Introduction

Easy reading guide

The Introduction presents an overview of the book. We spell out what this book contains, what it is good for – and what it is not! Rather than starting with technical details of set-theoretic methods, we put the content of the book into a broader context of current methodological debates. The Introduction will help the reader to find out whether, in general, this book might be interesting and, if so, which chapters in particular are most relevant for him/her.

In a first step, we show that notions of sets and their relations are more common in the social sciences than might probably be known. Then we describe Qualitative Comparative Analysis (QCA) as the most developed form of set-theoretic method. We spell out the defining features of QCA and how they differentiate it from other set-theoretic methods. In the next section, we explain the differences and similarities among the various forms of QCA. In the following section, we not only explain the structure of the book, but also provide details on how to use the book by addressing some of its features, such as the Easy reading guides, At-a-glance boxes, the Glossary, or the online material which contains chapter-by-chapter "how-to" sections and exercises.

In short, by reading this Introduction, readers should get a better understanding of what to expect from this book and how to use it in order to maximize its utility.

Set-theoretic approaches in the social sciences

Arguments about set relations are pervasive in the social sciences, but this is not always obvious. Take, for example, Brady's (2010) intriguing deconstruction of the widely debated claim that, in the 2000 US Presidential Election, George W. Bush lost about 10,000 votes because Al Gore had been declared the winner before the closure of the polling stations in those western counties of Florida that are on Central Standard Time (i.e., the Panhandle). This claim is made by Lott (2000), who arrived at this inference by estimating a "'difference-in-differences' form of regression analysis, based on data-set

observations" (Brady 2010: 238). Using causal-process observations, Brady cogently shows that this inference is "highly implausible" (241) and that, instead of 10,000 lost voters, a more adequate estimate would be a maximum of 224 or, even more realistically, 28 to 56 voters (NB: total voters, not percentage!). Brady successfully frames his debate of Lott as an argument in favor of causal-process observations – "diagnostic 'nuggets' of data that make a strong contribution to causal inference" (Brady 2010: 237).

Brady's argument is set-theoretic in nature (Goertz and Mahoney 2012). In essence, he claims that the set of *voters not voting for Bush due to the premature announcement of Gore as the winner* (Y) can only be very small because membership in this set requires simultaneous membership in several other sets. Such allegedly lost Bush voters must, of course, also be members of the set of *registered voters in the Panhandle counties* (P), who are also members of the set of voters who had *not yet voted* (V), and the set of voters who had *received the news through the media* (M). Using plausible arguments about the rough percentage of voters that tend to vote late and the percentage of voters listening to the media, Brady shows that the sets of P, V, and M are small and that, as a direct consequence of this, the set of Y must be even smaller. This is because membership in each of the three sets P, V, and M is necessary in order to be a member of set Y (Goertz and Mahoney 2012: 54–56).

This example illustrates that many arguments in the social sciences can be (re-)framed in terms of relations between sets. The notion of sets is not explicitly invoked in Brady's original analysis, and there is nothing wrong with this. We do claim, however, that an explicit framing of arguments in terms of set relations is often adequate and that, once set relations are invoked, set-theoretic methods provide a powerful toolkit for such analyses.

Different mathematical sub-disciplines provide the underpinnings for the vast majority of social science methods and techniques. Most of the well-known and commonly applied statistical methods in the social sciences are applications of probability calculus or matrix algebra to social science data. While most of these mathematical sub-disciplines might be remembered from school, set theory is less familiar to most people. Although formal logic, a close relative of set theory, is a well-studied system of thought in disciplines such as philosophy and mathematics, it currently plays only a marginal role in school education and social science methods training in many parts of the world. This is unfortunate, because, as shown, set-theoretic notions are invoked in social science research more often than is usually recognized. The notion of sets and their relations is almost unavoidably invoked when

forming concepts or when verbally formulating (causal) relations between social phenomena. This book is motivated by the belief that the study of set-theoretic relationships provides an important perspective on social science research problems, thus adding to the currently predominant correlational approaches.

What are set-theoretic methods? Implicitly or explicitly, they all share three features: first, they work with membership scores of cases in sets; second, they perceive relations between social phenomena as set relations; third, these set relations are interpreted in terms of sufficiency and necessity, as well as forms of causes that can be derived from them, such as so-called INUS and SUIN conditions (section 3.3.2). Let us discuss these three points individually.

First, the data on which set-theoretic methods operate are membership scores of cases in sets which represent social science concepts. For instance, France is an element of the set of *European Countries* whereas the USA is not. France's set membership score in this set is therefore 1, while that of the USA is 0. When we invoke the notion of sets, it might seem unavoidable that we perceive them as dichotomies. This is not the case, though. Even an apparently straightforward dichotomous concept such as the set of *European Countries* might not be clearly dichotomous at all – just think of the case of Turkey and the discussion it triggers about where the (geographic, cultural, economic, military, etc.) boundaries of Europe are.[1] In fact, for many social science concepts, it is difficult to perceive them as clear dichotomies, or *crisp sets*, in which cases can be assigned full (non-)membership scores. Luckily, set theory can go beyond crisp sets. In its *fuzzy set* version, it also allows for partial set membership. Cases are not forced to be either full members of the set of European countries, or full non-members of it, but can also be partial members. A case like Turkey would receive a partial (or fuzzy-set) membership score lower than 1 and higher than 0 in the set European countries. This fuzziness does not derive from imprecise *empirical* information about the case of Turkey – we can gather very detailed information of its geographical location, economic structure, etc. Instead, fuzziness stems from non-sharp *conceptual* boundaries inherent in the notion of European country. Virtually

[1] Even concepts which most clearly seem to be dichotomous can be problematic. Just think about EU membership, about which we would think that it is clearly dichotomous. However, on closer examination we see differences on some of the aspects that we would use to determine crisp set membership; for example, the UK is neither a member of the Schengen Protocol nor uses the euro. As such we might want to see the UK as a qualitatively different type of member than say, Luxembourg or Germany. Likewise, Switzerland is not a formal member of the EU and yet it adopts a huge share of European legislation, frequently word-for-word (Kux and Sverdrup 2000: 251), something which other non-members (such as India, the Ivory Coast, or Samoa) do not do.

all social science concepts have fuzzy boundaries, and fuzzy sets are a tool for numerically expressing that.

The second trait shared by all set-theoretic methods in the social sciences is that relations between social phenomena are perceived of as set relations. Take, for example, the empirical observation that all NATO members are democracies. Although it might not be obvious, this is a clear-cut example of a set-theoretic statement. The verbal descriptions "NATO members" and "democratic countries" both represent sets in which different cases have different membership scores. If we observe further that all NATO members are democracies, but that not all democracies are NATO members (think of Sweden or Japan), then the set of NATO members is a subset of the set of democratic countries. This, in turn, implies that the set of democratic countries is a superset of the set of NATO members.

This simple recasting of social science phenomena in terms of set relations might not seem very inspiring on its own, and it might rather come across as a simple play on words. This rephrasing, however, gains great analytic potential once we understand that subset relations are intimately linked to the ideas of sufficiency and necessity. This is the third aspect of set-theoretic methods: set relations are usually interpreted in terms of sufficient or necessary conditions, or of their more complex modifications INUS and SUIN,[2] either in a causal or a descriptive manner. Applied to our example, we can conclude that being a democracy is a necessary condition for being a NATO member, for the latter is a subset of the former. Statements about conditions being either necessary or sufficient abound in the social sciences. Gary Goertz, one of the pioneers in the empirical study of necessary conditions, counts not fewer than 150 hypotheses about necessary conditions in the field of international relations alone (Goertz 2003). Hypotheses about sufficient conditions are at least as widespread (Ragin 2000). However, often we do not recognize these claims immediately, since they are frequently hidden in verbal formulations that do not explicitly use the terms *necessity* or *sufficiency* (Mahoney 2004).

Suppose we claim that "Citizens of small, rural towns in the USA vote for the Republican Party." This relationship denotes a subset relation. The set of all small-town, rural voters (X) is a subset of all Republican voters (Y). This means that all cases which exhibit X (i.e., voters living in small, rural towns)

[2] INUS stands for Insufficient but Necessary part of a condition which is itself Unnecessary but Sufficient for the result (Mackie 1965: 246). SUIN instead stands for Sufficient but Unnecessary part of a factor that is Insufficient but Necessary for the result (Mahoney, Kimball, and Koivu 2009: 126). As we will describe in this book, both forms of causal factors represent advanced forms of causal complexity and refer to components that do not count as necessary or sufficient conditions when taken alone, but which play a subtle causal role in intricate combinations with other factors.

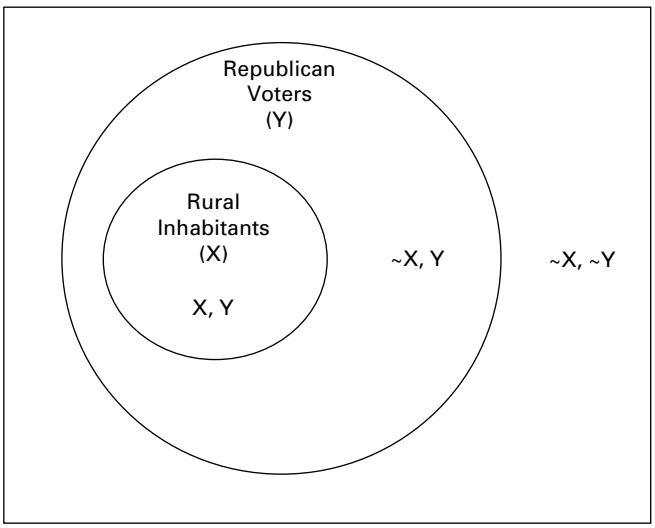

Figure 0.1 Venn diagram for relation of sufficiency

also exhibit Y (they cast their ballot for the Republican Party). This denotes the inner circle (X, Y) in the Venn diagram in Figure 0.1.

As we will learn throughout this book, such a pattern in the data can be interpreted to mean that X is sufficient for Y. Note that this statement does not tell us anything about the voting behavior of citizens *not* living in small, rural towns in the USA. They might be Republican voters (area ~X, Y) or they might not (~X, ~Y). Nor does the sufficiency claim entail that all voters for the Republican Party are living in small, rural towns. There are, of course, many non-rural voters of the Republican Party, as indicated by area ~X, Y of Figure 0.1. The point is, however, that such voters are irrelevant when it comes to corroborating the claim that living in a rural town is sufficient for voting for the Republican Party. The fact that there are other types of voters for the Republican Party simply indicates that there are other sufficient conditions for voting for the Republicans.

The intimate link between subset relations and the notions of necessity and sufficiency triggers several analytic consequences. For instance, saying that there is a sufficient (but not necessary) condition generally requires the existence of other sufficient conditions for the same outcome. This, in turn, means that by embracing a set-theoretic perspective on social science phenomena one unavoidably recognizes the existence of *equifinality*, i.e., a scenario in which alternative factors can produce the same outcome. Also, more often than not, in order to find perfect set relationships, one might

need to refer to combinations of various sets, where single conditions do not display their effect on their own, but only together with other conditions. For instance, it might be that only the set of young male inhabitants of rural towns vote Republican. Set theory is therefore also closely linked to the notion of *conjunctural causation*. Further, combining equifinality and conjunctural causation automatically implies the existence, and causal relevance, of the much-discussed INUS and SUIN conditions.

Yet another aspect of set theory consists of the *asymmetry* of concepts and causal relations. A set-theoretic perspective on concepts requires two separate definitions and operationalizations of concepts that in non-set-theoretic approaches are often not distinguished (Goertz and Mahoney 2012: chs. 9–13). For instance, an autocracy is not simply the opposite of a democracy. Richness is not simply the opposite of poverty. Consider, for instance, college students who are usually not "rich," but their non-membership in the set of rich persons does not imply that they are "poor." From this follows that we need two different sets to capture the two qualitatively different states of being rich and being poor. In most social science approaches, however, only one indicator is used – say, monthly disposable income – and the degree of richness (high or low, with low values on the richness scale being equal to poorness) inferred from this. The causal interpretation of asymmetry is that the explanation for the non-occurrence of the outcome cannot automatically be derived from the explanation for the occurrence of the outcome. For example, when trying to explain the conditions for successful democratization, we most likely will need to consider quite different conditions than a study that tries to understand failed democratization. In set-theoretic methods, there usually is no symmetry between the combinations of conditions for the occurrence of the outcome and its non-occurrence. This is a major difference from standard correlational methods (see also 3.3.3). We thus define set-theoretic methods as follows:

Set-theoretic methods are approaches to analyzing social reality in which (a) the data consists of set membership scores; (b) relations between social phenomena are modeled in terms of set relations; and (c) the results point to sufficient and necessary conditions and emphasize causal complexity in terms of INUS und SUIN causes.

Set-theoretic methods often come under different labels. They are sometimes called "Boolean methods" (Caramani 2009) or "logical methods" (Mill 1843). Rihoux and Ragin (2009) have coined the term "Configurational Comparative Methods" (CCM) in an attempt to find a name for a group of similar methods. By choosing the acronym CCM, they emphasize a feature that is shared

by all set-theoretic methods: they all understand the world in terms of configurations of conditions. We prefer the term set-theoretic methods because it is more encompassing and emphasizes the core analytic fact that all of them model social reality in terms of set-theoretic relations. It is the set-theoretic foundation from which all other features of this family of methods derive.

The use of set theory in the social sciences is not as new as it might seem. A closer look reveals that it provides the underlying logic for many, mostly qualitative approaches in the social sciences. As Mahoney notes, many comparative case-study approaches apply a set-theoretic reasoning in an informal and intuitive manner (Mahoney 2007: 135). One example for this is concept formation. If, for instance, we define a concept as the simultaneous presence of several phenomena – say, the concept of democracy being defined as the simultaneous presence of free elections and civil liberties – then we make use of set-theoretic logic: the set of all democracies is represented by the intersection of the set of countries that display free and fair elections with the set of countries that display civil liberties. Put differently, these are individually necessary and jointly sufficient elements of democracy. As Goertz (2006a) shows, adopting a set-theoretic perspective on concept definitions is often more in line with the underlying linguistic meaning conveyed by those definitions and also triggers important consequences for the data aggregation procedure. Rather than adding or averaging information across different dimensions of a concept, a set-theoretic perspective looks at necessary and sufficient components of a concept in order to maintain a strong link between the verbal meaning of a concept and its numerical representation. Ignoring this can lead to a severe misfit between the meaning of a concept and its operationalization. In our example, averaging the two indicators of free elections and civil liberties would mean that a totally illiberal country that happens to hold free elections would count as a half-democracy, whereas the set-theoretic approach would classify it as a non-democracy.

Set theory also provides a fruitful perspective on the creation of typologies (Elman 2005; George and Bennett 2005: ch. 11). Typologies can be seen as concepts for which information is not aggregated into a unidimensional scale of set membership (e.g., all countries being ranked in a way that represents their degree of membership in the concept of democracy), but where cases are classified on multiple dimensions. The example of the welfare state can help us to illustrate this point: countries differ not only in the (unidimensional) *degree* to which they provide welfare to their citizens but also in the (multidimensional) *type* of welfare state they have developed for this purpose. If, for the sake of illustration, we postulate that welfare states vary along two

dimensions – labor market protection and transfer payments – then there are four different ideal-typical forms of the welfare state: high labor market protection with high transfer payments; high labor market protection with low transfer payments; low labor market protection with high transfer payments; and low labor market protection with low transfer payments. As Kvist (2006) shows, a set-theoretic approach to forming and arguing about typologies can be very helpful, especially if we – as Kvist does – go beyond dichotomous (crisp) sets and work with fuzzy sets in which cases can have degrees of membership in each dimension.

Notions of set theory are also useful for those more ambitious social science practices that are designed to give a causal interpretation to patterns found in the data. Prominent examples are John Stuart Mill's methods (see, e.g., Mahoney 2003). The possibility of interpreting them in a set-theoretic manner is an aspect that has not received enough attention so far (Mahoney 2007: 134).

At-a-glance: set-theoretic approaches in the social sciences

Set-theoretic methods operate on membership scores of elements in sets; causal relations are modeled as subset or superset relations; **necessity**[3] and **sufficiency** or **INUS** and **SUIN** conditions are at the center of attention.

The use of set theory focuses attention on unraveling causally complex patterns in terms of **equifinality**, **conjunctural causation**, and **asymmetry**.

Set theory can be useful for concept formation, the creation of typologies, and causal analysis.

Qualitative Comparative Analysis as a set-theoretic approach and technique

Qualitative Comparative Analysis, commonly known under its acronym QCA, is the methodological tool that is perhaps most directly associated with set theory. QCA distinguishes itself from other set-theoretic approaches by the combined presence of the following features. First, it aims at a causal interpretation. This is not necessarily true for other set-theoretic approaches – just think of concept formation or the creation of typologies, which typically do not include any reference to an outcome (for two exceptions, Elman 2005 and George and Bennett 2005). Second, QCA makes use of so-called truth tables.

[3] All the terms that are further defined in the Glossary are printed in bold in the At-a-glance boxes.

This allows researchers to visualize and analyze central features of causal complexity, such as equifinality or conjunctural causation and the presence of INUS or SUIN conditions. Other set-theory based methods, such as Mill's methods or set-theory-based historical explanations (Mahoney, Kimball, and Koivu 2009), do not employ truth tables. Third, QCA approaches make use of the principles of logical minimization, a process by which the empirical information is expressed in a more parsimonious yet logically equivalent manner by looking for commonalities and differences among cases that share the same outcome. With a few exceptions (see Elman 2005), logical minimization does not play a role in the set-theoretic literature on typological theories (George and Bennett 2005: ch. 11); if it does, it is usually performed in an intuitive rather than formalized manner.

Large sections of this book are dedicated to explaining QCA, for it is arguably the most formalized and complete set-theoretic method. It requires more of a proper and systematic introduction in basic concepts from formal logic, set theory, and Boolean algebra than other set-theoretic methods. In addition, QCA can, and should, be performed with the help of specialized computer software. Related to this is the fact that most, if not all, other set-theoretic approaches can be interpreted as either specializations or extensions of specific elements of QCA. For instance, the use of set theory for classifying cases in multidimensional typologies can be interpreted as a specialized QCA without an outcome and thus without any causal interpretation. Yet other set-theoretic approaches are extensions of QCA. For instance, standard QCA has only indirect ways of including time as a causally relevant dimension into the analysis. Partially in response to this, Mahoney, Kimball, and Koivu (2009) have elaborated the conceptual foundations for combining historical explanations and set-theoretic reasoning. Similarly, Caren and Panofsky (2005) and Ragin and Strand (2008) have made specific suggestions for extending the QCA algorithm by allowing the order of events to matter causally. In short, by learning about the principles and practice of QCA, readers will learn about set-theoretic methods at large.

Figure 0.2 provides a graphical overview of our understanding of the different set-theoretic approaches in the social sciences and their relation to some other empirical comparative approaches. It shows that the umbrella term of set-theoretic methods covers several prominent and less prominent approaches to studying social reality. And QCA is just one of them.

The idea of making use of set theory for the interpretation and analysis of social science data in QCA has been put forward by the American social scientist Charles C. Ragin (1987, 2000, 2008). Interest in QCA has grown in

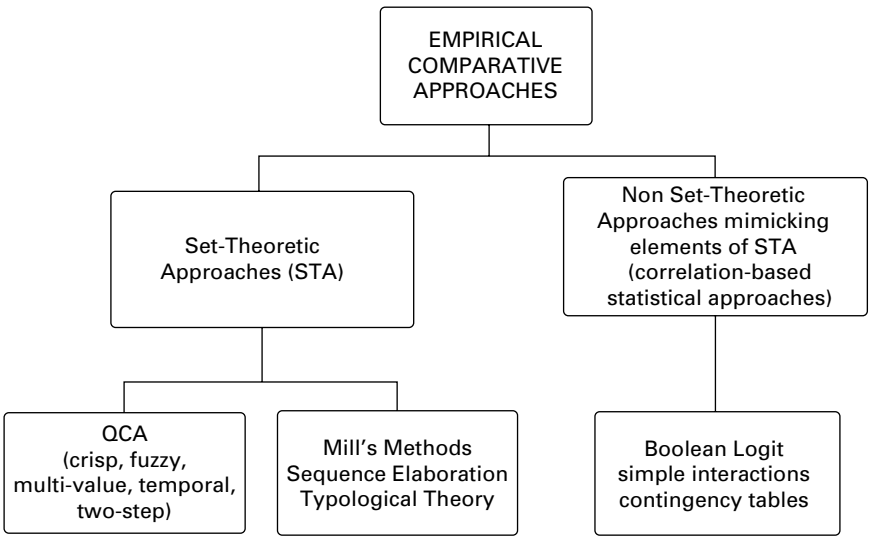

Figure 0.2 Set-theoretic approaches in the social sciences

recent years as comparative social science has revived fundamental debates on empirical social science methodology (e.g., King, Keohane, and Verba 1994; Gerring 2001, 2007; Brady and Collier 2004, 2010; or George and Bennett 2005; Gerring 2012; Goertz and Mahoney 2012). In this debate, QCA is often presented as a third way between quantitative statistical techniques and case-study methodology. By putting so much emphasis on QCA as a hybrid method that would, supposedly, combine the best of two worlds, and by focusing on the related claim that QCA is a method designed for analyzing mid-sized (that is, medium-N) datasets, its distinct characteristic as a set-theoretic method is often less widely recognized than it should be. As a matter of fact, in the early days, QCA's set-theoretic foundation was downplayed even by its inventor itself: Ragin's 1987 book, widely seen as the foundational work for QCA, does not mention set theory at all. All his later books have the term "set" in the title, though. Approaching QCA from a set-theoretic perspective has the double advantage of being able to explain its analytic features in a succinct manner and to unravel the fact that, contrary to widely held beliefs, QCA is not really a method invented *ex novo*, but makes use both of an established subfield in mathematics and of principles and practices well known in social science methodology.

Set-theoretic methods have a close affinity to case-oriented comparative approaches. As such, they cannot be seen only as data analysis techniques.