# The History of Mathematical Proof in Ancient Traditions

This radical, profoundly scholarly book explores the purposes and nature of proof in a range of historical settings. It overturns the view that the first mathematical proofs were in Greek geometry and rested on the logical insights of Aristotle by showing how much of that view is an artefact of nineteenth-century historical scholarship. It documents the existence of proofs in ancient mathematical writings about numbers, and shows that practitioners of mathematics in Mesopotamian, Chinese and Indian cultures knew how to prove the correctness of algorithms, which are much more prominent outside the limited range of surviving classical Greek texts that historians have taken as the paradigm of ancient mathematics. It opens the way to providing the first comprehensive, textually based history of proof.

*Jeremy Gray, Professor of the History of Mathematics, Open University*

'Each of the papers in this volume, starting with the amazing "Prologue" by the editor, Karine Chemla, contributes to nothing less than a revolution in the way we need to think about both the substance and the historiography of ancient non-Western mathematics, as well as a reconception of the problems that need to be addressed if we are to get beyond myth-eaten ideas of "unique Western rationality" and "the Greek miracle". I found reading this volume a thrilling intellectual adventure. It deserves a very wide audience.'

*Hilary Putnam, Cogan University Professor Emeritus, Harvard University*

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# Contents

# Figures

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# Note on references

The following books are frequently referred to in the notes. We use the following abbreviations to refer to them.

CG2004 Chemla, K. and Guo Shuchun (2004) *Les Neuf Chapitres: le classique mathématique de la Chine ancienne et ses commentaires*. Paris.

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N1999 Netz, R. (1999) *The Shaping of Deduction in Greek Mathematics*. Cambridge.

T1893/5 Tannery, P. (1893–5) *Diophanti Alexandrini opera omnia cum graecis commentariis, edidit et latine interpretatus*, vol. I: 1893; vol. II: 1895. Leipzig.

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