Quantum Models of Cognition and Decision

Much of our understanding of human thinking is based on probabilistic models. This innovative book by Jerome R. Busemeyer and Peter D. Bruza argues that, actually, the underlying mathematical structures from quantum theory provide a much better account of human thinking than traditional models. They introduce the foundations for modelling probabilistic-dynamic systems using two aspects of quantum theory. The first, "contextuality," is a way to understand interference effects found with inferences and decisions under conditions of uncertainty. The second, "quantum entanglement," allows cognitive phenomena to be modelled in non-reductionist ways. Employing these principles drawn from quantum theory allows us to view human cognition and decision in a totally new light. Introducing the basic principles in an easy-to-follow way, this book does not assume a physics background or a quantum brain and comes complete with a tutorial and fully worked-out applications in important areas of cognition and decision.

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CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town,
Singapore, São Paulo, Delhi, Dubai, Mexico City

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9781107011991

First published 2012

Printed in the United Kingdom at the University Press, Cambridge

A catalogue record for this publication is available from the British Library

Library of Congress Cataloguing in Publication data

Busemeyer, Jerome R.
Quantum models of cognition and decision / Jerome R. Busemeyer and Peter D. Bruza.
pages cm
Includes bibliographical references.
ISBN 978-1-107-01199-1 (Hardback)
1. Decision making–Mathematical models. 2. Statistical decision.
3. Cognition–Mathematical models. 4. Quantum theory.
I. Bruza, Peter David, 1962– II. Title.

QA279.4.B87 2012
530.12–dc23

2011051232

ISBN 978-1-107-01199-1 Hardback

《周易》有云：穷则变、变则通、通则久。

⟨⟨Yi Jing⟩⟩ Book states: ANY circumstance hitting a limit will begin to change.
Change will in turn lead to an unimpeded state, and then lead to continuity.

This book is dedicated to the person who inspired this amazing journey,
the first author's wife.

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Preface

Rationale

The purpose of this book is to introduce the application of quantum theory to cognitive and decision scientists. At first sight it may seem bizarre, even ridiculous, to draw a connection between cognition and decision making – research lying within the realm of day-to-day human behavior – on the one hand and quantum mechanics – a highly successful theory for modelling subatomic phenomena – on the other hand. Yet there are good scientific reasons for doing so, which is leading a growing number of researchers to examine quantum theory as a way to understand perplexing findings and stubborn problems within their own fields. Hence this book. Given the nascent state of this field, some words of justification are warranted. The research just mentioned is not concerned with modelling the brain using quantum mechanics, nor is it directly concerned with the idea of the brain as a quantum computer. Instead it turns to quantum theory as a fresh conceptual framework for explaining empirical puzzles, as well as a rich new source of alternative formal tools. To convey the idea that researchers in this area are not doing quantum mechanics, various modifiers have been proposed to describe this work, such as quantum-like models of cognition, cognitive models based on quantum structure, or generalized quantum models.

There are two aspects of quantum theory which open the door to addressing problems facing cognition and decision in a totally new light. The first is known as "contextuality" of judgments and decisions, which is captured in quantum theory by the idea of "interference": the context generated by making a first judgment or decision interferes with subsequent judgments or decisions to produce order effects, so that judgments and decisions are non-commutative. The second aspect relates to "quantum entanglement." Entanglement is a phenomenon whereby making an observation on one part of the system instantaneously affects the state in another part of the system, even if the respective systems are separated by space-like distances. The crucial point about entanglement relevant to this book is that entangled systems cannot be validly decomposed and modelled as separate subsystems. This opens the door to developing quantum-like models of cognitive phenomena which are not decompositional in nature. For example, the semantics of concept combinations would seem to be non-compositional, and quantum theory provides formal tools to model these as non-decomposable interacting systems. Similar applications appear in human memory. Most models consider words as separate entities – new models are made possible by going beyond this assumption and, for example, modelling a network of word associates as a non-decomposable system.

It is important to note the authors are agnostic toward the so-called "quantum mind" hypothesis, which assumes there are quantum processes going on in the brain. We motivate the use of quantum models as innovative abstractions of existing problems. That is all. These abstractions have the character of idealizations in the sense there is no claim as to the validity of the idealization "on the ground." For example, modelling concept combinations as quantum entangled particles involves no claim as to whether there is associated physical entanglement going on somewhere in the brain. This may seem like an easy way out, but is not that different than idealizations employed in other areas of science. For example, in neural dynamical models of the brain, continuous state and time differential equations are used to model growth of neural activation, even though actually there are only a finite number of neurons and each one only fires in an all or none manner. In short, we apply mathematical structures from quantum mechanics to cognition and decision without attaching the physical meaning attributed to them when applied to the human behavioral phenomena. In fact, many areas of inquiry that were historically part of physics are now considered part of mathematics, including complexity theory, geometry, and stochastic processes. Originally they were applied to physical entities and events. For geometry, this was shapes of objects in space. For stochastic processes, this was statistical mechanics of particles. Over time they became generalized and applied in other domains. Thus, what happens here with quantum mechanics mirrors the history of many, if not most, branches of mathematics.

The cognitive revolution that occurred in the 1960s was based on classical computational logic, and the connectionist/neural network movements of the 1970s were based on classical dynamic systems. These classical assumptions remain at the heart of both cognitive architecture and neural network theories, and they are so commonly and widely applied that we take them for granted and presume them to be obviously true. What are these critical but hidden assumptions upon which all traditional theories rely? Quantum theory provides a fundamentally different approach to logic, reasoning, probabilistic inference, and dynamic systems. For example, quantum logic does not follow the distributive axiom of Boolean logic; quantum probabilities do not obey the law of total probability; quantum reasoning does not obey the principle of monotonic reasoning; and quantum dynamics can evolve along several trajectories in parallel rather than be slave to a single trajectory as in classical dynamics. Nevertheless, human behavior itself does not obey all of these restrictions. This book will provide an exposition of the basic assumptions of classic versus quantum models of cognition and decision theories. These basic assumptions will be examined, side by side, in a parallel and elementary manner. For example, classical systems assume that measurement merely observes a preexisting property of a system; in contrast, quantum systems assume that measurement actively creates the existence of a property in a system. The logic and mathematical foundation of

Cambridge University Press
978-1-107-01199-1 - Quantum Models of Cognition and Decision
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classic and quantum theory will be laid out in a simple and elementary manner that uncovers the mysteries of both theories. Classic theory will emerge to be seen as a possibly overly restrictive case of the more general quantum theory. The fundamental implications of these contrasting assumptions will be examined closely with concrete examples and applications to cognition and decision making. New research programs in cognition and decision making, based on quantum theory, will be reviewed.

Book chapters

Chapter 1 provides the motivation for why one might be interested in applying quantum theory to cognition and decision making. In this chapter, we give a quick glance at several applications, including perception, conceptual judgments, decision making, and information retrieval. Also, this chapter briefly reviews some of the previous history and connections made between psychology and quantum physics and places the current ideas within this larger framework of research. Chapter 2 provides a simple and intuitive introduction to the basic axioms of quantum probability theory, alongside a comparison with the basic axioms of classic probability theory, and we also provide a clear *psychological* interpretation of the quantum axioms. The chapter includes simple numerical examples, calculations, and simple computer programs that provide clear and concrete ideas about how to use quantum theory to compute probabilities for cognitive and decision-making applications. Only linear algebra is needed for this introduction, which will be introduced and explained in a simple tutorial manner. No physics background is required. The next five chapters describe applications of the theory presented in Chapter 2. This includes applications to order effects on attitude judgments in Chapter 3, explanations for human probability judgment errors in Chapter 4, quantum models of conceptual combination judgments in Chapter 5, a detailed application of a quantum model to the conjoint memory recognition paradigm in Chapter 6, and a quantum model of the human mental lexicon in Chapter 7. Chapter 8 introduces the dynamic principles of quantum theory in a simple step-by-step manner with numerical examples and simple-to-use computer programs. This chapter also identifies fundamental differences between simple classical dynamic systems and quantum dynamic systems by presenting a parallel development of classic Markov and non-classic quantum processes. Chapter 9 applies the dynamic principles of the previous chapter to several paradoxical findings of decision making that cannot be easily explained by traditional decision models, including Markov models. Chapter 10 introduces some basic concepts of quantum computing and contrasts these ideas with production rule systems, connectionist networks, fuzzy set theory, and Bayesian inference theory. Computer code for analyzing various logic inference problems under uncertainty using quantum computing are provided. Chapter 11 introduces the problem of learning with quantum systems and reviews work on quantum neural networks. Finally, Chapter 12 summarizes the progress made toward applying quantum theory to cognitive and decision sciences thus far, and provides a view of future possibilities. This chapter also includes a debate with a skeptic (actually previous reviewers) about the advantages and disadvantages of using a quantum approach to cognition and decision making, as well as different ways to understand the biological basis of quantum computations by the brain. An appendix is included to review some additional mathematics needed for understanding and using more advance parts of quantum theory, and to present technical proofs that are too long to be included in the main text.

In our experience thus far, people either love or hate these ideas, but no one remains unaffected. We challenge you to make your own opinion.

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Acknowledgments

We were greatly helped by many of our colleagues who provided numerous useful comments, corrections, and thought-provoking dialogues in the subject area of this book. This includes Diederik Aerts, Sven Aerts, Harald Atmanspacher, Acacio de Barros, Reinhard Blutner, Charles Brainerd, Daoyi Dong, Riccardo Franco, Liane Gabora, Andrei Khrennikov, Kirsty Kitto, Peirofrancesco La Mura, Ariane Lambert-Mogiliansky, Shimon Malin, Cathy McEvoy, Douglas Nelson, Emmanual Pothos, Brentyn Ramm, Keith van Rijsbergen, Dawei Song, Laurianne Sitbon, Richard Shiffrin, Jennifer Trueblood, Giuseppe Vitiello, Zheng Wang, Dominic Widdows, John Woods, Eldad Yechiam, Guido Zuccon, and several other anonymous reviewers. The book is much improved by their efforts. The first author's efforts were supported by a grant from the United States National Science Foundation, and the second author's efforts were supported by a grant from the Australian Research Council.