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The basics of neutrino physics

Like the actors in ancient Greek tragedy and comedy, neutrinos play more than one role in the drama of the expanding universe. They couple to gravity and contribute to Einstein equations which rule the expansion dynamics. Furthermore, they interact in the primordial plasma with charged leptons and hadrons via electroweak interactions, until the rates for these processes become so low compared with the typical expansion rate that they *decouple* and start to propagate freely along geodesic lines. Any quantitative description of their role in cosmology thus requires several inputs from the theory of fundamental interactions, as well as a knowledge of their basic properties, such as masses and, in some cases, the features of neutrino flavour oscillations.

Neutrino interactions have been well understood since the first theory of β-decay proposed by Enrico Fermi in 1934, and now are succesfully and beautifully described by the unified picture of electroweak interactions. In the low energy limit the strength of these interactions is encoded in a single coupling, the Fermi coupling constant G_{F}, whose value, combined with the Newton constant, fixes the time of neutrino decoupling. From the strong experimental evidence in favour of neutrino oscillation, we also know that neutrinos are massive particles, and this, as we will see at length in the following, has a strong impact on how structures, i.e., inhomogeneities in the universe, grow on certain length scales.

As a viaticum for this journey in the land of neutrino cosmology, it seemed worth-while to the authors to provide the reader with certain minimal information on the basic properties of neutrinos, both at the level of their theoretical formulation and from the experimental point of view.

Unfortunately, to keep a self-contained summary of these topics reasonably short requires the reader to be acquainted with the basics of quantum field theory and of the gauge principle, which are treated in full detail in many excellent textbooks (e.g., Itzykson and Zuber, 1980; Halzen and Martin, 1984; Weinberg, 1995; Peskin and Schroeder, 1995). In case he or she is familiar with neutrino physics, it is then possible to skip this chapter, though it might be useful to go through it anyway to become familiar with our notation. For all other readers, the following sections can represent only a too-brief synthesis of the present understanding of neutrino properties, hopefully sufficient for them to comfortably read the rest of this book, and likewise hopefully to trigger their curiosity for a deeper understanding of neutrino physics.

Here is a summary of this introductory chapter. After a short review of the Standard Model of fundamental interactions, which covers only the details of its electroweak sector, we describe the main observable properties of neutrinos – interaction processes, Dirac and Majorana masses and flavour oscillations – including a summary of bounds on a certain class of exotic interactions which are beyond our present understanding of microscopic physics but are typically predicted by extensions of the Standard Model which represent its *ultraviolet completion*. We then give a résumé of experimental results on flavour oscillation experiments, laboratory neutrino mass bounds and neutrinoless double-β decay, the last being an *experimentum crucis* to test their Dirac or Majorana nature.

1.1 The electroweak Standard Model

Gauge symmetry has proven to be a powerful guideline to building up a satisfactory theory of fundamental interactions. Strong and electroweak interactions are described by a relativistic quantum field theory based on the gauge symmetry principle for the group $SU(3)_C \times SU(2)_L \times U(1)_Y$, where C, L and Y denote *colour*, *left-handed chirality* and *weak hypercharge* (Glashow, 1961; Weinberg, 1972; Salam, 1968; Fritzsch *et al.*, 1973; Gross and Wilczek, 1973; Politzer, 1973; Weinberg, 1973). The model is so successful that it is now usually referred to as the ‘Standard Model’ (SM) of elementary particles.

Whereas the strong sector $SU(3)_C$ symmetry remains unbroken, and hence is an exact symmetry at any energy level, the electroweak forces undergo spontaneous symmetry breaking via the Higgs mechanism, which reduces the symmetry of the model at low energies to $SU(2)_L \times U(1)_Y \to U(1)_Q$, with Q being the electric charge. In the following we will focus our attention on the electroweak sector only, because neutrinos, like all leptons, do not carry colour charge and are not strongly interacting.

The requirement that a field theory is gauge-invariant under a particular symmetry group strictly fixes the form of the interaction and the number of gauge bosons. Unfortunately, it leaves quite a high level of arbitrariness in the choice of the irreducible representations (IR) of the gauge group to accommodate fermions and the Higgs scalar bosons. The only constraint is provided by the cancellation of

Table 1.1 *Elementary fermions in the SM*

Generation	1st	2nd	3rd
quarks	u	c	t
	d	s	b
leptons	ν_e	ν_μ	ν_τ
	e^-	μ^-	τ^-

Table 1.2 *Electroweak quantum numbers of fermions in the SM*

Fermion IRs under $SU(2)_L \times U(1)_Y$			I	I_3	Y	Q
$L_{eL} \equiv \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$	$L_{\mu L} \equiv \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}$	$L_{\tau L} \equiv \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$	1/2	1/2 −1/2	−1	0 −1
$l_{eR} \equiv e_R$	$l_{\mu R} \equiv \mu_R$	$l_{\tau R} \equiv \tau_R$	0	0	−2	−1
$Q_{1L} \equiv \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$Q_{2L} \equiv \begin{pmatrix} c_L \\ s_L \end{pmatrix}$	$Q_{3L} \equiv \begin{pmatrix} t_L \\ b_L \end{pmatrix}$	1/2	1/2 −1/2	1/3	2/3 −1/3
$q^U_{uR} \equiv u_R$ $q^D_{dR} \equiv d_R$	$q^U_{cR} \equiv c_R$ $q^D_{sR} \equiv s_R$	$q^U_{tR} \equiv t_R$ $q^D_{bR} \equiv b_R$	0	0	4/3 −2/3	2/3 −1/3

the chiral anomaly, a condition which must be fulfilled if gauge symmetry should also be respected at the quantum level.

The currently known fermionic elementary particles (spin $s = 1/2$) are split into three generations of *quarks* and *leptons*; see Table 1.1. Each generation of fermions is described by the IRs of the electroweak gauge group as shown in Table 1.2, where we also report their charges.

By I we denote the weak isospin, which is 1/2 for $SU(2)_L$ doublets and 0 for singlets, respectively, whereas I_3 is its third component. The electric charge Q is given by the Gell-Mann–Nishijima relation $Q = I_3 + Y/2$.

The three-generation electroweak Lagrangian density is

$$\mathcal{L} = i\overline{L'_{\alpha L}} \not{D} L'_{\alpha L} + i\,\overline{Q'_{\alpha L}} \not{D} Q'_{\alpha L} + i\overline{l'_{\alpha R}} \not{D} l'_{\alpha R}$$
$$+ i\overline{q'^D_{\alpha R}} \not{D} q'^D_{\alpha R} + i\overline{q'^U_{\alpha R}} \not{D} q'^U_{\alpha R} - \frac{1}{4}\vec{F}_{\mu\nu} \cdot \vec{F}^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$
$$+ \left(D_\rho \Phi\right)^\dagger (D^\rho \Phi) + \mu^2 \Phi^\dagger \Phi - \lambda \left(\Phi^\dagger \Phi\right)^2$$

Table 1.3 *Electroweak quantum numbers of the Higgs doublet*

Higgs doublet	I	I_3	Y	Q
$\Phi(x) \equiv \begin{pmatrix} \phi_+(x) \\ \phi_0(x) \end{pmatrix}$	1/2	1/2 −1/2	+1	1 0

$$
\begin{aligned}
&- \left(Y'^{l}_{\alpha\beta} \, \overline{L'_{\alpha L}} \, \Phi \, l'_{\beta R} + Y'^{l*}_{\alpha\beta} \, \overline{l'_{\beta R}} \, \Phi^\dagger \, L'_{\alpha L} \right) \\
&- \left(Y'^{D}_{\alpha\beta} \, \overline{Q'_{\alpha L}} \, \Phi \, q'^{D}_{\beta R} + Y'^{D*}_{\alpha\beta} \, \overline{q'^{D}_{\beta R}} \, \Phi^\dagger \, Q'_{\alpha L} \right) \\
&- \left(Y'^{U}_{\alpha\beta} \, \overline{Q'_{\alpha L}} \, (i\sigma_2 \Phi^*) \, q'^{U}_{\beta R} + Y'^{U*}_{\alpha\beta} \, \overline{q'^{U}_{\beta R}} \, (-i\Phi^T \sigma_2) \, Q'_{\alpha L} \right),
\end{aligned} \tag{1.1}
$$

where Φ is the Higgs doublet, whose properties are reported in Table 1.3. σ_2 is a Pauli matrix, and α is the generation index. In the following, repeated indices are summed over, unless differently specified. The covariant derivative D_μ is defined as

$$
D_\mu \equiv \partial_\mu + i g \vec{A}_\mu \cdot \frac{\vec{\sigma}}{2} + i g' B_\mu \frac{Y}{2}, \tag{1.2}
$$

with $\vec{A}^\mu \equiv (A_1^\mu, A_2^\mu, A_3^\mu)$ and B^μ denoting the gauge boson fields of the $SU(2)_L$ and $U(1)_Y$ factors.

The canonical kinetic (and self-interacting for the $SU(2)_L$ factor) term for gauge bosons is written in terms of the electroweak tensors $\vec{F}^{\mu\nu} \equiv (F_1^{\mu\nu}, F_2^{\mu\nu}, F_3^{\mu\nu})$ and $B^{\mu\nu}$, where

$$
\begin{aligned}
F_a^{\mu\nu} &= \partial^\mu A_a^\nu - \partial^\nu A_a^\mu - g \sum_{b,c=1}^{3} \varepsilon_{abc} A_b^\mu A_c^\nu \\
B^{\mu\nu} &= \partial^\mu B^\nu - \partial^\nu B^\mu.
\end{aligned} \tag{1.3}
$$

In the expression (1.1), fermion fields are marked by a *prime* to denote that these fields in general are not mass eigenstates, as will be discussed in detail in the next sections. Equation (1.1) contains in the first two lines the kinetic and electroweak interaction terms for leptons and quarks and the pure gauge boson term, whereas the third line accounts for the Higgs sector responsible for the symmetry breaking. Finally, the last three lines correspond to the Yukawa terms characterized by the complex couplings $Y'^{l}_{\alpha\beta}$, $Y'^{D}_{\alpha\beta}$ and $Y'^{U}_{\alpha\beta}$. They are responsible for charged leptons and quark masses and mixing.

We note that in its minimal version, there are no right-handed neutrino states ν_R in the SM, which would be a singlet under all symmetry group factors. This implies that *active* neutrinos ν_L remain massless, because there are no mass terms which appear as a consequence of symmetry breaking, differently from charged leptons and quarks. The extension of the model to massive neutrinos will be discussed in the following.

From the first two lines of Eq. (1.1) one can extract the charged-current and neutral-current weak interaction Lagrangian densities, denoted by $\mathcal{L}_I^{(\mathrm{CC})}$ and $\mathcal{L}_I^{(\mathrm{NC})}$. In particular, one gets

$$\mathcal{L}_I^{(\mathrm{CC})} = -\frac{g}{2\sqrt{2}} J_W^\mu \, W_\mu + \mathrm{h.c.}, \tag{1.4}$$

where $J_W^\mu = J_{W,L}^\mu + J_{W,Q}^\mu$ and

$$\begin{aligned} J_{W,L}^\mu &= 2\left(\overline{\nu'_{eL}}\gamma^\mu e'_L + \overline{\nu'_{\mu L}}\gamma^\mu \mu'_L + \overline{\nu'_{\tau L}}\gamma^\mu \tau'_L\right) \\ J_{W,Q}^\mu &= 2\left(\overline{u'_L}\gamma^\mu d'_L + \overline{c'_L}\gamma^\mu s'_L + \overline{t'_L}\gamma^\mu b'_L\right). \end{aligned} \tag{1.5}$$

The gauge boson field $W^\mu \equiv (A_1^\mu - i A_2^\mu)/\sqrt{2}$ by definition annihilates a W^+ boson and creates a W^- boson. The neutral current density

$$\mathcal{L}_I^{(\mathrm{NC})} = -\frac{g}{2\cos\theta_W} J_Z^\mu \, Z_\mu + \mathrm{h.c.}, \tag{1.6}$$

where $J_Z^\mu = J_{Z,L}^\mu + J_{Z,Q}^\mu$ and

$$\begin{aligned} J_{Z,L}^\mu &= 2\left(g_L^\nu \overline{\nu'_{\alpha L}}\gamma^\mu \nu'_{\alpha L} + g_L^l \, \overline{l'_{\alpha L}}\gamma^\mu l'_{\alpha L} + g_R^l \, \overline{l'_{\alpha R}}\gamma^\mu l'_{\alpha R}\right) \\ J_{Z,Q}^\mu &= 2\left(g_L^U \, \overline{q'^U_{\alpha L}}\gamma^\mu q'^U_{\alpha L} + g_R^U \, \overline{q'^U_{\alpha R}}\gamma^\mu q'^U_{\alpha R} + g_L^D \, \overline{q'^D_{\alpha L}}\gamma^\mu q'^D_{\alpha L} + g_R^D \, \overline{q'^D_{\alpha R}}\gamma^\mu q'^D_{\alpha R}\right). \end{aligned} \tag{1.7}$$

The gauge field Z^μ is defined *via* the rotation $Z^\mu = \cos\theta_W \, A_3^\mu - \sin\theta_W \, B^\mu$, where $\tan\theta_W = g'/g$ and $e = g\sin\theta_W$. Finally, the couplings $g_L^{\nu,l,U,D}$ and $g_R^{l,U,D}$ are given by the relations

$$g_L^f = I_3^f - Q_f \sin^2\theta_W \tag{1.8}$$

$$g_R^f = -Q_f \sin^2\theta_W, \tag{1.9}$$

which are summarized in Table 1.4.

1.2 Spontaneous symmetry breaking and fermion masses

It is easy to see that a naive mass term such as $\overline{e_L} e_R + \mathrm{h.c.}$ is not allowed in the Lagrangian density (1.1) because it would spoil the symmetry invariance under the gauge group $SU(2)_L \times U(1)_Y$. However, when this group symmetry is broken,

Table 1.4 *Neutral-current couplings for the elementary fermions*

g_L	g_R
$g_L^\nu = \frac{1}{2}$	
$g_L^l = -\frac{1}{2} + \sin^2\theta_W$	$g_R^l = \sin^2\theta_W$
$g_L^U = \frac{1}{2} - \frac{2}{3}\sin^2\theta_W$	$g_R^U = -\frac{2}{3}\sin^2\theta_W$
$g_L^D = -\frac{1}{2} + \frac{1}{3}\sin^2\theta_W$	$g_R^D = \frac{1}{3}\sin^2\theta_W$

masses are produced via the celebrated Higgs–Englert–Brout–Guralnik–Hagen–Kibble mechanism (Englert and Brout, 1964; Guralnik *et al.*, 1964; Higgs, 1964a,b).

The dynamics of the Higgs field Φ is ruled by the term in $\mathcal{L}$

$$\mathcal{L}_{\rm H} = \left(D_\rho\Phi\right)^\dagger (D^\rho\Phi) - V(\Phi) = \left(D_\rho\Phi\right)^\dagger (D^\rho\Phi) + \mu^2\Phi^\dagger\Phi - \lambda\left(\Phi^\dagger\Phi\right)^2 . \tag{1.10}$$

In quantum field theory, the minimum of the potential defines the ground state around which the fields are expanded in terms of creation and annihilation operators. Quantum excitations on the ground state correspond to particle states. Note that only neutral fields with vanishing spin (scalar) may have nontrivial ground states; otherwise this would spoil the electric charge conservation and the invariance under spatial rotations. The Higgs field ground state value $\langle\Phi\rangle$, hereafter referred to as the *vacuum expectation value* (vev), can be written in the form

$$\langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}, \tag{1.11}$$

where v is a real positive quantity. By substituting $\langle\Phi\rangle$ in $V(\Phi)$ one gets the minimum of the energy density for $v = \sqrt{\mu^2/\lambda}$, and around the minimum, in the *unitary gauge*, the Higgs doublet reads

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v + H(x)\end{pmatrix}, \tag{1.12}$$

$H(x)$ being a real scalar field.

The value of v is known, because the first striking effect of symmetry breaking is that three gauge bosons become massive, the charged $W^\pm$ and Z. In particular, $m_W = gv/2 = m_Z\cos\theta_W$. From the experimental value of the Fermi constant $G_{\rm F}$ – see the next section – we get $v \sim 246$ GeV.

Substituting (1.12) in the Yukawa couplings reported in the last three lines of Eq. (1.1), we see how fermion mass terms are produced after the symmetry breaking. Let us consider, for example, the term of $\mathcal{L}$ coupling leptons with the Higgs field,

$$\mathcal{L}_{\mathrm{H},L} = -Y'^{l}_{\alpha\beta}\, \overline{L'_{\alpha L}}\, \Phi\, l'_{\beta R} + \text{h.c.} \tag{1.13}$$

Once the Higgs field is developed around its minimum, one gets

$$\mathcal{L}_{\mathrm{H},L} = -\frac{v + H(x)}{\sqrt{2}} \left(Y'^{l}_{\alpha\beta}\, \overline{l'_{\alpha L}}\, l'_{\beta R} + \text{h.c.} \right). \tag{1.14}$$

The term of Eq. (1.14) proportional to the vev provides the mass term for charged leptons, whereas the contribution proportional to $H(x)$ accounts for the trilinear coupling between charged leptons and the scalar boson H. Because the couplings Y'^{l} are generally not diagonal in the three-generations space, one must diagonalize them before interpreting (1.14) as a genuine mass term. A generic complex matrix such as Y'^{l} can be transformed into a diagonal form Y^{l} through a biunitary transformation

$$V_L^{l\dagger}\, Y'^{l}\, V_R^{l} = Y^{l} \qquad Y^{l}_{\alpha\beta} = y^{l}_{\alpha}\, \delta_{\alpha\beta}. \tag{1.15}$$

With the transformed leptonic fields defined as

$$l_R = V_R^{l\dagger} l'_R \quad \text{and} \quad l_L = V_L^{l\dagger} l'_L, \tag{1.16}$$

the term $\mathcal{L}_{\mathrm{H},L}$ can be rewritten as

$$\mathcal{L}_{\mathrm{H},L} = -\frac{v + H(x)}{\sqrt{2}} \sum_{\alpha} y_{l\alpha} \left(\overline{l_{\alpha L}}\, l_{\alpha R} + \text{h.c.} \right). \tag{1.17}$$

From (1.17) one gets $m_\alpha = y^{l}_{\alpha}\, v/\sqrt{2}$ for $\alpha = e, \mu, \tau$. In terms of these masses one can also rewrite the interaction term between charged leptons and $H(x)$ as

$$\mathcal{L}^{I}_{\mathrm{H},L} = -H(x) \sum_{\alpha} \frac{m_\alpha}{v} \left(\overline{l_{\alpha L}}\, l_{\alpha R} + \text{h.c.} \right), \tag{1.18}$$

which simply states that a heavier lepton is more strongly coupled to the Higgs field than a lighter one.

When the weak charged current $J^{\mu}_{\mathrm{W},L}$ is rewritten in terms of mass eigenstates $l_{\alpha L}$,

$$J^{\mu}_{\mathrm{W},L} = 2\, \overline{\nu'_{\alpha L}}\, (V_L)_{\alpha\beta}\, \gamma^{\mu}\, l_{\beta L}. \tag{1.19}$$

As long as neutrinos do not receive any mass term by the Higgs mechanism, because no right-handed partners ν_R have been introduced so far, we can redefine the neutrino field as $\nu_L = V_L^\dagger \, \nu'_L$ and get

$$J^\mu_{W,L} = 2\, \overline{\nu_{\alpha L}}\, \gamma^\mu \, l_{\alpha L}, \tag{1.20}$$

where by definition $l_{\alpha L} \equiv (e^-_L, \mu^-_L, \tau^-_L)$ and $\nu_{\alpha L} \equiv (\nu_{eL}, \nu_{\mu L}, \nu_{\tau L})$.

Concerning $J^\mu_{Z,L}$ of Eq. (1.7), one can easily see that by virtue of the unitarity of the matrices V^l_L and V^l_R it remains unchanged; hence one can simply replace the *primed* fields with the *unprimed* ones in (1.7).

For the Yukawa terms for quark fields one proceeds in the very same way. In the unitary gauge

$$\mathcal{L}_{\mathrm{H},Q} = -\frac{v + H(x)}{\sqrt{2}} \left(Y'^D_{\alpha\beta}\, \overline{q'^D_{\alpha L}}\, q'^D_{\beta R} + Y'^U_{\alpha\beta}\, \overline{q'^U_{\alpha L}}\, q'^U_{\beta R} + \text{h.c.} \right). \tag{1.21}$$

Thus, by simultaneously diagonalizing the matrices of Yukawa couplings $(Y'^D)_{\alpha\beta}$ and $(Y'^U)_{\alpha\beta}$ via biunitary transformations V^D_L, V^D_R, V^U_L and V^U_R and defining the transformed quark fields as

$$q^D_R = V^{D\dagger}_R q'^D_R \,, \qquad q^D_L = V^{D\dagger}_L q'^D_L \tag{1.22}$$

$$q^U_R = V^{U\dagger}_R q'^U_R \,, \qquad q^U_L = V^{U\dagger}_L q'^U_L \,, \tag{1.23}$$

we can rewrite the mass term in $\mathcal{L}_{\mathrm{H},Q}$ as

$$\mathcal{L}_{\mathrm{H},Q} = -\sum_{\alpha=d,s,b} m_\alpha \left(1 + \frac{H(x)}{v}\right) \overline{q^D_{\alpha L}}\, q^D_{\alpha R} - \sum_{\alpha=u,c,t} m_\alpha \left(1 + \frac{H(x)}{v}\right) \overline{q^U_{\alpha L}}\, q^U_{\alpha R} + \text{h.c.} \tag{1.24}$$

In this case, however, as all quarks are massive and have different masses, we have no freedom to arbitrarily rotate D or U quarks in the hadronic weak charged current

$$J^\mu_{W,Q} = 2\overline{q^U_{\alpha L}} \gamma^\mu \left(V^{U\dagger}_L V^D_L \right)_{\alpha\beta} q^D_{\beta L}. \tag{1.25}$$

The unitary matrix

$$V \equiv V^{U\dagger}_L V^D_L \tag{1.26}$$

is the Cabibbo–Kobayashi–Maskawa (CKM) mixing matrix (Cabibbo, 1963; Kobayashi and Maskawa, 1973), which depends upon three angles θ_{12} (the Cabibbo angle, θ_C, up to very small corrections), θ_{23} and θ_{13} and one phase δ,

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{1.27}$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$, and $0 \le \theta_{ij} \le \pi/2$. To see that V can always be reduced to this form one has to recall that an arbitrary 3×3 unitary matrix has nine real parameters, but we have to subtract five free parameters connected with the single and independent rephasing of quark fields (the global rephasing still remains a symmetry of the system). In the case of quarks the three mixing angles satisfy a hierarchical structure, $s_{12} = 0.22535 \pm 0.00065$ (Beringer *et al.*, 2012), $s_{23} \sim s_{12}^2$, $s_{13} \sim s_{12}^4$.

As can be proven by studying the CP transformation (charge conjugation and parity) of the SM Lagrangian density written in terms of mass eigenstates, the only possible source of CP violation is encoded in the presence of the complex phase δ in V (1.27). Because CP violation processes have been observed in the hadron phenomenology, this has been ascribed to the presence of a nonvanishing value of δ. Indeed, the preferred value for this parameter is $\sin\delta \sim 0.93$ (Beringer *et al.*, 2012). All CP-violating effects in the quark sector can be expressed in terms of a single parameter, the Jarlskog parameter, which is invariant under the phase convention of quark fields:

$$J = -\mathrm{Im}\left[V_{us} V_{cd} V_{cs}^* V_{ud}^*\right] = c_{12} c_{23} c_{13}^2 s_{12} s_{23} s_{13} s_\delta \sim 3 \times 10^{-5}. \tag{1.28}$$

In complete analogy to the leptonic case, one can show that unitarity of the matrices V_L^D, V_L^U, V_R^D, V_R^U implies that the expression for $J_{Z,Q}^\mu$ remains the same after the primed quark fields are replaced with the mass eigenstates.

1.3 The basic properties of neutrinos: interactions, masses and oscillations

1.3.1 Neutrino interactions in the low energy limit

The two terms in the SM Lagrangian density, $\mathcal{L}_I^{(\mathrm{CC})}$ and $\mathcal{L}_I^{(\mathrm{NC})}$ – see (1.4) and (1.6), respectively – describe a three-body process involving two fermions and $W^\pm$ and Z gauge bosons, whose mass is on the order of 100 GeV. Whenever the typical range for energies and momenta carried by the leptons (or quarks) is much smaller than this value, the gauge bosons produced in the trilinear vertex can only propagate as virtual particles. We will see that this is, for example, typically the case in almost all relevant cases in cosmology in which we will be interested in the following. The gauge propagators

$$G_{\mu\nu}^W(x - x') \equiv \langle 0|\mathrm{T}\, W_\mu(x) W_\nu^\dagger(x')|0\rangle = \lim_{\epsilon\to 0} i \int \frac{d^4 p}{(2\pi)^4} \frac{-g_{\mu\nu} + \frac{p_\mu p_\nu}{m_W^2}}{p^2 - m_W^2 + i\epsilon} e^{-ip\cdot(x-x')}$$
$$G_{\mu\nu}^Z(x - x') \equiv \langle 0|\mathrm{T}\, Z_\mu(x) Z_\nu^\dagger(x')|0\rangle = \lim_{\epsilon\to 0} i \int \frac{d^4 p}{(2\pi)^4} \frac{-g_{\mu\nu} + \frac{p_\mu p_\nu}{m_Z^2}}{p^2 - m_Z^2 + i\epsilon} e^{-ip\cdot(x-x')} \tag{1.29}$$

can then be considered in their short-range limit,

$$G^W_{\mu\nu}(x-x') \overset{p^\mu \ll m_W}{\longrightarrow} i\,\frac{g_{\mu\nu}}{m_W^2}\,\delta^4(x-x') \tag{1.30}$$

$$G^Z_{\mu\nu}(x-x') \overset{p^\mu \ll m_Z}{\longrightarrow} i\,\frac{g_{\mu\nu}}{m_Z^2}\,\delta^4(x-x'). \tag{1.31}$$

Hence, the weak charged-current and neutral-current processes at tree level in the low energy limit are described by the effective Lagrangians

$$\mathcal{L}^{(\mathrm{CC})}_{\mathrm{eff}} = -\frac{g^2}{8m_W^2} J^{\mu\dagger}_W J_{\mu W} = -\frac{G_\mathrm{F}}{\sqrt{2}} J^{\mu\dagger}_W J_{\mu W} \tag{1.32}$$

$$\mathcal{L}^{(\mathrm{NC})}_{\mathrm{eff}} = -\frac{g^2}{4\cos^2\theta_W m_Z^2} J^{\mu\dagger}_Z J_{\mu Z} = -2\frac{G_\mathrm{F}}{\sqrt{2}}\rho J^{\mu\dagger}_Z J_{\mu Z}, \tag{1.33}$$

where $G_\mathrm{F} \equiv \sqrt{2}g^2/(8m_W^2) = 1.166 \times 10^{-5}\ \mathrm{GeV}^{-2}$ is the Fermi constant and $\rho \equiv m_W^2/(m_Z^2\cos^2\theta_W)$, which is equal to unity in the SM.

The interaction terms $\mathcal{L}^{(\mathrm{CC})}_{\mathrm{eff}}$ and $\mathcal{L}^{(\mathrm{NC})}_{\mathrm{eff}}$ can mediate a set of purely four-lepton processes such as the ones reported in Tables 1.5 and 1.6. In the following we will treat only some of them in detail, but we will show how to use the results contained in Tables 1.5 and 1.6 for a simple generalization to all the others.

Let us consider in particular neutrino–electron elastic scattering, $\nu_e + e^- \to \nu_e + e^-$ and $\nu_{\mu(\tau)} + e^- \to \nu_{\mu(\tau)} + e^-$. From Eqs. (1.32) and (1.33) one can easily get the amplitudes

$$\mathcal{A}_{\nu_x e^- \to \nu_x e^-} = -\frac{G_\mathrm{F}}{\sqrt{2}}[\overline{u}_{\nu_x}\gamma^\rho(1-\gamma_5)u_{\nu_x}][\overline{u}_e\gamma_\rho(g^l_V - g^l_A\gamma_5)u_e] \tag{1.34}$$

$$\begin{aligned}\mathcal{A}_{\nu_e e^- \to \nu_e e^-} &= -\frac{G_\mathrm{F}}{\sqrt{2}}\left\{[\overline{u}_{\nu_e}\gamma^\rho(1-\gamma_5)u_e\right]\left\{[\overline{u}_e\gamma_\rho(1-\gamma_5)u_{\nu_e}]\right. \\ &\quad + [\overline{u}_{\nu_e}\gamma^\rho(1-\gamma_5)u_{\nu_e}][\overline{u}_e\gamma_\rho(g^l_V - g^l_A\gamma_5)u_e]\left.\right\} \\ &= -\frac{G_\mathrm{F}}{\sqrt{2}}[\overline{u}_{\nu_e}\gamma^\rho(1-\gamma_5)u_{\nu_e}][\overline{u}_e\gamma_\rho((1+g^l_V) - (1+g^l_A)\gamma_5)u_e],\end{aligned} \tag{1.35}$$

where $x = \mu, \tau$, $g^l_V \equiv g^l_L + g^l_R$, and $g^l_A \equiv g^l_L - g^l_R$ (see Table 1.4). Note that to obtain the final form of $\mathcal{A}_{\nu_e e^- \to \nu_e e^-}$ we have used one of the Fierz rearrangement formulas. In Tables 1.5 and 1.6 are reported the squared amplitudes for several pure weak leptonic processes at three level. The list is not complete, but the missing processes can be obtained using crossing symmetry.