

BLACK HOLES IN HIGHER DIMENSIONS

Black holes are one of the most remarkable predictions of Einstein's general relativity. Now widely accepted by the scientific community, most work has focused on black holes in our familiar four spacetime dimensions. In recent years, however, ideas in brane-world cosmology, string theory, and gauge/gravity duality have all motivated a study of black holes in more than four dimensions, with surprising results. In higher dimensions, black holes exist with exotic shapes and unusual dynamics.

Edited by leading expert Gary Horowitz, this exciting book is the first devoted to this new field. The major discoveries are explained by the people who made them: for example, Rob Myers describes the Myers–Perry solutions that represent rotating black holes in higher dimensions; Ruth Gregory describes the Gregory–Laflamme instability of black strings; and Juan Maldacena introduces gauge/gravity duality, the remarkable correspondence that relates a gravitational theory to nongravitational physics. There are two additional chapters on this duality, explaining how black holes can be used to describe relativistic fluids and aspects of condensed matter physics.

Accessible to anyone who has taken a standard graduate course in general relativity, this book provides an important resource for graduate students and for researchers in general relativity, string theory, and high energy physics.

GARY T. HOROWITZ is a professor of physics at the University of California, Santa Barbara. He has made numerous contributions to classical and quantum gravity. In particular, he (co-)discovered a class of higher-dimensional black holes called “black branes”. Professor Horowitz is a member of the National Academy of Science, a Fellow of the American Physical Society, and a member of the International Committee for the General Relativity and Gravitation Society. He has won the Xanthopoulos Prize in general relativity and has written over 150 research articles.

BLACK HOLES IN HIGHER DIMENSIONS

Edited by

GARY T. HOROWITZ



Cambridge University Press
 978-1-107-01345-2 — Black Holes in Higher Dimensions
 Edited by Gary T. Horowitz
 Frontmatter
[More Information](#)

CAMBRIDGE UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom
 One Liberty Plaza, 20th Floor, New York, NY 10006, USA
 477 Williamstown Road, Port Melbourne, VIC 3207, Australia
 314-321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi - 110025, India
 103 Penang Road, #05-06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org

Information on this title: www.cambridge.org/9781107013452

© Cambridge University Press 2012

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 2012

A catalogue record for this publication is available from the British Library

Library of Congress Cataloging in Publication data

Black holes in higher dimensions / edited by Gary T. Horowitz.

p. cm.

ISBN 978-1-107-01345-2 (hardback)

1. Black holes (Astronomy) – Mathematical models. 2. Hyperspace. I. Horowitz, Gary T., 1955–

QB843.B55.B5878 2012

523.8'875 – dc23 2011053168

ISBN 978-1-107-01345-2 Hardback

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication, and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.

Contents

<i>List of contributors</i>	<i>page</i> ix
<i>Preface</i>	xi
Part I Introduction	1
1 Black holes in four dimensions	3
GARY T. HOROWITZ	
1.1 Schwarzschild solution	3
1.2 Reissner–Nordström solution	6
1.3 Kerr solution	10
1.4 Black holes with nonzero cosmological constant	14
1.5 General properties of four-dimensional black holes	16
1.6 Black hole thermodynamics	20
Part II Kaluza–Klein theory	27
2 The Gregory–Laflamme instability	29
RUTH GREGORY	
2.1 Overview	29
2.2 Black holes in higher dimensions	30
2.3 A thermodynamic argument for instability	32
2.4 Perturbing the black string	33
2.5 Consequences of the instability	40
3 Final state of Gregory–Laflamme instability	44
LUIS LEHNER AND FRANS PRETORIUS	
3.1 Overview	44
3.2 Background	45
3.3 Numerical approach	46

vi	<i>Contents</i>	
3.4	Evolution of an unstable black string	54
3.5	Speculations and open questions	60
4	General black holes in Kaluza–Klein theory	69
	GARY T. HOROWITZ AND TOBY WISEMAN	
4.1	Energy in Kaluza–Klein theory	69
4.2	Homogeneous black hole solutions	72
4.3	Inhomogeneous black hole solutions	81
Part III	Asymptotically flat solutions	99
5	Myers–Perry black holes	101
	ROBERT C. MYERS	
5.1	Static black holes	101
5.2	Spinning black holes	103
5.3	Appendix A: Mass and angular momentum	126
5.4	Appendix B: A case study of $d = 5$	128
6	Black rings	134
	ROBERTO EMPARAN AND HARVEY S. REALL	
6.1	Introduction	134
6.2	Ring coordinates	135
6.3	Singly spinning black ring solution	138
6.4	Black ring with two angular momenta	145
6.5	Multiple black hole solutions	148
6.6	Stability	152
Part IV	General properties	157
7	Constraints on the topology of higher-dimensional black holes	159
	GREGORY J. GALLOWAY	
7.1	Introduction	159
7.2	Hawking’s theorem on black hole topology	160
7.3	Marginally outer-trapped surfaces	161
7.4	A generalization of Hawking’s theorem and some topological restrictions	164
7.5	The proof of Theorem 7.4.1	166
7.6	The borderline case	170
7.7	Effect of the cosmological constant	171
7.8	Further constraints on black hole topology	173
7.9	Final remarks	176

Contents

vii

8	Blackfolds	180
	ROBERTO EMPARAN	
8.1	Introduction	180
8.2	Effective theory for black hole motion	181
8.3	Effective blackfold theory	184
8.4	Stationary blackfolds, action principle, and thermodynamics	190
8.5	Examples of blackfold solutions	196
8.6	Gregory–Laflamme instability and black brane viscosity	204
8.7	Extensions	208
8.8	Appendix: Geometry of submanifolds	209
9	Algebraically special solutions in higher dimensions	213
	HARVEY S. REALL	
9.1	Introduction	213
9.2	CMPP classification	214
9.3	De Smet classification	224
9.4	Other approaches	227
9.5	Outlook	229
10	Numerical construction of static and stationary black holes	233
	TOBY WISEMAN	
10.1	Introduction	233
10.2	Static vacuum black holes	236
10.3	Stationary vacuum solutions	257
	Part V Advanced topics	271
11	Black holes and branes in supergravity	273
	DONALD MAROLF	
11.1	Introduction	273
11.2	Supergravity in eleven dimensions	274
11.3	M-branes: the BPS solutions	285
11.4	Kaluza–Klein and dimensional reduction	296
11.5	Branes in 9+1 type II supergravity	304
11.6	Some remarks on string perturbation theory	313
12	The gauge/gravity duality	325
	JUAN MALDACENA	
12.1	Scalar field in AdS	332
12.2	The $\mathcal{N} = 4$ super Yang–Mills/AdS ₅ $\times$ S ⁵ example	335
12.3	The spectrum of states or operators	342
12.4	The radial direction	343

13	The fluid/gravity correspondence	348
	VERONIKA E. HUBENY, SHIRAZ MINWALLA, MUKUND RANGAMANI	
13.1	Introduction	348
13.2	Relativistic fluid dynamics	353
13.3	Perturbative construction of gravity solutions	359
13.4	Results at second order	363
13.5	Specific fluid flows and their gravitational analogue	370
13.6	Extensions beyond conformal fluids	374
13.7	Relation to other developments	380
13.8	Summary	382
13.9	Epilogue: Einstein and Boltzmann	383
14	Horizons, holography and condensed matter	387
	SEAN A. HARTNOLL	
14.1	Introduction	387
14.2	Preliminary holographic notions	388
14.3	Brief condensed matter motivation	392
14.4	Holography with a chemical potential	396
14.5	The planar Reissner–Nordström–AdS black hole	397
14.6	Holographic superconductors	400
14.7	Electron stars	405
14.8	Dilatonic scalars: Lifshitz and beyond	410
14.9	Horizons, fractionalisation and black hole information	412
	<i>Index</i>	420

Contributors

Roberto Emparan

Institució Catalana de Recerca i Estudis Avançats (ICREA), Passeig Lluís Companys 23, E-08010 Barcelona, Spain; Departament de Física Fonamental and Institut de Ciències del Cosmos, Universitat de Barcelona, Martí i Franquès 1, E-08028 Barcelona, Spain

Gregory J. Galloway

Department of Mathematics, University of Miami, Coral Gables, FL 33124, USA

Ruth Gregory

Centre for Particle Theory, Durham University, Durham, DH1 3LE, UK

Sean A. Hartnoll

Department of Physics, Stanford University, Stanford, CA 94305, USA

Gary T. Horowitz

Department of Physics, University of California at Santa Barbara, Santa Barbara, CA 93106, USA

Veronika E. Hubeny

Centre for Particle Theory, Durham University, Durham, DH1 3LE, UK

Luis Lehner

Perimeter Institute, Waterloo, Ontario N2L 2Y5, Canada; Department of Physics, University of Guelph, Ontario N1G 2W1, Canada

Juan Maldacena

Institute for Advanced Study, Princeton, NJ 08540, USA

Donald Marolf

Department of Physics, University of California at Santa Barbara, Santa Barbara, CA 93106, USA

Shiraz Minwalla

Tata Institute of Fundamental Research, Homi Bhabha Rd, Mumbai 400005, India

Robert C. Myers

Perimeter Institute, Waterloo, Ontario N2L 2Y5, Canada

Frans Pretorius

Department of Physics, Princeton University, Princeton, NJ 08544, USA

Mukund Rangamani

Centre for Particle Theory, Durham University, Durham, DH1 3LE, UK

Harvey S. Reall

Department of Applied Mathematics and Theoretical Physics, Cambridge University, Cambridge, CB3 0WA, UK

Toby Wiseman

Theoretical Physics Group, Blackett Laboratory, Imperial College, London SW7 2AZ, UK

Preface

Black holes are one of the most striking predictions of general relativity. Their main classical properties were understood by the 1970s. In particular, it had been shown that they are very simple objects: stationary vacuum black holes are uniquely characterized by their mass and angular momentum. Astrophysical evidence for black holes has improved dramatically over the past decade and it is now widely accepted that black holes are ubiquitous in our universe.

Naturally enough, the work through the 1970s was focused on black holes in our familiar four spacetime dimensions ($D = 4$). Recently, there has been an explosion of interest in higher-dimensional black holes. There are at least four reasons for this.

(1) As mentioned above, black holes in $D = 4$ are special: they must be spherical, specified by a few parameters, always stable, etc. It is natural to ask whether these properties are characteristic of black holes in general or just a result of four spacetime dimensions.

(2) Recent brane-world ideas suggest that our familiar three spatial dimensions might just be a surface in a higher-dimensional space. In these theories, nongravitational forces are confined to the brane but gravity is higher dimensional. So black holes extend into the extra dimensions.

(3) String theory, one of the most promising approaches to quantum gravity, predicts that spacetime has more than four dimensions. This incorporates older ideas of unification based on the idea that extra dimensions are curled up into a small ball. So, in string theory we must consider higher-dimensional black holes.

(4) Gauge/gravity duality, which has emerged from string theory, relates certain strongly coupled nongravitational theories to higher-dimensional theories with gravity. Under this duality, thermal equilibrium in some $(3+1)$ -dimensional non-gravitational systems is described by a higher-dimensional black hole.

Over the past two decades we have learned that higher-dimensional black holes are much less constrained than their cousins in $D = 4$. They do not have to be

spherical, and they are not uniquely determined by their mass and angular momentum. In addition, they have more interesting dynamics, since some stationary black holes are unstable. This instability causes the horizon to pinch off in finite time, producing a singularity that is visible from infinity, i.e., a naked singularity. So while there is strong evidence that naked singularities cannot form generically in four-dimensional general relativity, they do form generically in higher dimensions.

Given all these exciting results, the time is right for a book devoted to higher-dimensional black holes and some of their applications. I have asked people who have made significant contributions to this subject to explain them for this edited volume. The book is not an exact-solutions manual, with all known $D > 4$ black hole solutions included. Since the field is still progressing, that would quickly become obsolete. Instead, while it includes the most important explicitly known solutions, it focuses on general properties of higher-dimensional black holes and their recent applications to other areas of physics.

The book is divided into five parts. The first gives a brief introduction, reviewing the familiar four-dimensional black holes. The next part is devoted to five-dimensional Kaluza–Klein theory. This is the oldest approach to higher dimensions, in which one adds a single compact extra dimension to general relativity. This is the simplest setting in which to introduce the Gregory–Laflamme instability (described in a chapter by Ruth Gregory) and the resulting pinch-off of the horizon (explained by Luis Lehner and Frans Pretorius). There is also a chapter (by Toby Wiseman and Gary Horowitz) on general Kaluza–Klein black holes, including solutions that are homogeneous in the extra dimension and those that are not.

In Part III some asymptotically flat higher dimensional black holes are described. The higher-dimensional analogues of the Kerr solution are Myers–Perry black holes (described in a chapter by Rob Myers). The first nonspherical black hole was the black ring discovered by (and explained by) Roberto Emparan and Harvey Reall. After these analytic solutions, the next part focuses on more general higher-dimensional black holes. There is a chapter by Greg Galloway on the possible topologies of higher-dimensional horizons, and a chapter by Roberto Emparan on blackfolds – a way of approximately constructing higher-dimensional black holes that realize some of these nontrivial topologies. Harvey Reall describes algebraically special solutions in higher dimensions and Toby Wiseman explains how to numerically construct general static and stationary black holes.

Essentially everything up to this point is based on the (higher-dimensional) Einstein equation with zero matter. The final section (which is more than a third of the book) adds some interesting matter fields and describes the recent applications of black holes to other areas of physics. It starts with a chapter by Don Marolf introducing the black holes and branes of ten- and eleven-dimensional supergravity. Juan Maldacena then explains the basic ideas behind the remarkable gauge/gravity

duality, which allows us to relate general relativity to nongravitational physics. Taking the long-wavelength limit of this duality yields a fluid/gravity correspondence (described in a chapter by Veronika Hubeny, Shiraz Minwalla, and Mukund Rangamani). Finally, Sean Hartnoll explains some aspects of the most recent application of gauge/gravity duality, namely to condensed matter physics.

A common theme is the connection between black hole horizons and fluid dynamics. This had already been noticed in the 1980s, with the development of the membrane paradigm, but it has become much more refined in recent years. Similarities to fluids are found in the way the horizon pinches off (Chapter 3), in the effective dynamics of black branes (Chapter 8) and, of course, the fluid/gravity correspondence (Chapter 13).

The book is pedagogical in style, and in most chapters it is assumed only that the reader has taken a standard course on general relativity. (Some chapters in the last part perhaps require a somewhat broader background.) The book should be a useful reference for students and researchers in string theory, high energy theory, and gravitational physics. Owing to the developments described in the final chapter, it may also be of interest to some condensed matter physicists.

I would like to thank the chapter authors for agreeing to contribute to this project and for writing excellent introductions or reviews of their respective topics. They made my job as editor as painless as possible. I also thank the National Science Foundation for providing funding for my research. Finally, I wish to thank my wife, Corinne, for her constant support.

Gary T. Horowitz