

ECONOPHYSICS OF INCOME AND WEALTH DISTRIBUTIONS

The distribution of wealth and income is never uniform, and philosophers and economists have tried for years to understand the reasons and formulate remedies for such inequalities. This book introduces the elegant and intriguing kinetic exchange models that physicists have developed to tackle these issues.

This is the first monograph in econophysics focused on the analyses and modelling of these distributions, and is ideal for physicists and economists. It explores the origin of economic inequality. It is written in simple, lucid language, with plenty of illustrations and in-depth analyses, making it suitable for researchers new to this field as well as more specialized readers.

BIKAS K. CHAKRABARTI is a Senior Professor of Theoretical Condensed Matter Physics at the Saha Institute of Nuclear Physics, and a visiting Professor of Economics at the Indian Statistical Institute. He has research interests in statistical physics, condensed matter physics, computational physics and econophysics.

ANIRBAN CHAKRABORTI is an Associate Professor at the Quantitative Finance Group, École Centrale Paris. He has research interests in statistical physics, econophysics and quantum physics.

SATYA R. CHAKRAVARTY is a Professor in the Economic Research Unit of the Indian Statistical Institute. His main areas of research interests are welfare economics, public economics, mathematical finance, industrial organization and game theory.

ARNAB CHATTERJEE is a Postdoctoral Researcher at Aalto University. He has research interests in statistical physics, and its application to condensed matter and social sciences.

ECONOPHYSICS OF INCOME AND WEALTH DISTRIBUTIONS

BIKAS K. CHAKRABARTI

Saha Institute of Nuclear Physics

ANIRBAN CHAKRABORTI

École Centrale Paris

SATYA R. CHAKRAVARTY

Indian Statistical Institute

ARNAB CHATTERJEE

Aalto University



CAMBRIDGE
UNIVERSITY PRESS

Cambridge University Press & Assessment

978-1-107-01344-5 — Econophysics of Income and Wealth Distributions

Bikas K. Chakrabarti, Anirban Chakraborti, Satya R. Chakravarty, Arnab Chatterjee

Frontmatter

[More Information](#)



CAMBRIDGE
UNIVERSITY PRESS

Shaftesbury Road, Cambridge CB2 8EA, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India

103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of Cambridge University Press & Assessment,
a department of the University of Cambridge.

We share the University's mission to contribute to society through the pursuit of
education, learning and research at the highest international levels of excellence.

www.cambridge.org

Information on this title: www.cambridge.org/9781107013445

© B. K. Chakrabarti, A. Chakraborti, S. R. Chakravarty and A. Chatterjee 2013

This publication is in copyright. Subject to statutory exception and to the provisions
of relevant collective licensing agreements, no reproduction of any part may take
place without the written permission of Cambridge University Press & Assessment.

First published 2013

A catalogue record for this publication is available from the British Library

Library of Congress Cataloging-in-Publication data

Chakrabarti, B. K. (Bikas K.), 1952–

Econophysics of income and wealth distributions / Bikas K. Chakrabarti,
Anirban Chakraborti, Satya R. Chakravarty, Arnab Chatterjee.

pages cm

Includes bibliographical references and index.

ISBN 978-1-107-01344-5

1. Income distribution. 2. Wealth. 3. Econophysics.

I. Chakraborti, Anirban. II. Chakravarty, Satya R. III. Chatterjee, Arnab. IV. Title.

HB523.C43 2013

339.2 – dc23 2012032861

ISBN 978-1-107-01344-5 Hardback

Cambridge University Press & Assessment has no responsibility for the persistence
or accuracy of URLs for external or third-party internet websites referred to in this
publication and does not guarantee that any content on such websites is, or will
remain, accurate or appropriate.

Contents

<i>Preface</i>	<i>page vii</i>
1 Introduction	1
2 Income and wealth distribution data for different countries	7
2.1 What are money, wealth and income?	7
2.2 Empirical analyses using data from earlier periods	8
2.3 Empirical analyses using data from recent periods	10
2.4 Measures of income inequality: Gini coefficient and Lorenz curve	28
3 Major socioeconomic modelling	35
3.1 Models of income distribution	35
3.2 Models of wealth distribution	36
3.3 Statistical equilibrium theory of markets	52
4 Market exchanges and scattering process	55
4.1 Gas-like models	55
4.2 Models with commodity	87
4.3 Models on networks	97
4.4 Models with debt	105
4.5 Models with tax	107
4.6 Other related models	108
5 Analytic structure of the kinetic exchange market models	114
5.1 Analytic results for the CC model	114
5.2 Analytic results for the CCM model	127
5.3 Analytic results for other models	139

vi	<i>Contents</i>	
6	Microeconomic foundation of the kinetic exchange models	150
6.1	Derivation of the basic kinetic exchange model	150
6.2	Production of a vector of commodities	156
6.3	A generalized version of the CC model	157
6.4	Inequality reversal	159
6.5	Global market	163
6.6	Steady-state distribution of money and price	164
6.7	Discussion	165
7	Dynamics: generation of income, inequality and development	168
7.1	The economic significance	168
7.2	Analysis of income distributions	169
7.3	Analysis of wealth distributions	182
8	Outlook	193
8.1	Chapters in a nutshell	194
8.2	Beyond income and wealth	195
8.3	Open problems and challenges	199
	<i>References</i>	201
	<i>Index</i>	213

Preface

An imbalance between rich and poor is the oldest and most fatal ailment of all republics.

Plutarch, ancient Greek biographer (c. 46–120 CE)

Why does this imbalance exist in the first place? Why are a few rich and many poor? For centuries we have borne the effects of this inequality. We know neither the cause nor the solution to this elusive problem. From philosophers to economists, many have vehemently tried for ages to understand the reasons and formulate remedies for such inequalities. No doubt, great efforts have been made to tackle this multifaceted problem, but the situation has been analogous to fighting the Greek mythological monster Hydra, who grows two heads in place of an injured one. Overcoming this problem, indeed, seems to be a Herculean task!

Heraclitus¹ said, ‘change is the only constant’. Putting our faith in him, one might have expected things to change drastically, and the inequality to even disappear at some point in time! Strangely, this has not been the case. We find that inequality has been a universal and robust phenomenon – not bound by either time or geography. Fortunately for scholars, it has a few statistical regularities, most of which have been recorded in the past 115 years or so. Owing to the seminal works of Pareto (1897) and Gibrat (1931), one can now identify certain regularities in the income and wealth distributions over a wide range of societies and time periods. Physicists have come up with some very elegant and intriguing kinetic exchange models in recent times to shed some light on these observations. Our intention is to describe these developments in this book.

Standard economic theory would like to consider that the activities of individual agents are driven by the utility maximization principle. The alternative picture proposed by physicists is that the agents can be simply viewed as gas particles exchanging ‘money’, in the place of energy, and trades as money (energy)

¹ Ancient Greek philosopher (c. 535–475 BCE).

conserving two-body scatterings, as in the entropy maximization-based kinetic theory of gases. This qualitative analogy seems to be quite old, and both economists and natural scientists had already noted it earlier in various contexts. However, this equivalence between the two maximization principles has gained firmer ground only recently.

When tested with empirical data from various countries, just pure kinetic exchange models fall short of accommodating the Pareto tail. However, the introduction of ‘saving propensity’ (in various forms) in such kinetic exchange models enables one to successfully explain several of the observed features, including the much desired Pareto tail. A direct link between the saving propensity distribution and the inequality can also be established. The subsequent developments in the analysis of these models further established many intriguing features in the observed data. The mathematical structures of these models and their economic implications are now being investigated extensively. As mentioned above, the discovery of the equivalence of the physical entropy and the utility or psychological satisfaction, and their corresponding maximization principles, marks the entry of the kinetic exchange models of market in the domain of macroeconomics.

Interestingly, the economic inequality is a natural outcome of this framework of stochastic kinetics of trading processes in the market, independent of any exogenous factors. Thus, the kinetic exchange models described in this book demonstrate how inequality may arise. They also indicate how its effects may be partially reduced by modifying the saving habits.

The book is organized as follows: the first chapter introduces the topic to the readers. In Chapter 2, a detailed presentation of the recorded data and analyses of the income and wealth distributions across various countries in different time periods is given. In Chapter 3, some of the major recent attempts to set up the physics-inspired many-body dynamical models for income or wealth exchanges, amongst the agents in the market or network, are discussed. In Chapter 4, the details of the numerical results for the kinetic exchange models for asset or income among the agents in the market are presented. Then, Chapter 5 gives the detailed analytical structure of such kinetic exchange models for the income and wealth distributions. Chapter 6 shows how, in two-person, two-commodity trading dynamics, the Cobb–Douglas utility maximization leads to the same kinetic exchange dynamics with uniform saving propensity, discussed in the earlier chapters. In Chapter 7, these kinetic exchange modelling approaches for income and wealth distributions leading to the economic inequalities are reviewed in terms of economics of income generation and development. Finally, we present an outlook with a brief summary of the chapters, a few discussions on new directions and open problems in the last chapter.

We are extremely grateful to all our collaborators: Urna Basu, Pratip Bhattacharyya, Anindya Sundar Chakrabarti, Guido Germano, Asim Ghosh,

Preface

ix

Sanchari Goswami, Els Heinsalu, Aymen Jedidi, Kimmo Kaski, Mehdi Lallouache, Subhrangshu Sekhar Manna, Sugata Marjit, Marco Patriarca, Srutarshi Pradhan, Parongama Sen, Sitabhra Sinha and Robin Stinchcombe, for their contributions to these developments. We acknowledge John Angle, J. Barkley Rosser Jr., Banasri Basu, Arnab Das, Kishore Dash, Deepak Dhar, Mauro Gallegati, Kausik Gangopadhyay, Abhijit Kargupta, Thomas Lux, Matteo Marsili, Pradeep K. Mohanty, Peter Richmond, Dietrich Stauffer, Victor M. Yakovenko and Sudhakar Yarlaga for their comments and criticisms on our work, from time to time. We are indebted to several scientists, especially Victor M. Yakovenko, who granted us the permission to use their works and figures liberally. We are grateful to Esteban Guevara and Gayatri Tilak for providing invaluable help during the preparation of the manuscript. We heartily thank Soumyajyoti Biswas, Anindya Sundar Chakrabarti, Marco LiCalzi and Gayatri Tilak for critical reading of the manuscript. We also express our gratitude to Anindya Sundar Chakrabarti for helping us rewrite Chapter 6. We are thankful to the Centre for Applied Mathematics and Computational Science Project of the Saha Institute of Nuclear Physics, Kolkata, for generous funding for the research activities in econophysics.

We hope that researchers, especially the younger ones, will find the ideas described in this book intriguing enough to inspire them to do further research and take up the Herculean challenge of solving this chronic problem, which is one of the pertinent sources of tragedy for human civilization.

Kolkata, India

Châtenay-Malabry, France

Kolkata, India

Espoo, Finland

*Bikas K. Chakrabarti**Anirban Chakraborti**Satya R. Chakravarty**Arnab Chatterjee*