software developed in his group are used throughout the world. He received numerous awards and was elected to the National Academy of Engineering (US) and is a Fellow of the Royal Academy of Engineering (UK).

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University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

4843/24, 2nd Floor, Ansari Road, Daryaganj, Delhi – 110002, India

79 Anson Road, #06-04/06, Singapore 079906

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www.cambridge.org
Information on this title: www.cambridge.org/9781107016880
10.1017/9781139061759

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First published 2017

Printed in the United Kingdom by TJ International Ltd. Padstow Cornwall

A catalogue record for this publication is available from the British Library.

Library of Congress Cataloging-in-Publication Data

Names: Borrelli, Francesco, author. | Bemporad, Alberto, author. | Morari, Manfred, author.
Title: Predictive control for linear and hybrid systems / Francesco Borrelli, University of California, Berkeley, Alberto Bemporad, IMT Institute for Advanced Studies, Manfred Morari, Swiss Federal Institute of Technology (ETH).
Description: New York : Cambridge University Press, 2017. | Includes bibliographical references. | Includes bibliographical references and index.
Identifiers: LCCN 2016042160| ISBN 9781107016880 (Hardback) | ISBN 9781107652873 (Paperback)
Subjects: LCSH: Predictive control.
Classification: LCC TJ217.6 .B67 2017 | DDC 629.8–dc23 LC record available at https://lccn.loc.gov/2016042160

ISBN 978-1-107-01688-0 Hardback
ISBN 978-1-107-65287-3 Paperback

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To

Maryan, Federica and Marina

and our families

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Preface

Dynamic optimization has become a standard tool for decision making in a wide range of areas. The search for the most fuel-efficient strategy to send a rocket into orbit or the most economical way to start up a chemical production facility can be expressed as dynamic optimization problems that are solved almost routinely nowadays.

The basis for these dynamic optimization problems is a dynamic model, for example,

$$x_{k+1} = g(x_k, u_k), \;\; x_0 = x(0)$$

that describes the evolution of the state x_k with time, starting from the initial condition $x(0)$, as it is affected by the manipulated input u_k. Here, $g(x, u)$ is some nonlinear function. Throughout the book we are assuming that this discrete-time description, i.e., the model of the underlying system, is available. The goal of the dynamic optimization procedure is to find the vector of manipulated inputs $U_N = [u_0', ..., u_{N-1}']'$ such that the objective function is optimized over some time horizon N, typically

$$\min_{U_N} \sum_{k=0}^{N-1} q(x_k, u_k) + p(x_N)$$

The terms $q(x, u)$ and and $p(x)$ are referred to as the *stage cost* and *terminal cost*, respectively. Many practical problems can be put into this form and many algorithms and software packages are available to determine the optimal solution vector U_N^*, the *optimizer*. The various algorithms exploit the structure of the particular problem, e.g., linearity and convexity, so that even large problems described by complex models and involving many degrees of freedom can be solved efficiently and reliably.

One difficulty with this idea is that, in practice, the sequence of $u_0, u_1, ...$, which is obtained by this procedure cannot be simply applied. The model of the system predicting its evolution is usually inaccurate and the system may be affected by external disturbances that may cause its path to deviate significantly from the one that is predicted. Therefore, it is common practice to measure the state after some time period, say one time step, and to solve the dynamic optimization problem

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again, starting from the measured state $x(1)$ as the new initial condition. This *feedback* of the measurement information to the optimization endows the whole procedure with a *robustness* typical for closed-loop systems.

What we have described above is usually referred to as Model Predictive Control (MPC), but other names like Open Loop Optimal Feedback and Reactive Scheduling have been used as well. Over the last 25 years MPC has evolved to dominate the process industry, where it has been employed for thousands of problems [241].

The popularity of MPC stems from the fact that the resulting operating strategy respects all the system and problem details, including interactions and constraints, something that would be very hard to accomplish in any other way.

Indeed, often MPC is used for the regulatory control of large multivariable linear systems with constraints, where the objective function is not related to an economical objective, but is simply chosen in a mathematically convenient way, namely quadratic in the states and inputs, to yield a "good" closed-loop response. Again, there is no other controller design method available today for such systems that provides constraint satisfaction and stability guarantees.

One limitation of MPC is that running the optimization algorithm on-line at each time step requires substantial time and computational resources. Today, fast computational platforms together with advances in the field of operations research and optimal control have enlarged in a very significant way the scope of applicability of MPC to fast-sampled applications. One approach is to use tailored optimization routines which exploit both the structure of the MPC problem and the architecture of the embedded computing platform to implement MPC in the order of milliseconds.

The second approach is to have the result of the optimization precomputed and stored for each x in the form of a look-up table or as an algebraic function $u_k = f(x(k))$ which can be easily evaluated. In other words, we want to determine the (generally nonlinear) feedback control law $f(x)$ that generates the optimal $u_k = f(x(k))$ explicitly and not just implicitly as the result of an optimization problem. It requires the solution of the Bellman equation and has been a long-standing problem in optimal control. A clean, simple solution exists only in the case of linear systems with a quadratic objective function, where the optimal controller turns out to be a linear function of the state (Linear Quadratic Regulator, LQR). For all other cases a solution of the Bellman equation was considered prohibitive except for systems of low dimension (2 or 3), where a look-up table can be generated by gridding the state space and solving the optimization problem off-line for each grid point.

The major contribution of this book is to show how the nonlinear optimal feedback controller can be calculated efficiently for some important classes of systems, namely linear systems with constraints and switched linear systems or, more generally, hybrid systems. Traditionally, the design of feedback controllers for linear systems with constraints, for example, antiwindup techniques, was *ad hoc* requiring both much experience and trial and error. Though significant progress has been achieved on antiwindup schemes over the last decade, these techniques deal with input constraints only and cannot be extended easily.

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The classes of constrained linear systems and linear hybrid systems treated in this book cover many, if not most, practical problems. The new design techniques hold the promise to lead to better performance and a dramatic reduction in the required engineering effort.

The book is structured in five parts.

- In the first part of the book (Part I) we recall the main concepts and results of convex and discrete optimization. Our intent is to provide only the necessary background for the understanding of the rest of the book. The material of this part follows closely the presentation from the following books and lecture notes: "Convex Optimization" by Boyd and Vandenberghe [65], "Nonlinear Programming Theory and Algorithms" by Bazaraa, Sherali and Shetty [27], "LMIs in Control" by Scherer and Weiland [258] and "Lectures on Polytopes" by Ziegler [296].

  Continuous problems as well as integer and mixed-integer problems are presented in Chapter 1. Chapter 1 also discusses the classical results of Lagrange duality. In Chapter 2, linear and quadratic programs are presented together with their properties and some fundamental results. Chapter 3 introduces algorithms for the solution of unconstrained and constrained optimization problems. We only discuss those that are important for the problems encountered in this book and explain the underlying concepts. Since polyhedra are the fundamental geometric objects used in this book, Part I closes with Chapter 4, where we introduce the main definitions and the algorithms, which describe standard operations on polyhedra.

- The second part of the book (Part II) is a self-contained introduction to multiparametric programming. In our framework, parametric programming is the main technique used to study and compute state feedback optimal control laws. In fact, we formulate the finite time optimal control problems as mathematical programs where the input sequence is the optimization vector. Depending on the dynamical model of the system, the nature of the constraints, and the cost function used, a different mathematical program is obtained. The current state of the dynamical system enters the cost function and the constraints as a parameter that affects the solution of the mathematical program. We study the structure of the solution as this parameter changes and we describe algorithms for solving multiparametric linear, quadratic and mixed integer programs. They constitute the basic tools for computing the state feedback optimal control laws for these more complex systems in the same way as algorithms for solving the Riccati equation are the main tools for computing optimal controllers for linear systems. In Chapter 5, we introduce the concept of multiparametric programming and we recall the main results of nonlinear multiparametric programming. Then, in Chapter 6, we describe three algorithms for solving multiparametric linear programs (mp-LP), multiparametric quadratic programs (mp-QP) and multiparametric mixed-integer linear programs (mp-MILP).

- In the third part of the book (Part III) we introduce the general class of optimal control problems studied in the book. Chapter 7 contains the

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basic definitions and essential concepts. Chapter 8 presents standard results on Linear Quadratic Optimal Control, while in Chapter 9, unconstrained optimal control problems for linear systems with cost functions based on 1 and ∞ norms are analyzed.

- In the fourth part of the book (Part IV) we focus on linear systems with polyhedral constraints on inputs and states. We start with a self-contained introduction to controllability, reachability and invariant set theory in Chapter 10. The chapter focuses on computational algorithms for constrained linear systems and constrained linear systems subject to additive and parametric uncertainty.

  In Chapter 11 we study *finite time and infinite time* constrained optimal control problems with cost functions based on 2, 1 and ∞ norms. We first show how to transform them into LP or QP optimization problems for a fixed initial condition. Then we show that the solution to all these optimal control problems can be expressed as a *piecewise affine* state feedback law. Moreover, the optimal control law is continuous and the value function is convex and continuous. The results form a natural extension of the theory of the Linear Quadratic Regulator to constrained linear systems.

  Chapter 12 presents the concept of MPC. Classical feasibility and stability issues are shown through simple examples and explained by using invariant set methods. Finally, we show how they can be addressed with a proper choice of the terminal constraints and the cost function.

  The result in Chapter 11 and Chapter 12 have important consequences for the implementation of MPC laws. Precomputing off-line the explicit piecewise affine feedback policy reduces the on-line computation for the receding horizon control law to a function evaluation, therefore avoiding the on-line solution of a mathematical program. However, the number of polyhedral regions of the explicit optimal control laws could grow exponentially with the number of constraints in the optimal control problem. Chapter 13 discusses approaches to define approximate explicit control laws of desired complexity that provide certificates of recursive feasibility and stability.

  Chapter 14 focuses on efficient on-line methods for the computation of MPC control laws. If the state-feedback solution is available explicitly, we present efficient on-line methods for the evaluation of explicit piecewise affine control laws. In particular, we present algorithms to reduce its storage demands and computational complexity. If the on-line solution of a quadratic or linear program is preferred, we briefly discuss how to improve the efficiency of a mathematical programming solver by exploiting the structure of the MPC control problem.

  Part IV closes with Chapter 15 where we address the robustness of the optimal control laws. We discuss min–max control problems for uncertain linear systems with polyhedral constraints on inputs and states and present an approach to compute their state feedback solutions. Robustness is achieved against additive norm-bounded input disturbances and/or polyhedral parametric uncertainties in the state space matrices.

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- In the fifth part of the book (Part V) we focus on linear hybrid systems. We give an introduction to the different formalisms used to model hybrid systems focusing on computation-oriented models (Chapter 16). In Chapter 17, we study finite time optimal control problems with cost functions based on 2, 1 and ∞ norms. The optimal control law is shown to be, in general, piecewise affine over nonconvex and disconnected sets. Along with the analysis of the solution properties, we present algorithms that compute the optimal control law for all the considered cases.

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Acknowledgments

- Large parts of the material presented in this book are extracted from the work of the authors Francesco Borrelli, Alberto Bemporad and Manfred Morari.
- Several sections contain the results of the PhD theses of Miroslav Baric, Mato Baotic, Fabio Torrisi, Domenico Mignone, Pascal Grieder, Eric Kerrigan, Tobias Geyer and Frank J. Christophersen.
- We are extremely grateful to Martin Herceg for his meticulous work on the Matlab examples and all the book figures.
- A special thanks goes to Michal Kvasnica, Stefan Richter, Valerio Turri, Thomas Besselmann and Rick Meyer for their help with the construction of the examples.
- Our gratitude also goes to the colleagues who have carefully read preliminary versions of the book and gave us suggestions on how to improve them. They include Dimitri Bertsekas, Miroslav Fikar, Paul Goulart, Per Olof Gutman, Diethard Klatte, Bill Levine, David Mayne and all the students of our classes on Model Predictive Control.
- The authors of the LaTex book style files and macros are Stephen Boyd and Lieven Vandenberghe. We are most grateful to Stephen and Lieven for sharing them with us.
- Over many years ABB provided generous financial support making possible the fundamental research reported in this book.
- The authors of Chapter 3 are Dr. Alexander Domahidi and Dr. Stefan Richter.
- The author of Chapter 13 is Professor Colin N. Jones.

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Symbols and Acronyms

Logic Operators and Functions

$A \Rightarrow B$	A implies B, i.e., if A is true then B must be true
$A \Leftrightarrow B$	A implies B and B implies A, i.e., A is true if and only if (iff) B is true

Sets

$\mathbb{R}$ ($\mathbb{R}_+$)	Set of (nonnegative) real numbers
$\mathbb{N}$ ($\mathbb{N}_+$)	Set of (nonnegative) integers
$\mathbb{R}^n$	Set of real vectors with n elements
$\mathbb{R}^{n\times m}$	Set of real matrices with n rows and m columns

Algebraic Operators and Matrices

A'	Transpose of matrix A
A^{-1}	Inverse of matrix A
$A^\dagger$	Generalized Inverse of A, $A^\dagger = (A'A)^{-1}A'$
$\det(A)$	Determinant of matrix A
$A \succ (\succeq) 0$	A symmetric positive (semi)definite matrix, $x'Ax > (\geq) 0, \forall x \neq 0$
$A \prec (\preceq) 0$	A symmetric negative (semi)definite matrix, $x'Ax < (\leq) 0, \forall x \neq 0$
A_i	i-th row of matrix A
x_i	i-th element of vector x
$x \in \mathbb{R}^n$, $x > 0$ ($x \geq 0$)	True iff $x_i > 0$ ($x_i \geq 0$) $\forall\ i = 1, \ldots, n$
$x \in \mathbb{R}^n$, $x < 0$ ($x \leq 0$)	True iff $x_i < 0$ ($x_i \leq 0$) $\forall\ i = 1, \ldots, n$
$\lvert x\rvert$, $x \in \mathbb{R}$	Absolute value of x
$\lVert x\rVert$	Any vector norm of x
$\lVert x\rVert_2$	Euclidian norm of vector $x \in \mathbb{R}^n$, $\lVert x\rVert_2 = \sqrt{\sum_{i=1}^n \lvert x_i\rvert^2}$

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$\|x\|_1$	Sum of absolute elements of vector $x \in \mathbb{R}^n$, $\|x\|_1 = \sum_{i=1}^n \|x_i\|$
$\|x\|_\infty$	Largest absolute value of the vector $x \in \mathbb{R}^n$, $\|x\|_\infty = \max_{i\in\{1,\ldots,n\}} \|x_i\|$
$\|S\|_\infty$	Matrix ∞-norm of $S \in \mathbb{C}^{m\times n}$, i.e., $\|S\|_\infty = \max_{i\in\{1,\ldots,m\}} \sum_{j=1}^n \|s_{i,j}\|$
$\|S\|_1$	Matrix 1-norm of $S \in \mathbb{C}^{m\times n}$, i.e., $\|S\|_1 = \max_{j\in\{1,\ldots,n\}} \sum_{i=1}^m \|s_{i,j}\|$
$\mathbf{I}$	Identity matrix
$\mathbf{1}$	Vector of ones, $\mathbf{1} = [1\ 1\ \ldots\ 1]'$
$\mathbf{0}$	Vector of zeros, $\mathbf{0} = [0\ 0\ \ldots\ 0]'$

Set Operators and Functions

$\emptyset$	The empty set
$:$	"Such that"
$\partial\mathcal{P}$	The boundary of $\mathcal{P}$
$\mathrm{int}(\mathcal{P})$	The interior of $\mathcal{P}$, i.e., $\mathrm{int}(\mathcal{P}) = \mathcal{P} \setminus \partial\mathcal{P}$
$\|\mathcal{P}\|$	The cardinality of $\mathcal{P}$, i.e., the number of elements in $\mathcal{P}$
$\mathcal{P} \cap \mathcal{Q}$	Set intersection $\mathcal{P} \cap \mathcal{Q} = \{x\ : x \in \mathcal{P}$ and $x \in \mathcal{Q}\}$
$\mathcal{P} \cup \mathcal{Q}$	Set union $\mathcal{P} \cup \mathcal{Q} = \{x\ :\ x \in \mathcal{P}$ or $x \in \mathcal{Q}\}$
$\bigcup_{r\in\{1,\ldots,R\}} \mathcal{P}_r$	Union of R sets $\mathcal{P}_r$, i.e., $\bigcup_{r\in\{1,\ldots,R\}} \mathcal{P}_r = \{x : x \in \mathcal{P}_0$ or $\ldots$ or $x \in \mathcal{P}_R\}$
$\mathcal{P}^c$	Complement of the set $\mathcal{P}$, $\mathcal{P}^c = \{x : x \notin \mathcal{P}\}$
$\mathcal{P} \setminus \mathcal{Q}$	Set difference $\mathcal{P} \setminus \mathcal{Q} = \{x : x \in \mathcal{P}$ and $x \notin \mathcal{Q}\}$
$\mathcal{P} \subseteq \mathcal{Q}$	The set $\mathcal{P}$ is a subset of $\mathcal{Q}$, $x \in \mathcal{P} \Rightarrow\ x \in \mathcal{Q}$
$\mathcal{P} \subset \mathcal{Q}$	The set $\mathcal{P}$ is a strict subset of $\mathcal{Q}$, $x \in \mathcal{P} \Rightarrow\ x \in \mathcal{Q}$ and $\exists x \in (\mathcal{Q} \setminus \mathcal{P})$
$\mathcal{P} \supseteq \mathcal{Q}$	The set $\mathcal{P}$ is a superset of $\mathcal{Q}$
$\mathcal{P} \supset \mathcal{Q}$	The set $\mathcal{P}$ is a strict superset of $\mathcal{Q}$
$\mathcal{P} \ominus \mathcal{Q}$	Pontryagin difference $\mathcal{P} \ominus \mathcal{Q} = \{x : x + q \in \mathcal{P},\ \forall q \in \mathcal{Q}\}$
$\mathcal{P} \oplus \mathcal{Q}$	Minkowski sum $\mathcal{P} \oplus \mathcal{Q} = \{x + q : x \in \mathcal{P},\ q \in \mathcal{Q}\}$
$f(x)$	With abuse of notation denotes the value of the function f at x or the function f, $f :\ x \to f(x)$. We use the notation $f : \mathbb{R}^n \to \mathbb{R}^s$ to mean that f is a $\mathbb{R}^s$-valued function on some subset of $\mathbb{R}^n$, its domain, which we denote by dom f
$f(x)$ continuous and positive definite	$f : \mathbb{R}^n \to \mathbb{R}$ continuous for all $x \in \mathbb{R}^n$, $f(0) = 0$ and $f(x) > 0\ \forall x \in \mathbb{R}^n \setminus \{0\}$
$f(x) \succ 0$	$f(x)$ continuous and positive definite
$f(x)$ continuous and positive semi-definite	$f : \mathbb{R}^n \to \mathbb{R}$ continuous for all $x \in \mathbb{R}^n$, $f(x) \geq 0\ \forall x \in \mathbb{R}^n$
$f(x) \succeq 0$	$f(x)$ continuous and positive semi-definite

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Cambridge University Press
978-1-107-01688-0 — Predictive Control for Linear and Hybrid Systems
Francesco Borrelli , Alberto Bemporad , Manfred Morari
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Acronyms

ARE	Algebraic Riccati Equation
CLQR	Constrained Linear Quadratic Regulator
CFTOC	Constrained Finite Time Optimal Control
CITOC	Constrained Infinite Time Optimal Control
DP	Dynamic Program(ming)
LMI	Linear Matrix Inequality
LP	Linear Program(ming)
LQR	Linear Quadratic Regulator
LTI	Linear Time Invariant
MILP	Mixed Integer Linear Program
MIQP	Mixed Integer Quadratic Program
MPC	Model Predictive Control
mp-LP	multiparametric Linear Program
mp-QP	multiparametric Quadratic Program
PWA	Piecewise Affine
PPWA	Piecewise Affine on Polyhedra
PWP	Piecewise Polynomial
PWQ	Piecewise Quadratic
QP	Quadratic Program(ming)
RHC	Receding Horizon Control
rhs	right-hand side
SDP	Semi Definite Program(ming)

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