Structural Geology Algorithms

Vectors and Tensors

State-of-the-art analysis of geological structures has become increasingly quantitative, but traditionally, graphical methods are used in teaching and in textbooks. Now, this innovative lab book provides a unified methodology for problem solving in structural geology using linear algebra and computation. Assuming only limited mathematical training, the book builds from the basics, providing the fundamental background mathematics, and demonstrating the application of geometry and kinematics in geoscience without requiring students to take a supplementary mathematics course.

Starting with classic orientation problems that are easily grasped, the authors then progress to more fundamental topics of stress, strain, and error propagation. They introduce linear algebra methods as the foundation for understanding vectors and tensors. Connections with earlier material are emphasized to allow students to develop an intuitive understanding of the underlying mathematics before introducing more advanced concepts. All algorithms are fully illustrated with a comprehensive suite of online MATLAB® functions, which build on and incorporate earlier functions, and which also allow users to modify the code to solve their own structural problems.

Containing 20 worked examples and over 60 exercises, this is the ideal lab book for advanced undergraduates or beginning graduate students. It will also provide professional structural geologists with a valuable reference and refresher for calculations.

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Preface

Structural geology has been taught, largely unchanged, for the last 50 years or more. The lecture part of most courses introduces students to concepts such as stress and strain, as well as more descriptive material like fault and fold terminology. The lab part of the course usually focuses on practical problem solving, mostly traditional methods for describing quantitatively the geometry of structures. While the lecture may introduce advanced concepts such as tensors, the lab commonly trains the student to use a combination of graphical methods, such as orthographic or spherical projection, and a variety of plane trigonometry solutions to various problems. This leads to a disconnect between lecture concepts that require a very precise understanding of coordinate systems (e.g., tensors) and lab methods that appear to have no common spatial or mathematical foundation. Students have no chance to understand that, for example, seemingly unconnected constructions such as down-plunge projections and Mohr circles share a common mathematical heritage: They are both graphical representations of coordinate transformations. In fact, it is literally impossible to understand the concept of tensors without understanding coordinate transformations. And yet, we try to teach students about tensors without teaching them about the most basic operations that they need to know to understand them.

The basic math behind all of these seemingly diverse topics consists of linear algebra and vector operations. Many geology students learn something about vectors in their first two semesters of college math, but are seldom given the opportunity to apply those concepts in their chosen major. Fewer students have learned linear algebra, as that topic is often reserved for the third or fourth semester math. Nonetheless, these basic concepts needed for an introductory structural geology course can easily be mastered without a formal course; we assume no prior knowledge of either. On one level, then, this book teaches a consistent approach to a subset of structural geology problems using linear algebra and vector operations. This subset of structural geology problems coincides with those that are usually treated in the lab portion of a structural geology course.

The linear algebra approach is ideally suited to computation. Thirty years after the widespread deployment of personal computers, most labs in structural geology teach students increasingly arcane graphical methods to solve problems. Students are taught the operations needed to solve orientation problems on a stereonet, but that does not teach them the mathematics of rotation. Thus, a stereonet, either digital or analog version, is nothing more than a graphical black box. When the time comes for the student to solve a more involved problem – say, the rotation of principal stresses into a fault plane coordinate system – how will they know how to proceed? Thus, on another level, one can look at this book as a structural geology lab manual for the twenty-first century, one that teaches students how to solve problems by computation rather than by graphical manipulation.

The concept of a twenty-first century lab manual is important because this book is not a general structural geology text. We make no attempt to provide an understanding of deformation, rather we focus on how to describe and analyze structures quantitatively. Nonetheless, the background and approach is common to that of modern continuum mechanics treatments. As such, the book would make a fine accompaniment to recent structural texts such as Pollard & Fletcher (2005) or Fossen (2010).

Chapter 1 provides an overview of problem solving in structural geology and presents some classical orientation problems commonly found in the lab portion of a structural geology course. Throughout the chapter (and the book) we make only a brief attempt to explain why a student might want to carry out a particular calculation; instead we focus on how to solve it. Chapters 2 and 3 focus on the critically important topic of coordinate systems and coordinate transformations. These topics are essential to the understanding of vectors and tensors. Chapter 4 presents a review (for some students, at least) of basic matrix operations and indicial notation, shorthand that makes it easy to see the essence of an operation without getting bogged down in the details. Then, in Chapter 5, we address head on the topic of what, exactly, is a tensor as well as essential operations for analyzing tensors. With this background, we venture on to stress in Chapter 6 and deformation in Chapters 7 to 11. In the final chapter, we address a topic that all people solving problems quantitatively should know how to do: error analysis. All chapters are accompanied by well-known examples from structural geology, as well as exercises that will help students grasp these operations. Allmendinger was the principal author of chapters 1–9, Cardozo of chapters 11–12, and Fisher of chapter 10. All authors contributed algorithms, which were implemented in MATLAB® by Cardozo. Any bug reports should be sent to him.

Many of the exercises involve computation, which is the ideal way to learn the linear algebra approach. Some of the exercises in the earlier chapters can be solved using a spreadsheet program, but, as the exercises get more complicated and the programs more complex, we clearly need a more functional approach. Throughout the book, we provide code snippets that follow the syntax of MATLAB® functions. MATLAB is a popular scientific computing platform that is specifically oriented towards linear algebra operations. MATLAB is an interpreted language (i.e., no compilation needed) that is easy to program, and from which results are easily obtained in numerical and graphical form. Teaching the basic syntax of MATLAB is beyond the scope of this book, but the basic concepts should be familiar to anyone who is conversant with any programming language. The first author programs in FORTRAN and the second in Objective C, however, neither has trouble reading the MATLAB code. Additionally, the code snippets are richly commented to help even the novice reader capture the basic approach. Many of these code snippets come directly from programs by the first two authors, which are widely used by structural geologists. Thus, on a third level, this book can be viewed as a sort of "Numerical Recipes" (Press *et al.*, 1986) for structural geology.

Many colleagues and students have helped us to learn these methods and have influenced our own teaching of these topics. Foremost among them is Win Means, whose own little book, *Stress and Strain* (Means, 1976), unfortunately now out-of-print, was the first introduction that many of our generation had to this approach. Win was kind enough to read an earlier copy of this manuscript. Allmendinger was first introduced to these methods through a class that used Nye's excellent and concise treatment (Nye, 1985). Classes, and many discussions, with Ray

Fletcher, Arvid Johnson, and David Pollard about structural geology were fundamental to forming his worldview. We thank generations of our students and colleagues who have learned these topics from us and have, through painful experience, exposed the errors in our problem sets and computer code. Allmendinger would especially like to thank Ben Brooks, Trent Cladouhos, Ernesto Cristallini, Stuart Hardy, Phoebe Judge, Jack Loveless, Randy Marrett, and Alan Zehnder for sharing many programming adventures. He is particularly grateful to the US National Science Foundation for supporting his research over the years, much of which led to the methods described here. Cardozo would like to thank Alvar Braathen, Haakon Fossen, and Jan Tveranger for their interest in the description and modeling of structures, and Sigurd Aanonsen for introducing inverse problems. Our families have suffered, mostly silently, with our long hours spent programming, not to mention in preparation of this manuscript.