Theory of Conditional Games

Game theory explains how to make good choices when different decision makers have conflicting interests. The classical approach assumes that decision makers are committed to making the best choices for themselves regardless of the effect on others, but such an approach is less appropriate when cooperation, compromise, and negotiation are important. This book describes conditional games, an extension of game theory that accommodates multiple stakeholder decision-making scenarios where cooperation and negotiation are significant issues and where notions of concordant group behavior are important. Using classical binary preference relations as a point of departure, the book extends the concept of a preference ordering that permits stakeholders to modulate their preferences as functions of the preferences of others. As these conditional preferences propagate through a group of decision makers, they create social bonds that lead to notions of group concordance.

This book is intended for all students and researchers of decision theory and game theory, including students in artificial intelligence (especially multiagent systems and distributed control), economics, management science, psychology, analytic philosophy, and applied mathematics.

WYNN C. STIRLING is a professor of Electrical and Computer Engineering and Dean of Graduate Studies at Brigham Young University. His current research interests include multiagent decision theory, estimation theory, information theory, and stochastic processes. He is the author or co-author of more than seventy publications, including the graduate text *Mathematical Methods and Algorithms for Signal Processing* (with Todd Moon) and the monograph *Satisficing Games and Decision Making*.

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WYNN C. STIRLING
Brigham Young University

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To Tony, Dave, and Kate

Natural science is an expansion of observing; technology, of contriving; mathematics, of understanding.

— Michael Polanyi

Personal Knowledge (University of Chicago Press, 1958)

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Preface

Hypothesen sind Netze, nur der wird fangen, der auswirft.
Theories are nets: only he who casts will catch.
— *Novalis (Friedrich von Hardenberg)*
Dialogen und Monolog, 1798

John Dewey observed that "In scientific inquiry, the criterion of what is taken to be settled, or to be knowledge, is being *so* settled that it is available as a resource in further inquiry; not being settled in such a way as not to be subject to revision in further inquiry" [emphasis in original, Dewey, 1938, pp. 8–9]. Game theory has been successfully applied to the subject matter of general economic theory, particularly for competitive and market-driven scenarios where individual rationality dominates. It is firmly established; it is settled. In fact, it is *so* settled that it is available as a resource for further inquiry into the issue of multistakeholder decision making for scenarios where social relationships extend beyond self-interest.

The net cast by classical game theory, however, is designed to capture the essential characteristics of multistakeholder decision scenarios where all participants possess categorical (unconditional) preference orderings and are committed to achieving the best individual outcomes for themselves. This book revises game theory to cast a wider net designed to capture, in addition, decision-making scenarios where cooperation, compromise, negotiation, and altruism are significant issues, and where notions of concordant group behavior are important. Using classical binary preference relations as a point of departure, the book extends the concept of a preference ordering to permit stakeholders to modulate their preferences as functions of the preferences of others. As these conditional preferences propagate through a group of autonomous decision makers, they create social bonds that lead naturally to notions of group concordance. This concept is formalized by defining a conditional game as a set of players, their feasible actions, and their conditional preference orderings.

Conditional game theory permits the development of (a) solution concepts that simultaneously account for both individual preferences and group concordance; (b) negotiation protocols that permit each stakeholder to achieve its security level while maximizing the concordance of the group; (c) a theory to characterize the intrinsic ability of a group to coordinate; (d) a mechanism to combine the deterministic and stochastic components of decision making into a single unified utility structure; and (e) rigorously defined concepts of multiple-agent satisficing that relax the brittle demands of optimization and provide decision makers with the flexibility to achieve solutions that accommodate multiple points of view.

Motivation

Historically, game theory has mainly been the purview of the social sciences (chiefly economics and political science). Recently, however, engineering and computer science have employed game theory as a useful framework within which to model multiagent decision making and control. Thus, the book is written to both the social science and the engineering/computer science audiences. To appeal to multiple disciplines, it focuses on the foundational assumptions that underlie the formulation of decision-making methodologies and does not delve deeply into practical applications or case studies.

Although they share similar mathematics, social science and engineering/computer science differ significantly in the way game theory is used. On the one hand, social science uses game theory as an *analysis* tool with which to explain, predict, justify, and recommend human action. But the solution does not dictate action – it is not causal. At best it is a reasonable approximation under controlled circumstances. When observed behavior deviates from the behavior predicted by the game, it is necessary to invoke ex post psychological or sociological arguments that, while not part of the mathematical model, are necessary to explain the behavior.

On the other hand, engineers and computer scientists use game theory for *synthesis*: to design and construct artificial decision-making agents. When used this way, the solution does indeed dictate behavior – it is causal. Whereas a model used for analysis can be a convenient fiction, a model used for synthesis creates its own reality. A multiagent system cannot rely on ex post psychological or sociological interpretations to explain inappropriate behavior. In particular, if notions of social behavior are relevant, then they must be explicitly encoded into the model.

The intended readership

This book is intended for all students of decision theory and game theory. Researchers and students in distributed artificial intelligence, distributed control, and multiagent systems will find the material covered in this book to be quite different from the treatments that are directed to computer science and engineering audiences, and thus may supplement standard game theoretic–based treatments as applied to those disciplines. Also, researchers and students in the social sciences will benefit from exposure to a perspective of game theory that is not heavily emphasized in the conventional social science literature, namely, extending beyond considerations of individually rational behavior. In addition, since this book touches on such philosophical issues as rationality and sociality, it should be of interest to philosophers and other students of rational choice theory.

Structure of the book

Chapter 1 moves beyond classical notions of rationality and argues that social influence relationships that exist among stakeholders must be accounted for explicitly, and that concepts of individual and group rationality must be reconciled if game theory is to be successfully applied to engineering and artificial intelligence decision problems.

Chapter 2 extends game theory by replacing ex ante categorical preference orderings, which unconditionally characterize each stakeholder's interests before the game is played, with preference orderings that permit stakeholders to condition their preferences on the preferences of others, thereby dynamically modulating their preferences as the game is played. These conditional preferences propagate throughout the group to create a social bond among its members, thereby making it possible to define preference orderings for both the individuals and the group. If the preferences of all stakeholders are categorical, however, then no notion of group preference exists, and the situation reverts to a conventional game. Thus, conditional game theory is a true generalization of the classical theory.

The extension to conditional games makes it possible to introduce theoretical developments that are not permitted within the classical structure. **Chapter 3** provides an expanded solution concept that simultaneously accounts for both group and individual interests. This new concept provides a framework for the negotiation of a compromise that allows each agent to reconcile its own interests with the interests of others and the group.

Chapter 4 introduces a formal notion of coordinatability. Drawing on concepts from Shannon information theory, it presents a notion of coordination capacity that provides a quantified measure of the intrinsic ability of a group to function in a coherent, harmonious way, independently of the notions of rational behavior that are adopted by the players. The coordination capacity essentially provides an ecological measure of how well an organization is able to function in its environment.

Chapter 5 offers a unified treatment of the preferential and probabilistic aspects of a multistakeholder decision problem in the presence of uncertainty. The result is the definition of a combined utility-probability network that possesses all of the mathematical properties of a standard Bayesian network. This structure enables the symmetric representation of the deterministic and random components of a decision problem, which facilitates the calculation of expected utility.

Chapter 6 generalizes the notion of satisficing game theory, which was first presented in my earlier book, *Satisficing Games and Decision Making* (Cambridge University Press, 2003). This theory presents an alternative to the classical approach of seeking the best and only the best solution to a multistakeholder decision problem by defining a set of solutions that satisfy a rigorous notion of what it means to be good enough. This approach provides increased flexibility in choosing a solution that is acceptable to both the individuals and the group.

Chapter 7 provides a number of examples that illustrate the use of conditional game theory, and **Chapter** 8 is an invitation to explore the theory of conditional games more deeply, with the hope that other researchers will contribute to its further theoretical development and to useful applications.

The usage of personal pronouns

Depending on the context, a stakeholder may be a person, an artificial agent, an attribute, or a random phenomenon. To accommodate this diversity, I have adopted the convention of using "it" for third-person singular pronouns unless the gender of the antecedent is obvious from the context.

Acknowledgments

Developing the ideas expressed in this book has been a joy. Although it is a relief to bring this project to conclusion, I cannot help but reflect on the sentiment expressed by Gauss: "It is not knowledge, but the act of learning, not possession but the act of getting there, which grants the greatest enjoyment." The seed ideas for this book arise from what may seem to be an unlikely source for an engineer: the writings of the distinguished epistemologist Isaac Levi, who insists

that attention should be focused on the improvement of knowledge, not merely its justification. I have transmigrated his theory of epistemic utility from the epistemological domain to the praxeological domain, and have extended it to the multiagent (multivariate) case. His emphasis on formulating a mathematical framework for dynamically synthesizing knowledge resonates with the engineering concept of designing and synthesizing mathematical decision-making devices.

Many colleagues have contributed to the ideas contained in this book. Of particular note are three of my former graduate students, Darryl Morrell, Michael Goodrich, and Matthew Nokleby, whose creativity and enthusiasm have sustained me many times, and whose ideas are integral to this theory. I have also greatly benefited from my collaborations with Richard Frost, Harold Miller, Todd Moon, Dennis Packard, and Teppo Felin. The research environment and support offered by Brigham Young University and, particularly, by my colleagues in the Electrical and Computer Engineering Department, who have graciously tolerated my unorthodox research agenda, are greatly appreciated. A special thanks goes to Laura Rawlins of BYU's Faculty Editing Service, whose careful reading of the manuscript provided many much needed clarifications, and to Lauren Cowles and David Jou of Cambridge University Press for their support and patience.

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WYNN C. STIRLING
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