

Cambridge University Press

978-1-107-01199-1 - Quantum Models of Cognition and Decision

Jerome R. Busemeyer and Peter D. Bruza

Frontmatter

[More information](#)

## Quantum Models of Cognition and Decision

Much of our understanding of human thinking is based on probabilistic models. This innovative book by Jerome R. Busemeyer and Peter D. Bruza argues that, actually, the underlying mathematical structures from quantum theory provide a much better account of human thinking than traditional models. They introduce the foundations for modelling probabilistic-dynamic systems using two aspects of quantum theory. The first, “contextuality,” is a way to understand interference effects found with inferences and decisions under conditions of uncertainty. The second, “quantum entanglement,” allows cognitive phenomena to be modelled in non-reductionist ways. Employing these principles drawn from quantum theory allows us to view human cognition and decision in a totally new light. Introducing the basic principles in an easy-to-follow way, this book does not assume a physics background or a quantum brain and comes complete with a tutorial and fully worked-out applications in important areas of cognition and decision.

**Jerome R. Busemeyer** is a Professor in the Department of Psychological and Brain Sciences at Indiana University, Bloomington, USA. His research includes mathematical models of learning and decision making, and he has formulated a dynamic theory of human decision making called decision field theory. Professor Busemeyer has published over 100 articles in cognitive and decision science journals, including *Psychological Review*, and was Chief Editor of the *Journal of Mathematical Psychology* from 2005 to 2010.

**Peter D. Bruza** is a Professor in the Faculty of Science and Technology at Queensland University of Technology, Brisbane, Australia. His research intersects information retrieval, cognitive science, and applied logic. He is a pioneer and co-instigator of the field of quantum interaction (QI) and serves on the steering committee of the quantum interaction symposia. Professor Bruza also serves on the editorial boards of *Information Retrieval*, *Journal of Applied Logic*, *The Logic Journal of the IGPL*, and the “Information Science and Knowledge Management” book series.

Cambridge University Press

978-1-107-01199-1 - Quantum Models of Cognition and Decision

Jerome R. Busemeyer and Peter D. Bruza

Frontmatter

[More information](#)

---

Cambridge University Press

978-1-107-01199-1 - Quantum Models of Cognition and Decision

Jerome R. Busemeyer and Peter D. Bruza

Frontmatter

[More information](#)

---

# Quantum Models of Cognition and Decision

Jerome R. Busemeyer

Peter D. Bruza



CAMBRIDGE  
UNIVERSITY PRESS

Cambridge University Press  
 978-1-107-01199-1 - Quantum Models of Cognition and Decision  
 Jerome R. Busemeyer and Peter D. Bruza  
 Frontmatter  
[More information](#)

CAMBRIDGE UNIVERSITY PRESS  
 Cambridge, New York, Melbourne, Madrid, Cape Town,  
 Singapore, São Paulo, Delhi, Dubai, Mexico City

Cambridge University Press  
 The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

[www.cambridge.org](http://www.cambridge.org)  
 Information on this title: [www.cambridge.org/9781107011991](http://www.cambridge.org/9781107011991)

© Jerome R. Busemeyer and Peter D. Bruza 2012

This publication is in copyright. Subject to statutory exception  
 and to the provisions of relevant collective licensing agreements,  
 no reproduction of any part may take place without  
 the written permission of Cambridge University Press.

First published 2012

Printed in the United Kingdom at the University Press, Cambridge

*A catalogue record for this publication is available from the British Library*

*Library of Congress Cataloguing in Publication data*

Busemeyer, Jerome R.  
 Quantum models of cognition and decision / Jerome R. Busemeyer and Peter D. Bruza.  
 pages cm

Includes bibliographical references.  
 ISBN 978-1-107-01199-1 (Hardback)

1. Decision making—Mathematical models. 2. Statistical decision.
  3. Cognition—Mathematical models. 4. Quantum theory.
- I. Bruza, Peter David, 1962– II. Title.

QA279.4.B87 2012  
 530.12—dc23

2011051232

ISBN 978-1-107-01199-1 Hardback

Cambridge University Press has no responsibility for the persistence or  
 accuracy of URLs for external or third-party internet websites referred to  
 in this publication, and does not guarantee that any content on such  
 websites is, or will remain, accurate or appropriate.

Cambridge University Press

978-1-107-01199-1 - Quantum Models of Cognition and Decision

Jerome R. Busemeyer and Peter D. Bruza

Frontmatter

[More information](#)

---

《周易》有云：穷则变、变则通、通则久。

《Yi Jing》 Book states: ANY circumstance hitting a limit will begin to change.  
Change will in turn lead to an unimpeded state, and then lead to continuity.

This book is dedicated to the person who inspired this amazing journey,  
the first author's wife.

Cambridge University Press

978-1-107-01199-1 - Quantum Models of Cognition and Decision

Jerome R. Busemeyer and Peter D. Bruza

Frontmatter

[More information](#)

---

## Contents

Preface	xi
Acknowledgments	xv
1 Why use quantum theory for cognition and decision? Some compelling reasons	1
1.1 Six reasons for a quantum approach to cognition and decision	1
1.2 Four examples from cognition and decision	8
1.3 Some history and a broader picture	20
2 What is quantum theory? An elementary introduction	28
2.1 Geometric approach	29
2.2 Path diagram approach	49
2.3 Matrix algebra	57
2.4 Linear algebra	77
2.5 Quantum axioms	88
2.6 Some further reading on quantum theory	98
3 What can quantum theory predict? Predicting question order effects on attitudes	99
3.1 A simple example	100
3.2 Empirical tests of reciprocity	104
3.3 General quantum model	108
3.4 Order effect predictions	111
3.5 Concluding thoughts	115
4 How to apply quantum theory? Accounting for human probability judgment errors	117
4.1 Conjunction and disjunction errors	117
4.2 Order effects on inference	131
4.3 Compatibility and quantum rationality	141
5 Quantum-inspired models of concept combinations	143
5.1 Concept combinations and cognition	143
5.2 Non-compositional models of concept combinations based in quantum interference	146
5.3 Concept combinations modelled as composite quantum systems	148

Cambridge University Press

978-1-107-01199-1 - Quantum Models of Cognition and Decision

Jerome R. Busemeyer and Peter D. Bruza

Frontmatter

[More information](#)

viii

Contents

5.4	Probabilistic approaches for analyzing the compositionality of concept combinations	156
5.5	Empirical examples of the non-compositionality of concept combinations	162
6	An application of quantum theory to conjoint memory recognition	169
6.1	Episodic overdistribution effect	169
6.2	Classic probability	171
6.3	Cognitive models	173
6.4	Path diagram quantum model	177
6.5	Comparison of models	183
6.6	Concluding comments	184
7	Quantum-like models of human semantic space	185
7.1	The human mental lexicon	185
7.2	Words, context and Hilbert space	191
7.3	An analysis of spooky-activation-at-a-distance in terms of a composite quantum system	194
7.4	The quantum mechanics of semantic space	200
7.5	The distance between semantic spaces	209
8	What about quantum dynamics? More advanced principles	211
8.1	Bistable perception application	212
8.2	Categorization decision-making application	231
8.3	Random walk signal detection model	242
9	What is the quantum advantage? Applications to decision making	254
9.1	Allais and Ellsberg paradoxes	255
9.2	The disjunction effect	261
9.3	Markov and quantum models of the disjunction effect	267
9.4	Dynamic consistency	280
9.5	Interference of categorization on decisions	288
9.6	Concluding comments	290
10	How to model human information processing using quantum information theory	291
10.1	Information represented by qubits	292
10.2	Rules formed by control U gates	295
10.3	Evaluations represented by plus–minus states	307
10.4	State–action sequences	314
10.5	Concluding comments	320
11	Can quantum systems learn? Quantum updating	321
11.1	Quantum state updating based on experience	321
11.2	Weight updating based on gradient descent learning	327

Cambridge University Press  
978-1-107-01199-1 - Quantum Models of Cognition and Decision  
Jerome R. Busemeyer and Peter D. Bruza  
Frontmatter  
[More information](#)

---

Contents	ix
11.3 Quantum reinforcement learning	332
11.4 Summary of learning models	337
12 What are the future prospects for quantum cognition and decision?	338
12.1 Contributions to cognition and decision	338
12.2 Directions for future research	342
12.3 Practical applications of quantum cognition	344
12.4 Quantum rationality	347
12.5 Neuroscience and quantum theory	349
12.6 The hard problem of consciousness	353
12.7 Can only misbegotten fallacies result from a marriage between quantum and cognition?	357
Appendices	361
A Notation	361
B Density matrix representation of states	362
C Analysis of question order effects using density matrices	365
D Pauli matrices	367
E Quantum random walk	370
F Alternative quantum models of the disjunction effect	371
G Computer programs	378
References	385
Index	403

Cambridge University Press

978-1-107-01199-1 - Quantum Models of Cognition and Decision

Jerome R. Busemeyer and Peter D. Bruza

Frontmatter

[More information](#)

---

## Preface

### Rationale

The purpose of this book is to introduce the application of quantum theory to cognitive and decision scientists. At first sight it may seem bizarre, even ridiculous, to draw a connection between cognition and decision making – research lying within the realm of day-to-day human behavior – on the one hand and quantum mechanics – a highly successful theory for modelling subatomic phenomena – on the other hand. Yet there are good scientific reasons for doing so, which is leading a growing number of researchers to examine quantum theory as a way to understand perplexing findings and stubborn problems within their own fields. Hence this book. Given the nascent state of this field, some words of justification are warranted. The research just mentioned is not concerned with modelling the brain using quantum mechanics, nor is it directly concerned with the idea of the brain as a quantum computer. Instead it turns to quantum theory as a fresh conceptual framework for explaining empirical puzzles, as well as a rich new source of alternative formal tools. To convey the idea that researchers in this area are not doing quantum mechanics, various modifiers have been proposed to describe this work, such as quantum-like models of cognition, cognitive models based on quantum structure, or generalized quantum models.

There are two aspects of quantum theory which open the door to addressing problems facing cognition and decision in a totally new light. The first is known as “contextuality” of judgments and decisions, which is captured in quantum theory by the idea of “interference”: the context generated by making a first judgment or decision interferes with subsequent judgments or decisions to produce order effects, so that judgments and decisions are non-commutative. The second aspect relates to “quantum entanglement.” Entanglement is a phenomenon whereby making an observation on one part of the system instantaneously affects the state in another part of the system, even if the respective systems are separated by space-like distances. The crucial point about entanglement relevant to this book is that entangled systems cannot be validly decomposed and modelled as separate subsystems. This opens the door to developing quantum-like models of cognitive phenomena which are not decompositional in nature. For example, the semantics of concept combinations would seem to be non-compositional, and quantum theory provides formal tools to model these

Cambridge University Press

978-1-107-01199-1 - Quantum Models of Cognition and Decision

Jerome R. Busemeyer and Peter D. Bruza

Frontmatter

[More information](#)

as non-decomposable interacting systems. Similar applications appear in human memory. Most models consider words as separate entities – new models are made possible by going beyond this assumption and, for example, modelling a network of word associates as a non-decomposable system.

It is important to note the authors are agnostic toward the so-called “quantum mind” hypothesis, which assumes there are quantum processes going on in the brain. We motivate the use of quantum models as innovative abstractions of existing problems. That is all. These abstractions have the character of idealizations in the sense there is no claim as to the validity of the idealization “on the ground.” For example, modelling concept combinations as quantum entangled particles involves no claim as to whether there is associated physical entanglement going on somewhere in the brain. This may seem like an easy way out, but is not that different than idealizations employed in other areas of science. For example, in neural dynamical models of the brain, continuous state and time differential equations are used to model growth of neural activation, even though actually there are only a finite number of neurons and each one only fires in an all or none manner. In short, we apply mathematical structures from quantum mechanics to cognition and decision without attaching the physical meaning attributed to them when applied to the human behavioral phenomena. In fact, many areas of inquiry that were historically part of physics are now considered part of mathematics, including complexity theory, geometry, and stochastic processes. Originally they were applied to physical entities and events. For geometry, this was shapes of objects in space. For stochastic processes, this was statistical mechanics of particles. Over time they became generalized and applied in other domains. Thus, what happens here with quantum mechanics mirrors the history of many, if not most, branches of mathematics.

The cognitive revolution that occurred in the 1960s was based on classical computational logic, and the connectionist/neural network movements of the 1970s were based on classical dynamic systems. These classical assumptions remain at the heart of both cognitive architecture and neural network theories, and they are so commonly and widely applied that we take them for granted and presume them to be obviously true. What are these critical but hidden assumptions upon which all traditional theories rely? Quantum theory provides a fundamentally different approach to logic, reasoning, probabilistic inference, and dynamic systems. For example, quantum logic does not follow the distributive axiom of Boolean logic; quantum probabilities do not obey the law of total probability; quantum reasoning does not obey the principle of monotonic reasoning; and quantum dynamics can evolve along several trajectories in parallel rather than be slave to a single trajectory as in classical dynamics. Nevertheless, human behavior itself does not obey all of these restrictions. This book will provide an exposition of the basic assumptions of classic versus quantum models of cognition and decision theories. These basic assumptions will be examined, side by side, in a parallel and elementary manner. For example, classical systems assume that measurement merely observes a preexisting property of a system; in contrast, quantum systems assume that measurement actively creates the existence of a property in a system. The logic and mathematical foundation of

Cambridge University Press

978-1-107-01199-1 - Quantum Models of Cognition and Decision

Jerome R. Busemeyer and Peter D. Bruza

Frontmatter

[More information](#)

Preface

xiii

classic and quantum theory will be laid out in a simple and elementary manner that uncovers the mysteries of both theories. Classic theory will emerge to be seen as a possibly overly restrictive case of the more general quantum theory. The fundamental implications of these contrasting assumptions will be examined closely with concrete examples and applications to cognition and decision making. New research programs in cognition and decision making, based on quantum theory, will be reviewed.

## Book chapters

Chapter 1 provides the motivation for why one might be interested in applying quantum theory to cognition and decision making. In this chapter, we give a quick glance at several applications, including perception, conceptual judgments, decision making, and information retrieval. Also, this chapter briefly reviews some of the previous history and connections made between psychology and quantum physics and places the current ideas within this larger framework of research. Chapter 2 provides a simple and intuitive introduction to the basic axioms of quantum probability theory, alongside a comparison with the basic axioms of classic probability theory, and we also provide a clear *psychological* interpretation of the quantum axioms. The chapter includes simple numerical examples, calculations, and simple computer programs that provide clear and concrete ideas about how to use quantum theory to compute probabilities for cognitive and decision-making applications. Only linear algebra is needed for this introduction, which will be introduced and explained in a simple tutorial manner. No physics background is required. The next five chapters describe applications of the theory presented in Chapter 2. This includes applications to order effects on attitude judgments in Chapter 3, explanations for human probability judgment errors in Chapter 4, quantum models of conceptual combination judgments in Chapter 5, a detailed application of a quantum model to the conjoint memory recognition paradigm in Chapter 6, and a quantum model of the human mental lexicon in Chapter 7. Chapter 8 introduces the dynamic principles of quantum theory in a simple step-by-step manner with numerical examples and simple-to-use computer programs. This chapter also identifies fundamental differences between simple classical dynamic systems and quantum dynamic systems by presenting a parallel development of classic Markov and non-classic quantum processes. Chapter 9 applies the dynamic principles of the previous chapter to several paradoxical findings of decision making that cannot be easily explained by traditional decision models, including Markov models. Chapter 10 introduces some basic concepts of quantum computing and contrasts these ideas with production rule systems, connectionist networks, fuzzy set theory, and Bayesian inference theory. Computer code for analyzing various logic inference problems under uncertainty using quantum computing are provided. Chapter 11 introduces the problem of learning with quantum systems and reviews work on quantum neural networks. Finally, Chapter 12 summarizes the progress made toward applying quantum theory to cognitive and decision sciences thus far,

Cambridge University Press

978-1-107-01199-1 - Quantum Models of Cognition and Decision

Jerome R. Busemeyer and Peter D. Bruza

Frontmatter

[More information](#)

xiv

Preface

and provides a view of future possibilities. This chapter also includes a debate with a skeptic (actually previous reviewers) about the advantages and disadvantages of using a quantum approach to cognition and decision making, as well as different ways to understand the biological basis of quantum computations by the brain. An appendix is included to review some additional mathematics needed for understanding and using more advanced parts of quantum theory, and to present technical proofs that are too long to be included in the main text.

In our experience thus far, people either love or hate these ideas, but no one remains unaffected. We challenge you to make your own opinion.

Jerome R. Busemeyer, Indiana University, USA

Peter Bruza, Queensland University of Technology, Australia

Cambridge University Press

978-1-107-01199-1 - Quantum Models of Cognition and Decision

Jerome R. Busemeyer and Peter D. Bruza

Frontmatter

[More information](#)

## Acknowledgments

We were greatly helped by many of our colleagues who provided numerous useful comments, corrections, and thought-provoking dialogues in the subject area of this book. This includes Diederik Aerts, Sven Aerts, Harald Atmanspacher, Acacio de Barros, Reinhard Blutner, Charles Brainerd, Daoyi Dong, Riccardo Franco, Liane Gabora, Andrei Khrennikov, Kirsty Kitto, Peirofrancesco La Mura, Ariane Lambert-Mogiliansky, Shimon Malin, Cathy McEvoy, Douglas Nelson, Emmanuel Pothos, Brentyn Ramm, Keith van Rijsbergen, Dawei Song, Laurianne Sitbon, Richard Shiffrin, Jennifer Trueblood, Giuseppe Vitiello, Zheng Wang, Dominic Widdows, John Woods, Eldad Yechiam, Guido Zuccon, and several other anonymous reviewers. The book is much improved by their efforts. The first author's efforts were supported by a grant from the United States National Science Foundation, and the second author's efforts were supported by a grant from the Australian Research Council.