1

Introduction

Ill fares the land, to hastening ills a prey,
Where wealth accumulates, and men decay.
Oliver Goldsmith,
Anglo-Irish writer (1730–74)

It would be difficult to find any society or country where income or wealth is equally distributed among its people. Socioeconomic inequality is not limited to modern times; it has been a persistent fact, and a constant source of irritation to most, since time immemorial.

The issue of inequality in terms of income and wealth is perhaps the most fiercely debated subject in economics. Economists and philosophers have spent much time on the normative aspects of this problem (Rawls 1971; Scruton 1985; Sen 1999; Foucault 2003). The direct and indirect effects of inequality on society have also been studied extensively. In particular, the effects of inequality on the growth of the economy (Benabou 1994; Aghion *et al.* 1999; Barro 1999; Forbes 2000) and on the econopolitical scenario (Blau and Blau 1982; Alesina and Rodrik 1992; Alesina and Perotti 1996; Benabou 2000) have attracted major attention. Relatively less emphasis has been put on the sources of the problem itself. There are several non-trivial issues and open questions related to this observation: How are income and wealth distributed? What are the forms of the distributions? Are they universal, or do they depend upon specific conditions of a country? Perhaps the most important question is: if inequality is universal (as some of its gross features indicate), then what is the reason for such universality?

Such questions have intrigued many personalities in the past. More than a century ago, Pareto made extensive studies in Europe and found that wealth distribution follows a power law tail for the richer sections of society (Pareto 1897), known now as the Pareto law. Separately, Gibrat (1931) worked on the same problem, and he proposed a 'law of proportionate effect'. Much later, Champernowne also considered this problem systematically and came up with a probabilistic theory to justify Pareto's claim (Champernowne 1953; Champernowne and Cowell 1998).

It was subsequently found in numerous studies that the distributions of income and wealth indeed possess some globally stable and robust features (for a review, see Yakovenko and Barkley Rosser 2009). In general, the bulk of the distribution of both income and wealth seems to fit both the log-normal and the gamma distributions reasonably well. Economists usually prefer the log-normal distribution (Gini 1921; Montroll and Shlesinger 1982), whereas statisticians (Hogg *et al.* 2007) and, more recently, physicists (Chatterjee *et al.* 2005b; Chatterjee and Chakrabarti 2007b; Yakovenko and Barkley Rosser 2009) tend to rely more on alternate forms such as the gamma distribution (for the probability density) or the Gibbs/exponential distribution (for the cumulative distribution). The upper end of the distribution, that is, the tail of the distribution, is agreed to be described well by a power law, as was found by Pareto.

These observed regularities in the income distribution may thus indicate a 'natural' law of economics. The distribution of income $P(x)$ is defined as follows: $P(x)dx$ is the probability that, in the 'equilibrium' or 'steady state' of the system,[1] a randomly chosen person would be found to have income between x and $x + dx$. Detailed empirical analyses of the income distribution so far indicate

$$P(x) \sim x^n \exp(-x/T), \quad \text{for} \quad x < x_c, \tag{1.1}$$

and

$$P(x) \sim x^{-\alpha-1}, \quad \text{for} \quad x \geq x_c, \tag{1.2}$$

where n and α are two exponents, and T denotes a scaling factor. The latter exponent α is called the Pareto exponent and its value ranges between 1 and 3 (e.g. Aoyama *et al.* 2000; Sinha 2006). A historical account of Pareto's data and that from recent sources can be found in Richmond *et al.* (2006). The crossover point x_c is extracted from the numerical fittings of the initial gamma distribution form to the eventual power law tail. One often fits the region below x_c to a log-normal form: $\log P(x) = \text{const} - (\log x)^2$. As mentioned before, although this form is often preferred by economists, the statisticians and physicists think that the gamma distribution form fits better with the data (see Salem and Mount 1974; Hogg *et al.* 2007; Yakovenko and Barkley Rosser 2009). Figure 1.1 shows the features of the cumulative income or wealth distribution.

Most of the empirical analyses, especially with recent income data, have been extensively reviewed in Chapter 2. Compared with the empirical work done on

[1] We will often be using the terms 'equilibrium' or 'steady state' interchangeably in this book; strictly speaking, for systems that are 'non-ergodic', one can only write 'steady state'.

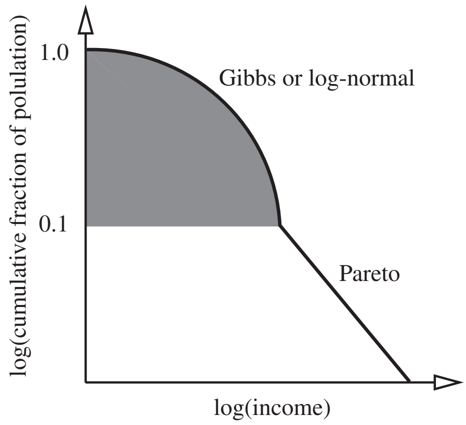

Figure 1.1 When one plots the cumulative wealth (income) distribution against the wealth (income), almost 90–95% of the population fits the Gibbs distribution, or is often fitted also to the log-normal form (Gibrat law), as indicated by the shaded region in the distribution; for the remaining (very rich) 5–10% of the population in any country, the number density falls off with their wealth (income) much more slowly, following a power law (Pareto law). It is found that approximately 40–60% of the total wealth of any economy is possessed by 5–10% of the people in the Pareto tail.

income distribution, relatively fewer studies have looked at the distribution of wealth, which consists of the net value of assets (financial holdings and/or tangible items) owned at a given instant. The lack of an easily available data source for measuring wealth, analogous to income tax returns for measuring income, means that one has to resort to indirect methods. Again, the general feature observed in the limited empirical study of wealth distribution, as presented in Chapter 2, is that of a power law behaviour for the wealthiest 5–10% of the population, and exponential or log-normal distribution for the rest of the population. The Pareto exponent, as measured from the wealth distribution, is always found to be lower than that for the income distribution, which is consistent with the general observation that, in market economies, wealth is much more unequally distributed than income (Samuelson 1998).

It is interesting to note that, when one shifts attention from the income of individuals to the income of companies, one still observes the power law tail. A study of the income distribution of Japanese firms (Aoyama *et al.* 2000; see also Aoyama *et al.* 2011) concluded that it follows a power law (with exponent value near unity, which is also often referred to as the Zipf law). Similar observation has been reported for the income distribution of companies in the USA (Axtell 2001).

These strikingly robust features of the distribution $P(x)$, in income or wealth, seem to be well established from the analyses of the enormous amount of data available today. Is it plausible that this only reflects a basic natural law, with simple physical explanation? Many econophysicists actually believe so. According to these proponents, the regular patterns observed in the income (and wealth) distribution are indeed indicative of a natural law for the statistical properties of a many-body dynamical system representing the entire set of economic interactions in a society, analogous to those previously derived for gases and liquids. By viewing the economy as a 'thermodynamic' system (Chakrabarti and Marjit 1995; Drăgulescu and Yakovenko 2000; Hayes 2002; Patriarca *et al.* 2010), one can liken income distribution to the distribution of energy among the particles in a gas. Several attempts by statisticians (e.g. Angle 1986, 2006) and economists (Bennati 1988a,b, 1993) also provide impetus to this interdisciplinary approach.

In particular, a class of kinetic exchange models (Chakraborti and Chakrabarti 2000; Chatterjee *et al.* 2003, 2004; Chakrabarti and Chatterjee 2004) have provided a simple mechanism for understanding the unequal accumulation of assets. While being simple from the perspective of economics, they have the benefit of gripping a key factor – savings – in socioeconomic interactions, which results in very different societies converging to similar forms of unequal distribution.

These simple microeconomic models, with a large number of 'agents' and the 'asset' transfer equations among the agents owing to 'trading' in such an economy, closely resemble the process of 'energy' transfer owing to 'collisions' among 'particles' like those in a thermodynamic system of ideal gas. In these models, the system is assumed to be made up of N agents with assets $\{x_i \geq 0\}$ $(i = 1, 2, \ldots, N)$. At every trade, an agent j exchanges a part Δx with another agent k chosen randomly. The total asset $X = \sum_i x_i$ is constant, as well as the average asset $\langle x \rangle = X/N$. After the exchange the new values x'_j and x'_k are $(x'_j, x'_k \geq 0)$

$$\left.\begin{aligned} x'_j &= x_j - \Delta x, \\ x'_k &= x_k + \Delta x. \end{aligned}\right\} \tag{1.3}$$

The form of the function $\Delta x = \Delta x(x_j, x_k)$ defines the underlying dynamics of the model. Figure 1.2 shows the schematic picture that captures the essence of these models.

The steady-state distribution for a system with pure random asset exchange is an exponential one, as was found by Gibbs 100 years ago (e.g. Chatterjee and Chakrabarti 2007b; Yakovenko and Barkley Rosser 2009). However, the introduction of 'saving propensity' (Chakraborti and Chakrabarti 2000) brought forth the gamma-like feature of the distribution $P(x)$ and such a random exchange model with uniform saving propensity for all agents was subsequently shown to be equivalent to a commodity clearing market in which each agent maximizes his/her own utility (Chakrabarti and Chakrabarti 2009). A further modification of the model

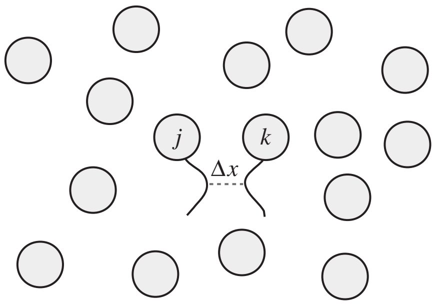

Figure 1.2 The kinetic exchange models prescribe a microscopic interaction between two units analogously to a kinetic model of gas in which, during an elastic collision, two generic particles j and k exchange an energy amount Δx, as in Eq. (1.3). Reproduced from Patriarca *et al.* (2010).

produces (Chatterjee *et al.* 2004) a power law for the upper or tail end of the distribution of money, as has been found empirically.

Several analytical aspects of this class of models have been studied (e.g. Ispolatov *et al.* 1998; Düring *et al.* 2008; Garibaldi and Scalas 2010; Lallouache *et al.* 2010b; Toscani and Brugna 2010). It is noteworthy that, at present, this is the only known class of models which, starting from the microeconomics of utility maximization and solving for the resultant dynamical equations in the line of rigorously established statistical physics, can quite reliably reproduce the major empirical features of income and wealth distributions in economies.

These developments have, of course, not gone without criticism (e.g. Hogan 2005; Lux 2005; Gallegati *et al.* 2006), and subsequent rebuttal (Richmond *et al.* 2006). In view of the embarrassing failure of mainstream economic schools to anticipate or correctly analyse the recent economic crisis, there has been some recent interest by the mainstream economic schools to revisit such physically motivated models of the market dynamics and their solutions (e.g. Lux and Westerhoff 2009).

The successive chapters of this book will review in detail the various aspects mentioned above. In Chapter 2, a detailed presentation of the recorded data and analyses of the income and wealth distributions across countries at different periods of time is given. The generic feature is, of course, as indicated in Fig. 1.1.

In Chapter 3, we discuss some of the major recent attempts to set up the physics-inspired many-body dynamical models for income or wealth exchanges among the agents in the market or network. Attempts are also made to compare the results with the established economic laws for the flow of money and the empirically observed distributions in society. In Chapter 4, we discuss in detail the numerical results for the kinetic exchange models for assets or income among the agents in the market. This development follows closely the century-old kinetic theory of gases, and models each trade as money (energy equivalent) conserving two-body collision leading to many-body steady-state or equilibrium distributions of money. As mentioned above, incorporation of saving propensity in the dynamics gives gamma-like distributions, while the dispersion in saving propensity among the agents leads to the Pareto tail from those gamma-like distributions. Chapter 5 gives the detailed analytical structure of such kinetic exchange models for income and wealth distributions. While the kinetic exchange dynamics discussed in Chapters 4 and 5 can essentially be viewed as 'entropy maximization' dynamics, it is shown to be equivalent to that following a utility maximization principle, as well. Chapter 6 shows how, in a two-person two-commodity trading dynamics, the Cobb–Douglas utility maximization leads to the same kinetic exchange dynamics with uniform saving propensity discussed in earlier chapters. These two maximization principles of physics and economics lead to identical dynamical equations. In Chapter 7, these econophysics models for income and wealth distributions leading to economic inequalities are reviewed from the perspective of economics of income generation and development. Extensive discussions on the various economic inequality indices, following the income and wealth distributions obtained in earlier chapters, are given here to cast these developments in proper economic perspectives. Finally, we present an outlook in Chapter 8, with a brief summary of the chapters and a few discussions on new directions, challenges and open problems.

2

Income and wealth distribution data for different countries

Investigations over more than a century and the recent availability of electronic databases of income and wealth distribution (ranging from a national sample survey of household assets to the income tax return data available from governmental agencies) have revealed some remarkable features. Irrespective of many differences in culture, history, social structure, indicators of relative prosperity (such as gross domestic product or infant mortality) and, to some extent, the economic policies followed in different countries, the income distribution seems to follow a particular *universal* pattern, as does the wealth distribution: after an initial rise, the number density of people rapidly decays with their income, the bulk described by a Gibbs or log-normal distribution crossing over at the very high income range (for 5–10% of the richest members of the population) to a power law, as shown in Fig. 1.1. The power law in income and wealth distribution is called the *Pareto law*, after the Italian sociologist and economist Vilfredo Pareto. The log-normal part is named as the *Gibrat law*, after the French economist Robert Gibrat. This seems to be a universal feature: from ancient Egyptian society (Abul-Magd 2002) through nineteenth-century Europe (Pareto 1897; Champernowne 1953) to modern Japan (Chatterjee *et al.* 2005b; Chakrabarti *et al.* 2006). The same is true across the globe today: from the advanced capitalist economy of USA (Chatterjee *et al.* 2005b; Kar Gupta 2006a; Richmond *et al.* 2006) to the developing economy of India (Sinha 2006).

A historical account of the empirical data analyses, followed by an account of research using recent sources, will be presented in this chapter. Country-wise studies at various time periods will be also presented. Measures of income inequality in terms of the Gini coefficient and other indices will be briefly reviewed.

2.1 What are money, wealth and income?

Let us start by considering the basic economic quantities: money, wealth and income. A common definition of *money* suggests that money is ‘a commodity accepted by general consent as a medium of economic exchange'.[1] In fact, money circulates from one economic agent (which can represent an individual, firm, country, etc.) to another, thus facilitating trade. It is 'something which all other goods or services are traded for' (for details, see Shostak 2000). Throughout history various commodities have been used as money, for these cases termed 'commodity money', which include for example rare seashells or beads and cattle (such as the cow in India). Recently, 'commodity money' has been replaced by other forms referred to as 'fiat money', which have gradually become the most common ones, such as metal coins and paper notes. Nowadays, other forms of money, such as electronic money, have become the most frequent form used to carry out transactions. In any case the most relevant points about money employed are its basic functions, which according to standard economic theory are:

- to serve as a medium of exchange, which is universally accepted in trade for goods and services;
- to act as a measure of value, making possible the determination of the prices and the calculation of costs, or profit and loss;
- to serve as a standard of deferred payments, i.e. a tool for the payment of debt or the unit in which loans are made and future transactions are fixed;
- to serve as a means of storing wealth not immediately required for use.

A related feature relevant for the present investigation is that money is the medium in which prices or values of all commodities as well as costs, profits and transactions can be determined or expressed. *Wealth* is usually understood as things that have economic utility (monetary value or value of exchange), or material goods or property; it also represents the abundance of objects of value (or riches) and the state of having accumulated these objects; for our purpose, it is important to bear in mind that wealth can be measured in terms of money. Also *income*, defined in Case and Fair (2008) as 'the sum of all the wages, salaries, profits, interests payments, rents and other forms of earnings received . . . in a given period of time', is a quantity which can be measured in terms of money (per unit time).[2]

2.2 Empirical analyses using data from earlier periods

It was first observed by Pareto (1897) that in an economy the higher end of the distribution of income $f(x)$ follows a power law,

$$P(x) \sim x^{-1-\alpha}, \tag{2.1}$$

[1] In *Encyclopædia Britannica*. Retrieved 18 June 2012 from Encyclopædia Britannica Online.
[2] See Chakraborti *et al.* (2011).

with α, now known as the Pareto exponent, estimated by him to be $\alpha \approx 3/2$. For the last hundred years the value of $\alpha \sim 3/2$ seems to have changed little in time and across the various capitalist economies (see Yakovenko and Barkley Rosser 2009, and references therein). The normalized Pareto distribution has the form

$$P(x) \sim \begin{cases} F(x) & \text{for } x < x_c, \\ \frac{\alpha x_c^{\alpha}}{x^{1+\alpha}} & \text{for } x \geq x_c, \end{cases} \tag{2.2}$$

where $F(x)$ is sometimes assumed to be $x^n \exp(-x/T)$ in the research communities; n and T are two constants. The above distribution has a scale x_c, which denotes a crossover from one kind of distribution to another, separating the low and middle wealth from the large wealth regime.

Gibrat (1931) clarified that Pareto's law is valid only for the high-income range, whereas for the middle-income range he suggested that the income distribution is described by a log-normal probability density

$$P(x) \sim \frac{1}{x\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{\log^2(x/x_0)}{2\sigma^2}\right\}, \tag{2.3}$$

where $\log(x_0) = \langle\log(x)\rangle$ is the mean value of the logarithmic variable and $\sigma^2 = \langle[\log(x) - \log(x_0)]^2\rangle$ is the corresponding variance. The factor $\beta = 1/\sqrt{2\sigma^2}$, also known as the Gibrat index, measures the equality of the distribution. Empirically, β is known to lie between 2 and 3 (Souma 2002).[3]

Abul-Magd (2002) studied the wealth distribution in ancient Egypt. Excavations of the ancient Egyptian city Akhetaten, which was populated for only a brief period during the fourteenth century BC, have yielded a distribution of the house areas. Abul-Magd assumed that the house area is a measure of the wealth of its inhabitants, and made a comparative study of the wealth distributions in ancient and modern societies. According to his analysis of the wealth distribution in Akhetaten, the best-fit value of the Pareto exponent for the resulting distribution is $\alpha = 1.59 \pm 0.19$, which agrees very well with the values of the Pareto index obtained for modern societies.

Hegyi *et al.* (2007) also found a power law tail for the wealth distribution of aristocratic families in medieval Hungary. Hegyi *et al.* assumed that the number of serf families belonging to a noble is a measure of the corresponding wealth. He obtained a Pareto law for such a society with Pareto index $\alpha = 0.92$, which is smaller than the values reported for studies of the current period. The results obtained are plotted in Fig. 2.1.

[3] Historically speaking, Gibrat also analysed the firm size distribution and he proposed a 'law of proportionate effect'. This stated that a small change in a quantity is independent of the quantity itself. Thus, the distribution of a quantity $dz = dx/x$ should be Gaussian, and hence x as log-normal. A random variable is said to be log-normally distributed if its logarithm is normally distributed, also now known as *Gibrat's law*.

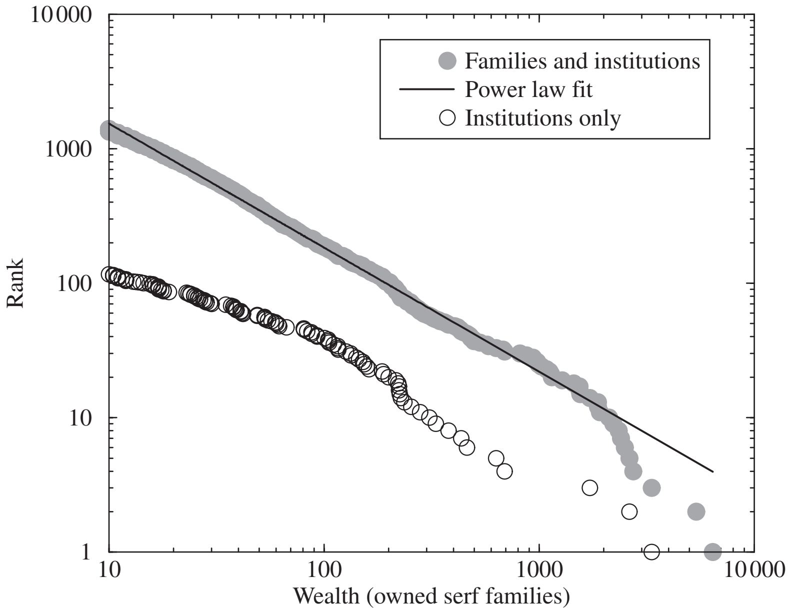

Figure 2.1 The rank of the top 8% aristocratic families and institutions as a function of their estimated total wealth on a double-logarithmic scale. Estimations made for the Hungarian noble society in the year 1550, in which the total wealth of a family is taken as the number of owned serf families. The power law fit suggests a Pareto index $\alpha = 0.92$. Reproduced from Hegyi *et al.* (2007).

Souma (2001) investigated the Japanese personal income distribution in the high-income range over the 112 years 1887–1998, and that in the middle-income range over the 44 years 1955–98. His data analysis revealed that the personal income followed the log-normal distribution (Gibrat) with a power law tail (Pareto). Since the same behaviour was observed in the analysis for the different years, it can be considered a statistical regularity. Figures 2.2 and 2.3 show the variations of the exponents that were obtained from the analyses of the log-normal distributions (for Gibrat index β) and the power law tails (for Pareto index α).

2.3 Empirical analyses using data from recent periods

According to Pareto (1897), ‘the society is not homogeneous’. Hence, Pareto and many others were of the opinion that the distribution of income in a particular society would be an excellent indicator of non-homogeneity of any society. Thus, it would be interesting to study the empirical data and analyse the money, wealth and income distributions. Unfortunately, empirical data of money and wealth are rather scarce, and more data are available for the distribution of income from tax