Random Matrix Methods for Wireless Communications

Blending theoretical results with practical applications, this book provides an introduction to random matrix theory and shows how it can be used to tackle a variety of problems in wireless communications. The Stieltjes transform method, free probability theory, combinatoric approaches, deterministic equivalents, and spectral analysis methods for statistical inference are all covered from a unique engineering perspective. Detailed mathematical derivations are presented throughout, with thorough explanations of the key results and all fundamental lemmas required for the readers to derive similar calculus on their own. These core theoretical concepts are then applied to a wide range of real-world problems in signal processing and wireless communications, including performance analysis of CDMA, MIMO, and multi-cell networks, as well as signal detection and estimation in cognitive radio networks. The rigorous yet intuitive style helps demonstrate to students and researchers alike how to choose the correct approach for obtaining mathematically accurate results.

Romain Couillet is an Assistant Professor at the Chair on System Sciences and the Energy Challenge at Supélec, France. Previously he was an Algorithm Development Engineer for ST-Ericsson, and he received his PhD from Supélec in 2010.

Mérouane Debbah is a Professor at Supélec, where he holds the Alcatel-Lucent Chair on Flexible Radio. He is the recipient of several awards, including the 2007 General Symposium IEEE Globecom best paper award and the Wi-Opt 2009 best paper award.

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Romain Couillet and Mérouane Debbah
École Supérieure d'Électricité, Gif sur Yvette, France

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To my family,
– Romain Couillet

To my parents,
– Mérouane Debbah

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Preface

More than sixty years have passed since the 1948 landmark paper of Shannon providing the capacity of a single antenna point-to-point communication channel. The method was based on information theory and led to a revolution in the field, especially on how communication systems were designed. The tools then showed their limits when we wanted to extend the analysis and design to the multiterminal multiple antenna case, which is the basis of the wireless revolution since the nineties. Indeed, in the design of these networks, engineers frequently stumble on the scalability problem. In other words, as the number of nodes or bandwidth increase, problems become harder to solve and the determination of the precise achievable rate region becomes an intractable problem. Moreover, engineering insight progressively disappears and we can only rely on heavy simulations with all their caveats and limitations. However, when the system is sufficiently large, we may hope that a macroscopic view could provide a more useful abstraction of the network. The properties of the new macroscopic model nonetheless need to account for microscopic considerations, e.g. fading, mobility, etc. We may then sacrifice some structural details of the microscopic view but the macroscopic view will preserve sufficient information to allow for a meaningful network optimization solution and the derivation of insightful results in a wide range of settings.

Recently, a number of research groups around the world have taken this approach and have shown how tools borrowed from physical and mathematical frameworks, e.g. percolation theory, continuum models, game theory, electrostatics, mean field theory, stochastic geometry, just to name a few, can capture most of the complexity of dense random networks in order to unveil some relevant features on network-wide behavior.

The following book falls within this trend and aims to provide a comprehensive understanding on how random matrix theory can model the complexity of the interaction between wireless devices. It has been more than fifteen years since random matrix theory was successfully introduced into the field of wireless communications to analyze CDMA and MIMO systems. One of the useful features, especially of the large dimensional random matrix theory approach, is its ability to predict, under certain conditions, the behavior of the empirical eigenvalue distribution of products and sums of matrices. The results are striking in terms of accuracy compared to simulations with reasonable matrix sizes, and the theory has been shown to be an efficient tool to predict the behavior of wireless systems with only few meaningful parameters. Random matrix theory is also increasingly making its way into the statistical signal processing field with the generalization of detection and inference methods, e.g. array processing, hypothesis tests, parameter estimation, etc., to the multi-variate case. This comes as a small revolution in modern signal processing as legacy estimators, such as the MUSIC method, become increasingly obsolete and unadapted to large sensing arrays with few observations.

The authors are confident and have no doubt on the usefulness of the tool for the engineering community in the upcoming years, especially as networks become denser. They also think that random matrix theory should become sooner or later a major tool for electrical engineers, taught at the graduate level in universities. Indeed, engineering education programs of the twentieth century were mostly focused on the Fourier transform theory due to the omnipresence of frequency spectrum. The twenty-first century engineers know by now that space is the next frontier due to the omnipresence of spatial spectrum modes, which refocuses the programs towards a Stieltjes transform theory.

We sincerely hope that this book will inspire students, teachers, and engineers, and answer their present and future problems.

Romain Couillet and Mérouane Debbah

Acknowledgments

This book is the fruit of many years of the authors' involvement in the field of random matrix theory for wireless communications. This topic, which has gained increasing interest in the last decade, was brought to light in the telecommunication community in particular through the work of Stephen Hanly, Ralf Müller, Shlomo Shamai, Emre Telatar, David Tse, Antonia Tulino, and Sergio Verdú, among others. It then rapidly grew into a joint research framework gathering both telecommunication engineers and mathematicians, among which Zhidong Bai, Vyacheslav L. Girko, Leonid Pastur, and Jack W. Silverstein.

The authors are especially indebted to Prof. Silverstein for the agreeable time spent discussing random matrix matters. Prof. Silverstein has a very insightful approach to random matrices, which it was a delight to share with him. The general point of view taken in this book is mostly influenced by Prof. Silverstein's methodology. The authors are also grateful to the many colleagues working in this field whose knowledge and wisdom about applied random matrix theory contributed significantly to its current popularity and elegance. This book gathers many of their results and intends above all to deliver to the readers this simplified approach to applied random matrix theory. The colleagues involved in long and exciting discussions as well as collaborative works are Florent Benaych-Georges, Pascal Bianchi, Laura Cottatellucci, Maxime Guillaud, Walid Hachem, Philippe Loubaton, Mylène Maïda, Xavier Mestre, Aris Moustakas, Ralf Müller, Jamal Najim, and Øyvind Ryan.

Regarding the book manuscript itself, the authors would also like to sincerely thank the anonymous reviewers for their wise comments which contributed to improve substantially the overall quality of the final book and more importantly the few people who dedicated a long time to thoroughly review the successive drafts and who often came up with inspiring remarks. Among the latter are David Gregoratti, Jakob Hoydis, Xavier Mestre, and Sebastian Wagner.

The success of this book relies in a large part on these people.

Romain Couillet and Mérouane Debbah

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Acronyms

AWGN	additive white Gaussian noise
BC	broadcast channel
BPSK	binary pulse shift keying
CDMA	code division multiple access
CI	channel inversion
CSI	channel state information
CSIR	channel state information at receiver
CSIT	channel state information at transmitter
d.f.	distribution function
DPC	dirty paper coding
e.s.d.	empirical spectral distribution
FAR	false alarm rate
GLRT	generalized likelihood ratio test
GOE	Gaussian orthogonal ensemble
GSE	Gaussian symplectic ensemble
GUE	Gaussian unitary ensemble
i.i.d.	independent and identically distributed
l.s.d.	limit spectral distribution
MAC	multiple access channel
MF	matched-filter
MIMO	multiple input multiple output
MISO	multiple input single output
ML	maximum likelihood
LMMSE	linear minimum mean square error
MMSE	minimum mean square error
MMSE-SIC	MMSE and successive interference cancellation
MSE	mean square error
MUSIC	multiple signal classification
NMSE	normalized mean square error
OFDM	orthogonal frequency division multiplexing

OFDMA	orthogonal frequency division multiple access
p.d.f.	probability density function
QAM	quadrature amplitude modulation
QPSK	quadrature pulse shift keying
ROC	receiver operating characteristic
RZF	regularized zero-forcing
SINR	signal-to-interference plus noise ratio
SISO	single input single output
SNR	signal-to-noise ratio
TDMA	time division multiple access
ZF	zero-forcing

Notation

Linear algebra

$\mathbf{X}$	Matrix
$\mathbf{I}_N$	Identity matrix of size $N \times N$
X_{ij}	Entry (i, j) of matrix $\mathbf{X}$ (unless otherwise stated)
$(X)_{ij}$	Entry (i, j) of matrix $\mathbf{X}$
$[X]_{ij}$	Entry (i, j) of matrix $\mathbf{X}$
$\{f(i, j)\}_{i,j}$	Matrix with (i, j) entry $f(i, j)$
$(X_{ij})_{i,j}$	Matrix with (i, j) entry X_{ij}
$\mathbf{x}$	Vector (column by default)
$\mathbf{x}^*$	Vector of the complex conjugates of the entries of $\mathbf{x}$
x_i	Entry i of vector $\mathbf{x}$
$F^{\mathbf{X}}$	Empirical spectral distribution of the Hermitian $\mathbf{X}$
$\mathbf{X}^{\mathsf{T}}$	Transpose of $\mathbf{X}$
$\mathbf{X}^{\mathsf{H}}$	Hermitian transpose of $\mathbf{X}$
$\operatorname{tr} \mathbf{X}$	Trace of $\mathbf{X}$
$\det \mathbf{X}$	Determinant of $\mathbf{X}$
$\operatorname{rank}(\mathbf{X})$	Rank of $\mathbf{X}$
$\Delta(\mathbf{X})$	Vandermonde determinant of $\mathbf{X}$
$\|\mathbf{X}\|$	Spectral norm of the Hermitian matrix $\mathbf{X}$
$\operatorname{diag}(x_1, \ldots, x_n)$	Diagonal matrix with (i, i) entry x_i
$\ker(\mathbf{A})$	Null space of the matrix $\mathbf{A}$, $\ker(\mathbf{A}) = \{\mathbf{x}, \mathbf{A}\mathbf{x} = 0\}$
$\operatorname{span}(\mathbf{A})$	Subspace generated by the columns of the matrix $\mathbf{A}$

Real and complex analysis

$\mathbb{N}$	The space of natural numbers
$\mathbb{R}$	The space of real numbers
$\mathbb{C}$	The space of complex numbers
A^*	The space $A \setminus \{0\}$
x^+	Right-limit of the real x
x^-	Left-limit of the real x

$(x)^+$	For $x \in \mathbb{R}$, $\max(x, 0)$
$\operatorname{sgn}(x)$	Sign of the real x
$\Re[z]$	Real part of z
$\Im[z]$	Imaginary part of z
z^*	Complex conjugate of z
i	Square root of -1 with positive imaginary part
$f'(x)$	First derivative of the function f
$f''(x)$	Second derivative of the function f
$f'''(x)$	Third derivative of the function f
$f^{(p)}(x)$	Derivative of order p of the function f
$\\|f\\|$	Norm of a function $\\|f\\| = \sup_x \|f(x)\|$
$1_A(x)$	Indicator function of the set A
	$1_A(x) = 1$ if $x \in A$, $1_A(x) = 0$ otherwise
$\delta(x)$	Dirac delta function, $\delta(x) = 1_{\{0\}}(x)$
$\Delta(x\|A)$	Convex indicator function
	$\Delta(x\|A) = 0$ if $x \in A$, $\Delta(x\|A) = \infty$ otherwise
$\operatorname{Supp}(F)$	Support of the distribution function F
$x_1, x_2, \ldots$	Series of general term x_n
$x_n \to \ell$	Simple convergence of the series $x_1, x_2, \ldots$ to ℓ
$x_n = o(y_n)$	Upon existence, $x_n/y_n \to 0$ as $n \to \infty$
$x_n = O(y_n)$	There exists K, such that $x_n \leq K y_n$ for all n
$n/N \to c$	As $n \to \infty$ and $N \to \infty$, $n/N \to c$
$\mathcal{W}(z)$	Lambert-W function satisfying $\mathcal{W}(z)e^{\mathcal{W}(z)} = z$
$\operatorname{Ai}(x)$	Airy function
$\Gamma(x)$	Gamma function, $\Gamma(n) = (n-1)!$ for n integer

Probability theory

$(\Omega, \mathcal{F}, P)$	Probability space Ω with σ-field $\mathcal{F}$ and measure P
$P_X(x)$	Density of the random variable X
$p_X(x)$	Density of the scalar random variable X
$P_{(X_i)}(x)$	Unordered density of the random variable $X_1, \ldots, X_N$
$P^{\geq}_{(X_i)}(x)$	Ordered density of the random variable $X_1 \geq \ldots \geq X_N$
$P^{\leq}_{(X_i)}(x)$	Ordered density of the random variable $X_1 \leq \ldots \leq X_N$
μ_X	Probability measure of X, $\mu_X(A) = P(X(A))$
$\mu_{\mathbf{X}}$	Probability distribution of the eigenvalues of $\mathbf{X}$
$\mu^{\infty}_{\mathbf{X}}$	Probability distribution associated with the l.s.d. of $\mathbf{X}$
$P_X(x)$	Density of the random variable X, $P_X(x)dx = \mu_X(dx)$
$F_X(x)$	Distribution function of X (real), $F_X(x) = \mu_X((-\infty, x])$
$\operatorname{E}[X]$	Expectation of X, $\operatorname{E}[X] = \int_\Omega X(\omega)d\omega$

$\mathrm{E}[f(X)]$	Expectation of $f(X)$, $\mathrm{E}[f(X)] = \int_\Omega f(X(\omega))d\omega$
$\mathrm{var}(X)$	Variance of X, $\mathrm{var}(X) = \mathrm{E}[X^2] - \mathrm{E}[X]^2$
$X \sim \mathcal{L}$	X is a random variable with density $\mathcal{L}$
$\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$	Real Gaussian distribution of mean $\boldsymbol{\mu}$ and covariance $\boldsymbol{\Sigma}$
$\mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$	Complex Gaussian distribution of mean $\boldsymbol{\mu}$ and covariance $\boldsymbol{\Sigma}$
$\mathcal{W}_N(n, \mathbf{R})$	Real zero mean Wishart distribution with n degrees of freedom and covariance $\mathbf{R}$
$\mathcal{CW}_N(n, \mathbf{R})$	Complex zero mean Wishart distribution with n degrees of freedom and covariance $\mathbf{R}$
$Q(x)$	Gaussian Q-function, $Q(x) = P(X > x)$, $X \sim \mathcal{N}(0, 1)$
F^+	Tracy–Widom distribution function
F^-	Conjugate Tracy–Widom d.f., $F^-(x) = 1 - F^+(-x)$
$x_n \xrightarrow{\text{a.s.}} \ell$	Almost sure convergence of the series $x_1, x_2, \ldots$ to ℓ
$F_n \Rightarrow F$	Weak convergence of the d.f. series $F_1, F_2, \ldots$ to F
$X_n \Rightarrow X$	Weak convergence of the series $X_1, X_2, \ldots$ to the random X

Random Matrix Theory

$m_F(z)$	Stieltjes transform of the function F
$m_{\mathbf{X}}(z)$	Stieltjes transform of the eigenvalue distribution of $\mathbf{X}$
$\mathcal{V}_F(z)$	Shannon transform of the function F
$\mathcal{V}_{\mathbf{X}}(z)$	Shannon transform of the eigenvalue distribution of $\mathbf{X}$
$R_F(z)$	R transform of the function F
$R_{\mathbf{X}}(z)$	R transform of the eigenvalue distribution of $\mathbf{X}$
$S_F(z)$	S transform of the function F
$S_{\mathbf{X}}(z)$	S transform of the eigenvalue distribution of $\mathbf{X}$
$\eta_F(z)$	η-transform of the function F
$\eta_{\mathbf{X}}(z)$	η-transform of the eigenvalue distribution of $\mathbf{X}$
$\psi_F(z)$	ψ-transform of the function F
$\psi_{\mathbf{X}}(z)$	ψ-transform of the eigenvalue distribution of $\mathbf{X}$
$\mu \boxplus \nu$	Additive free convolution of μ and ν
$\mu \boxminus \nu$	Additive free deconvolution of μ and ν
$\mu \boxtimes \nu$	Multiplicative free convolution of μ and ν
$\mu \boxslash \nu$	Multiplicative free deconvolution of μ and ν

Topology

A^c	Complementary of the set A
$\#A$	Cardinality of the discrete set A
$A \oplus B$	Direct sum of the spaces A and B
$\oplus_{1 \leq i \leq n} A_i$	Direct sum of the spaces A_i, $1 \leq i \leq n$

$\langle x, A \rangle$	Norm of the orthogonal projection of x on the space A

Miscellaneous

$x \triangleq y$	x is defined as y
$\mathrm{sgn}(\sigma)$	Signature (or parity) of the permutation σ, $\mathrm{sgn}(\sigma) \in \{-1, 1\}$