An Introduction to Category Theory

Category theory provides a general conceptual framework that has proved fruitful in subjects as diverse as geometry, topology, theoretical computer science and foundational mathematics. Here is a friendly, easy-to-read textbook that explains the fundamentals at a level suitable for newcomers to the subject.

Beginning postgraduate mathematicians will find this book an excellent introduction to all of the basics of category theory. It gives the basic definitions; goes through the various associated gadgetry, such as functors, natural transformations, limits and colimits; and then explains adjunctions. The material is slowly developed using many examples and illustrations to illuminate the concepts explained. Over 200 exercises, with solutions available online, help the reader to access the subject and make the book ideal for self-study. It can also be used as a recommended text for a taught introductory course.

An Introduction to Category Theory

HAROLD SIMMONS
University of Manchester

CAMBRIDGE UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town,
Singapore, São Paulo, Delhi, Tokyo, Mexico City

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9781107010871

Original published 2011

Printed in the United Kingdom at the University Press, Cambridge

A catalogue record for this publication is available from the British Library

Library of Congress Cataloguing in Publication data
Simmons, Harold
An introduction to category theory / Harold Simmons.
p. cm.
ISBN 978-1-107-01087-1 (hardback)
1. Categories (Mathematics) I. Title.
QA169.S56 2011
512′.62–dc23
2011021721

ISBN 978-1-107-01087-1 Hardback
ISBN 978-0-521-28304-5 Paperback

Additional resources for this publication at www.cambridge.org/simmons

Contents

Preface

As it says on the front cover this book is an introduction to Category Theory. It gives the basic definitions; goes through the various associated gadgetry such as functors, natural transformations, limits and colimits; and then explains adjunctions. This material could be developed in 50 pages or so, but here it takes some 220 pages. That is because there are many examples illustrating the various notions, some rather straightforward, and others with more content. More importantly, there are also over 200 exercises. And perhaps even more importantly, solutions to these exercises are available online.

The book is aimed primarily at the beginning graduate student, but that does not mean that other students or professional mathematicians will not find it useful. I have designed the book so that it can be used by a single student or small group of students to learn the subject on their own. The book will make a suitable text for a reading group. The book does not assume the reader has a broad knowledge of mathematics. Most of the illustrations use rather simple ideas, but every now and then a more advanced topic is mentioned. The book can also be used as a recommended text for a taught introductory course.

Every mathematician should at least know of the existence of category theory, and many will need to use categorical notions every now and then. For those groups this is the book you should have. Other mathematicians will use category theory every day. That group has to learn the subject sometime, and this is the book to start that process. Of course, the more advanced topics are not dealt with here.

The book has been developed over quite a few years. Several short courses of about 10 hours have been taught (not always by me) using some of the material. In 2007, 2008, and 2009 I gave a course over the web to about a dozen universities. This was part of MAGIC, the

Mathematics Access Grid Instruction and Collaboration

cooperative of quite a few University Departments of Mathematics in England and Wales. That was an interesting experience and helped me to split the material into small chunks each of the right length to fit into one hour. (The course is still being taught but someone else has taken over the wand.) Of course, the order in which material is taught need not be the same as the written account.

As someone once said, Mathematics is not a spectator sport. To learn and understand Mathematics you have to get stuck in and get your hands dirty. You have to do the calculations, manipulations, and proofs yourself, not just read the stuff and pretend you understand it. Thus I have included over 200 exercises to help with this process. I have also written a more or less complete set of solutions to these exercises. But these are not available in the book, for it is too easy simply to look up a solution. When you can't see how to do it you have to sweat a bit to find a solution. Someone else once said that horses sweat, gentlemen perspire, and ladies glow. However, I can't remember meeting many horses who could do mathematics all that well. In other words, although effort is important to learn mathematics you also need something else. You need help every now and then. That is why there are exercises *and* solutions. These solutions are available at

`www.cambridge.org/simmons`

The book is divided into six Chapters, each chapter is divided into several Sections, and a few of these are divided into Blocks (Subsections). Each chapter contains a list of Items, that is Definitions, Lemmas, Theorems, Examples, and so on. These are numbered by section. Thus item $X.Y.Z$ is in Chapter X, Section Y, and is the Zth item in that section. Where a section is divided into blocks the items are still numbered by the parent section.

Each section contains a selection of Exercises. These are numbered separately throughout the section. Thus Exercise $X.Y.Z$ is in Chapter X, Section Y, and is the Zth exercise of that section. Again, where a section is divided into blocks the exercises are still numbered by the parent section.

Occasionally you will see a word or two IN THIS FONT. This is a mention of a NOTION that is dealt with in more detail later. You should remember to come back to this place when you understand the notion.

There are several other books available on this subject. Some of these are introductory texts and some are more advanced. I have listed some of them in the bibliography. None of these are needed when reading this book, but some will certainly help broaden and advance your understanding of the subject. I have refrained from passing comment on these books, for I know that different people have different tastes. However, you should look around for different accounts. Some of these will help.

I first became aware of Category Theory in 1965 during a Summer Meeting in Leicester (England). Since then I have been trying to learn and understand the subject. It is patently obvious to me that Category Theory is a very useful tool. It helps to organize many parts of mathematics. It can sort out the 'routine' aspects of a proof and isolate the 'essential content' of the result. In some ways that is why Eilenberg and MacLane invented the subject. However, I am not one of those 42ers who think that Category Theory is the essential foundations for Mathematics, Life, and Everything. Of course Category Theory is something that every mathematician should know something about, but there are other things as well.

Many people have influenced this book. For several years Andrea Schalk has used the material to teach an introductory course. Hugh Steele, Roman Krenický, and Francisco Lobo have pointed out and sometimes corrected my eccentricities. And Wolfy has guided me through some of the deeper mysteries of LaTeX. Where would we be without the wonderful LaTeX?

There may still be mistakes, inaccuracies, or garbled bits in the book. I would be quite happy to pass on the blame, but I won't. I am not a politician. I am responsible for everything inside the cover. The outside cover is the work of others.

Any book of this kind must contain many diagrams, some of which must commute. I have used Paul Taylor's diagram package to do this job. If you don't know this package then I recommend you have a look at it. I have also used his lesser known tree drawing package at one place.

At Cambridge University Press my contact, Silvia Barbina, has been very helpful. I once taught her a little bit about football (and, as she reminded me, some Model Theory). Silvia has made writing this final version very enjoyable. She has kept me on the straight and narrow, so I didn't wander off to do something else. In her charming Italian style she asked me (instructed me) to cut out all the jokes. This was quite difficult since some of the official categorical terminology is a joke, but I have done my best.

Clare Dennison and Lucy Edwards oversaw the production period (getting my raw code converted into the material you have in front of you). Siriol Jones copy-edited the book and corrected many of my silly mistakes. I thank them all. Roger Astley was chief pie-man for the whole project.

On a more personal level I am very grateful to Bobby Manc and what he is achieving. I hope he continues for quite some time. The Lodge (Appleby Lodge) is at last getting back to what it should be. Ruth Maddocks kept me cheerful. She made me the odd cup of tea. A very odd cup of tea.

Enjoy yourself and learn something at the same time.