QUANTUM SOCIAL SCIENCE

Written by world experts in the foundations of quantum mechanics and its applications to social science, this book shows how elementary quantum mechanical principles can be applied to decision making paradoxes in psychology, and used in modeling information in finance and economics.

The book starts with a thorough overview of some of the salient differences between classical, statistical, and quantum mechanics. It presents arguments on why quantum mechanics can be applied outside of physics and defines quantum social science. The issue of the existence of quantum probabilistic effects in psychology, economics, and finance is addressed and basic questions and answers are provided. Aimed at researchers in economics and psychology, as well as physics, basic mathematical preliminaries and elementary concepts from quantum mechanics are defined in a self-contained way.

EMMANUEL HAVEN is a Professor at the School of Management, University of Leicester, UK. He has published numerous articles in a variety of fields, such as operations research, economics, and finance.

ANDREI KHRENNIKOV is a Professor of Applied Mathematics at Linnaeus University, Sweden, and Director of the International Centre for Mathematical Modelling in Physics, Engineering, Economics and Cognitive Science.

Cambridge University Press
978-1-107-01282-0 - Quantum Social Science
Emmanuel Haven, and Andrei Khrennikov
Frontmatter
More information

QUANTUM SOCIAL SCIENCE

EMMANUEL HAVEN
University of Leicester

AND

ANDREI KHRENNIKOV
Linnaeus University

CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town,
Singapore, São Paulo, Delhi, Mexico City

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9781107012820

First published 2013

Printed and bound in the United Kingdom by the MPG Books Group

A catalogue record for this publication is available from the British Library

Library of Congress Cataloguing in Publication data
Haven, Emmanuel, 1965–
Quantum social science / Emmanuel Haven and Andrei Yu. Khrennikov.
p. cm.
Includes bibliographical references and index.
ISBN 978-1-107-01282-0 (hbk.)
1. Social sciences – Mathematical models. 2. Quantum theory.
I. Khrennikov, A. IU. (Andrei IUr'evich), 1958– II. Title.
H61.25.H38 2012
300.1′53012 – dc23 2012027126

ISBN 978-1-107-01282-0 Hardback

To our lovely Wives – Irina and Sophie

To our lovely Children – Anton, Nath, and Sam

Contents

Cambridge University Press
978-1-107-01282-0 - Quantum Social Science
Emmanuel Haven, and Andrei Khrennikov
Frontmatter
More information

Foreword

This new book by Emmanuel Haven and Andrei Khrennikov argues that information processing in social systems can to a degree be formalized with the mathematical apparatus of quantum mechanics. This is a novel approach. Understanding decision making is a central objective of economics and finance and the quantum like approach proposed here, is used as a tool to enrich the formalism of such decision making. Emmanuel and Andrei argue for instance that probability interference can be used to explain the violation of the law of total probability in well known paradoxes like the Ellsberg decision making paradox.

Emmanuel and Andrei's book forms one of the very first contributions in a very novel area of research. I hope this book can open the road for many new books to come. More new results are needed, especially in the area of decision making.

H. Eugene Stanley
William Fairfield Warren Distinguished Professor;
Professor of Physics; Professor of Chemistry;
Professor of Biomedical Engineering;
Professor of Physiology (School of Medicine)
Director, Center for Polymer Studies,
Department of Physics, Boston University

By chance a few days before Andrei Khrennikov and Emmanuel Haven asked me to write this Foreword to their new book *Quantum Social Science,* I was browsing the collected works of Wolfgang Pauli, *Writings on Physics and Philosophy,* eds. Charles P. Enz and Karl von Meyenn, Springer (1994). I was just coming off a busy semester, including teaching a rather advanced course on harmonic analysis and quantum physics. To those erstwhile Ph.D. students in mathematics and physics, I had found myself counseling them with utterances such as "look, all physicists need to think semi-classically or even classically," or "you have to do something, you cannot just say it is all random motion," or "Heisenberg didn't really understand mathematics, but his intuition was sufficient to guide him."

Therefore I was very pleased to see Haven and Khrennikov also going to some of Pauli's thoughts in their Preface. Pauli, one of the greatest thinkers on quantum mechanics, was often preoccupied with the interaction of experiment with observer, and in analogy with the interaction of the conscious with the unconscious. Pauli's advocacy of the coupling of objective quantum physics to the subjective, e.g. psychic, was patterned upon Bohr's fundamental notion of complementarity. Two mutually contradictory concepts, e.g. those of particle and wave, may co-exist.

Indeed, quantum mechanics has forced upon us a new reality, possessing many co-existing dualities. One has the Schrödinger picture of differential equations describing all the chemical elements upon which the universe depends, and the Heisenberg picture stressing more the probabilistic nature of scattering interactions. The two pictures were more or less reconciled by Born in 1926, with his concept of probability wave. I have reviewed the Born probability interpretation of quantum mechanics from its inception to the present in K. Gustafson, *The Born Rule*, AIP Proceedings 962 (2007) pp. 98–107. I detailed in that review how often the great pioneers of quantum theory had to resort to reasonings of classical physics. So one should not think that quantum mechanics is all " hocus-pocus." Quantum mechanics is grounded in reality.

On the other hand, it is quite important to stress that the Born interpretation places the physics into an abstract configuration space, and not in real 3d space. As a consequence, from then on one must rely on the mathematics. Quantum mechanics has generated some very powerful mathematics. Ideally, this then should be coupled with new quantum-like thinking that one will not find in classical physics. It is the authors' intention in the present book to apply these powerful new mathematical tools and the evolving new non-classical quantum intuition to social science, behavioral economics, decision theory, and financial engineering.

Both authors already have considerable experience in this endeavor. Andrei Khrennikov is the founder of the celebrated series of annual quantum physics conferences held in Växjö Sweden for the last dozen years. At those conferences Emmanuel Haven from the economics side has joined with Khrennikov in recent years to organize special sessions on the subject matter of this book. Khrennikov has previously put forth his thinking in two books, *Information Dynamics in Cognitive, Psychological and Anomalous Phenomena*, Kluwer (2004), and *Ubiquitous Quantum Structure: From Psychology to Finance,* Springer (2010). Haven brings to the present book more expertise in economics and finance.

Overall, one could describe their basic approach as that of embedding situations from the social or economic sciences into a quantum mechanical context and then using the methods of the latter to obtain new insights and results for the former.

Such approach presumes of the reader a substantial knowledge of both contexts, that of quantum mechanics, and that of the particular social field of application. That is asking a lot.

I chose to address this issue, that of more needed interdisciplinary competence in education, science, and the general public, in my recent autobiography *The Crossing of Heaven: Memoirs of a Mathematician*, Springer (2012). I have come to the conclusion that we must invoke and enforce a new term, that of Multidisciplinarity. Interdisciplinarity is a weak word. It implies that one is less than one hundred percent committed to each of the two fields. Or that one is slightly weak in one's own field and leaning on an expert from the other field, who is probably a bit weak also in his field. I have worked successfully in several fields of science and I can assure you that you should plan on becoming an expert also in "the other field," and that will take you, say, at least five years before you have a chance of becoming competitive there.

Thus a collateral message of this foreword is that of advancing the concept and indeed the cause of creating more multidisciplinarity in our future mathematicians, physicists, social scientists, and, in a more general sense, throughout the educated public. A tall order! But great opportunities will open up to those who are strong enough.

This book by Haven and Khrennikov is a move in that direction, a pioneering effort.

Karl Gustafson
Professor Of Mathematics
University of Colorado at Boulder

Preface

The current level of specialization of knowledge in a variety of fields of inquiry may make it quite challenging for a researcher to be at the same time a "developer" and a "tester" of a theory. Although a theory can exist without a necessary clear and obvious practical end goal, the ultimate test of the validity of a theory (whether it is situated in the exact or social sciences) will always be how measurement can "confirm" or dislodge a theory.

This book is largely dedicated to the *development* of a theory. We will be the very first to accept the accusation that the duo "theory-test" is widely absent in this work, and we believe it necessary to make this statement at the very beginning.

This book is about a very counter-intuitive development. We want to use a physics machinery which is meant to explain sub-atomic behavior, in a setting which is at the near opposite end of the size spectrum, i.e. the world as we know and live it through our senses. We may know about the sub-atomic world, but we do not have human experience of the sub-atomic world. Do we have credible and provable stories which can explain how the sub-atomic engages into the mechanics of the statistical macro-world? Probably not. Why do we bother then about being so exotic? The interested reader will want us to provide for a satisfactory answer to this obvious question, and we want to leave it up to him or her to decide whether we have *begun*, via the medium of this book, to convince that the level of "exoticality" (and "yes" how exotic is that word?) is sensibly less than anticipated. We can possibly give a glimmer of "hope," even at this early stage. Consider the words of one of the towering giants of physics of the twentieth century – Wolfgang Pauli. In an unpublished essay by Pauli, entitled "Modern examples of 'background physics'," which is reproduced in Meier* (pp. 179–196), we can read Pauli's words (Meier* (p. 185)): "Complementarity in physics . . . has a very close analogy with the terms 'conscious' and 'unconscious' in psychology in

* Meier C. A. (2001). *Atom and Archetype: The Pauli/Jung Letters, 1932–1958.* Princeton University Press.

that any ‘observation’ of unconscious contents entails fundamentally indefinable repercussions of the conscious on these very contents.” The words of Pauli are important. They show there is promise for a connection between “concepts” of utmost importance in two very different sciences: complementarity in quantum physics and “complementarity” between consciousness and unconsciousness in psychology.

In this book, we intend to give the reader a flavor of an intellectual development which has taken shape over several years via the usual media many academics use: conference presentations and academic articles. The theory presented here is nowhere complete but we strongly believe that it merits presentation in book form.

The models presented in this book can be called “quantum-like.” They do not have a direct relation to quantum physics. We emphasize that in our approach, the quantum-like behavior of human beings is not a consequence of quantum physical processes in the brain. Our basic premise is that information processing by complex social systems can be described by the mathematical apparatus of quantum mechanics. We present quantum-like models for the financial market, behavioral economics, and decision making.

Connecting exact science with social science is not an easy endeavor. What reveals to be most difficult is to dispel an intuition that somehow there *should* exist a natural bridge between physics and the modeling of social systems. This is a very delicate issue. As we have seen above it is possible to think of “complementarity” as a concept which could bridge physics and psychology. However, in some specific areas of social systems, the “physics equivalent” of the obtained results may have very little meaning.

It is our sincere hope that with this book we can convince the brave reader that the intuition of the authors is not merely naive, but instead informative. Hence, may we suggest that “reading on” is the command of the moment? Let the neurons fire!

Acknowledgements

Luigi Accardi and A. Khrennikov and M. Ohya (2009). Quantum Markov model for data from Shafir-Tversky experiments in cognitive psychology. *Open Systems & Information Dynamics*, 16(4), 378–383. This material is reproduced with permission of World Scientific Publishing Co Pte Ltd.

Masanari Asano and M. Ohya and A. Khrennikov (2011). Quantum-like model for decision making process in two players game. *Foundations of Physics*, 41, 538–548. Foundations of Physics by Springer New York LLC. Reproduced with permission of Springer New York LLC in the format reuse in a book/textbook via Copyright Clearance Center.

Masanari Asano and M. Ohya and Y. Tanaka and I. Basieva and A. Khrennikov (2011). Quantum like model of brain's functioning: decision making from decoherence. *Journal of Theoretical Biology*, 281, 63–64. Journal of Theoretical Biology by Academic Press. Reproduced with permission of Academic Press in the format reuse in a book/textbook via Copyright Clearance Center.

Sheldon Goldstein (2010). Bohmian mechanics and quantum information. *Foundations of Physics*, 40, 335–355. © Springer Science+Business Media, LLC 2009. Quotes reproduced with kind permission from Springer Science+Business Media: Foundations of Physics.

Emmanuel Haven (2005). Analytical solutions to the backward Kolmogorov PDE via an adiabatic approximation to the Schrodinger PDE. *Journal of Mathematical Analysis and Applications*, 311, 442–443. © 2005. Reprinted with Permission from Elsevier Inc.

Emmanuel Haven (2008). Private information and the "information function": a survey of possible uses. *Theory and Decision*, 64, 200–201. © Springer 2007. Reprinted with kind permission from Springer Science+Business Media.

Emmanuel Haven (2008). The variation of financial arbitrage via the use of an information wave function. *International Journal of Theoretical Physics*, 47, 196–199. © Springer Science+Business Media, LLC 2007. Reprinted with kind permission from Springer Science+Business Media, LLC 2007.

Emmanuel Haven (2010). The Blackwell and Dubins theorem and Rényi's amount of information measure: some applications. *Acta Applicandae Mathematicae*, 109, 751–757. © Springer Science+Business Media B.V. 2008. Reprinted with kind permission from Springer Science+Business Media B.V. 2008.

Emmanuel Haven (2011). Itô's lemma with quantum calculus: some implications. *Foundations of Physics*, 41, 3, 530–536. © Springer Science+Business Media, LLC 2010. Reprinted with kind permission from Springer Science+Business Media.

Peter Holland (2000). *The Quantum Theory of Motion: An Account of the de Broglie–Bohm Causal Interpretation of Quantum Mechanics*. © Cambridge University Press 1995. Reproduced with permission.

Hiromu Ishio and E. Haven (2009). Information in asset pricing: a wave function approach. *Annalen der Physik*, 18, 1, 40–42. © 2009 Wiley-VCH Verlag GmbH & Co. KGaA, Weinheim. Reprinted with kind permission from J. Wiley.

Max Jammer (1974). *The Philosophy of Quantum Mechanics*. New York: J. Wiley. This material is reproduced with permission of John Wiley & Sons, Inc.

Andrei Khrennikov (2010). *Ubiquitous Quantum Structure: From Psychology to Finance, Prisoner's Dilemma*. © Springer-Verlag, Berlin, Heidelberg 2010. Reprinted with kind permission from Springer Science+Business Media.

Andrei Khrennikov (2011). Quantum-like model of processing of information in the brain based on classical electromagnetic field. *Biosystems*, 105, 251–259. Biosystems by Elsevier Science. Reproduced with permission of Elsevier Science.

C. A. Meier (2001). *Atom and Archetype: The Pauli/Jung Letters, 1932–1958.* © Princeton University Press 2001. Reprinted with permission.

Michael A. Morrison (1990). *Understanding Quantum Physics: A User's Manual*. Volume 1. 1st edition. © 1990. Reprinted by permission of Pearson Education Inc., Upper Saddle River, NJ.

Salih Neftci (2000). *An Introduction to the Mathematics of Financial Derivatives*, pp. 346–358. © Elsevier 2000.

Hal R. Varian (1992). *Microeconomic Analysis*. 3rd edition. © 1992, 1984, 1978 W. W. Norton and Company, Inc. Used by Permission of W. W. Norton and Company Inc.

List of symbols

Some mathematics symbols used in the book

- $\mathbb{R}$: space of real numbers
- $\mathbb{C}$: space of complex numbers
- $\rho(.,.)$: probability density function (2 dimensional)
- $\rho(t,.,.)$: time-dependent probability density function
- $L_2(\mathbb{R}^3)$: space of square integrable complex valued functions $\psi : \mathbb{R}^3 \to \mathbb{C}$
- $H = L_2(\mathbb{R}^3)$: complex Hilbert space with a scalar product
- $l.i.m$: limit in the mean square sense
- $\mathcal{P} = (\Omega, F, P)$: $\mathcal{P}$ is a probability space and points ω of Ω (which is a non-empty set) are said to be elementary events. F is a so-called σ-algebra and P is a probability measure
- $m.s.$: mean square
- $\delta(x - x_0)$: Dirac δ-"function"
- $\mathbf{P}^{b|a}$: matrix of transition probabilities
- $d_q f(x)$: q differential of a function $f(x)$
- $d_h f(x)$: h differential of a function $f(x)$

Some physics symbols used in the book

- m: mass
- a: acceleration
- f: force acting on particle
- V: real potential function
- ϕ, S: phase of a wave function
- ν: frequency
- t: time
- ∇V: gradient of the real potential function

- p: momentum
- q: position
- $\mathcal{H}(.,.)$: Hamiltonian function
- $\{f, g\}$: Poisson bracket of two functions f and g on an N particle phase space
- $\{f_1, f_2\}$: Poisson bracket for a pair of classical observables f_1, f_2
- $\phi(t, x, y, z,)$: field state at instant t of vector with coordinates x, y and z
- $E(t, x, y, z,)$: electrical field at instant t of vector with coordinates x, y and z
- $B(t, x, y, z,)$: magnetic field at instant t of vector with coordinates x, y and z
- h: Planck's constant
- $\hbar$: rationalized Planck constant
- $\Delta E_{ij} = E_i - E_j$: discrete portion of energy
- L: angular momentum of an electron
- I: intensity of the electromagnetic field
- $A = (a_{ij})$: Hermitian matrix
- $\widehat{\mathcal{H}}$: Hermitian matrix representing the energy observable (quantum Hamiltonian)
- $\hat{q}$: position operator
- $\hat{p}$: momentum operator
- σ_x: standard deviation of position
- σ_p: standard deviations of momentum
- Δ_{q_j}: Laplace operator
- $\psi(t, q)$: probability amplitude on time, t, and position, q
- Γ: phase space of hidden states
- $|\psi\rangle$: element of the Hilbert space H : a ket vector
- $\langle\phi|$: element of the dual space H^*, the space of linear continuous functionals on H: a bra vector
- $\langle\psi_1|\widehat{w}\psi_2\rangle$: Dirac braket, where ψ_1^* denotes the complex conjugate of ψ_1 and $\widehat{w}$ acts on the state function ψ_2.
- k: wave number
- $A(k)$: amplitude function of wave number k
- $\langle p\rangle$: average momentum
- Q: quantum potential
- $\mathbf{P}(.|C)$: conditional probability dependent on the context, C
- D_+: mean forward derivative
- D_-: mean backward derivative

Some economics/finance symbols used in the book

- σ: volatility
- $\alpha(\sigma)$: drift function of volatility
- $\beta(\sigma)$: diffusion function of volatility

- dX, dz, dW: Wiener process
- $\overrightarrow{q} = (q_1, q_2 \ldots q_n)$: n-dimensional price vector
- m_j: number of shares of stock j
- $T_j(t)$: market capitalization of trader j at time t
- $V(q_1, \ldots, q_n)$: interactions between traders as well as interactions from other macro-economic factors
- Π: portfolio value
- F: financial option price
- S: stock price
- $\Delta = \frac{\partial F}{\partial S}$: delta of the option
- f_u; f_d: intrinsic values of the option when the price of the asset is respectively going up and down
- $E(r)$: expected return
- $\delta\Pi$: discrete change in the value of the portfolio, Π
- μ: expected return
- dF: infinitesimal change in F (the option price)
- r_f: risk free rate of interest
- $\phi(S, t)$: part of the premium invested in the stock, S
- S_T: asset price at the expiration of the option contract
- S_0: asset price at the inception of the option contract
- $P(., .|., .)$: conditional probability distribution
- $E[S_T|I_t]$: conditional expectation of a stock price at time $T > t$, given the information you have at time t
- $E(e^{Y_t\lambda})$: moment generating function, λ is some arbitrary parameter, and Y_t follows a probability density function (pdf) with mean μt and $\sigma^2 t$
- $E^{\widetilde{P}}[., .]$: expectation with respect to a risk neutral probability measure $\widetilde{P}$
- $E^P[., .]$: expectation with respect to a probability measure P
- C_t: option call value at time t
- P_t: option put value at time t
- $\overrightarrow{\Phi} = (\Phi_1, \Phi_2, \ldots, \Phi_K)$: K-dimensional state price vector
- $\overrightarrow{D_1}, \ldots, \overrightarrow{D_K}$: security price vector at time t_1, if the market is, respectively, in state $1, \ldots, K$
- λ: Lagrangian multiplier
- $E(u(W))$: expected utility of wealth, W
- $\succ$: preference relation
- $\succeq$: weak preference relation
- β_i: CAPM - Beta of asset i