Gauge/Gravity Duality

Foundations and Applications

Gauge/gravity duality creates new links between quantum theory and gravity. It has led to new concepts in mathematics and physics, and provides new tools for solving problems in many areas of theoretical physics. This book is the first comprehensive textbook on this important topic, enabling graduate students and researchers in string theory and particle, nuclear and condensed matter physics to become acquainted with the subject.

Focusing on the fundamental aspects as well as on applications, this textbook guides readers through a thorough explanation of the central concepts of gauge/gravity duality. For the AdS/CFT correspondence, it explains in detail how string theory provides the conjectured map. Generalisations to less symmetric cases of gauge/gravity duality and their applications are then presented, in particular to finite temperature and density, hydrodynamics, QCD-like theories, the quark–gluon plasma and condensed matter systems. The textbook features a large number of exercises, with solutions available online at www.cambridge.org/9781107010345.

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Preface

Gauge/gravity duality is a major new development within theoretical physics. It brings together string theory, quantum field theory and general relativity, and has applications to elementary particle, nuclear and condensed matter physics. Gauge/gravity duality is of fundamental importance since it provides new links between quantum theory and gravity which are based on string theory. It has led to both new insights about the structure of string theory and quantum gravity, and new methods and applications in many areas of physics. In a particular case, the duality maps strongly coupled quantum field theories, which are generically hard to describe, to more tractable classical gravity theories. In this way, it provides a wealth of applications to strongly coupled systems. Examples include theories similar to low-energy quantum chromodynamics (QCD), the theory of strong interactions in elementary particle physics, and models for quantum phase transitions relevant in condensed matter systems.

Gauge/gravity duality realises the *holographic principle* and is therefore referred to as *holography*. The holographic principle states that the entire information content of a quantum gravity theory in a given volume can be encoded in an effective theory at the boundary surface of this volume. The theory describing the boundary degrees of freedom thus encodes all information about the bulk degrees of freedom and their dynamics, and vice versa. The holographic principle is of very general nature and is expected to be realised in many examples. In many of these cases, however, the precise form of the boundary theory is unknown, so that it cannot be used to describe the bulk dynamics.

String theory, however, gives rise to a precise realisation of the holographic principle, in which both bulk and boundary theory are known: this is gauge/gravity duality. In this case, a quantum field theory at the boundary, which involves a gauge symmetry, is conjectured to be equivalent to a theory involving gravity in the bulk. Moreover, string theory provides many examples of *dualities*: a physical theory may generically have different equivalent formulations which are referred to as being dual to each other. Two formulations are equivalent if there is a one-to-one map between the states in each of them, and the dynamics are the same. Duality is particularly useful if physical processes are hard to calculate in one formulation, but easy to obtain in another. An example of a duality of this type is a map between two equivalent formulations in different coupling constant regimes. For instance, in a particular limit gauge/gravity duality maps a strongly coupled gauge theory, which generally is hard to describe, to a weakly coupled gravity theory, in which it is much more straightforward to perform explicit calculations.

The most prominent and best understood example of gauge/gravity duality is the *AdS/CFT correspondence*, the celebrated proposal by Maldacena. The AdS/CFT correspondence is characterised by a very high degree of symmetry. Here, ‘AdS’ stands for

Anti-de Sitter space and ‘CFT’ for conformal field theory. The field and gravity theories involved in the AdS/CFT correspondence display both supersymmetry and conformal symmetry. These symmetries are realised by the isometries of Anti-de Sitter space and further internal spaces on the one hand, and by the covariance of the quantum fields on the other. The high degree of symmetry allows for very non-trivial tests of the duality conjecture. These have led in particular to an increased understanding of the mathematical properties of $\mathcal{N} = 4$ Super Yang–Mills theory, the four-dimensional superconformal quantum field theory which is the most studied example of the AdS/CFT correspondence.

Motivated by the successes of the AdS/CFT correspondence in its original form, many physicists have begun to ask the question whether the AdS/CFT correspondence can be used to shed new light onto open problems in theoretical physics which are linked to strong coupling. There are many important strongly coupled systems in physics. However, although approaches to describing subsets of their properties exist, there is no general method to calculate their observables which as well established and ubiquitous as perturbation theory is for weakly coupled systems. Consequently, new ideas for describing strongly coupled systems are very welcome, and generalisations of the AdS/CFT correspondence to *gauge/gravity dualities* have made useful contributions to new descriptions of at least some aspects of strongly coupled systems. The best established example is given by the combination of gauge/gravity duality methods with linear response theory, for describing transport processes.

There are many interesting phenomena of strong coupling which have been investigated using gauge/gravity duality. These include the description of theories related to QCD at low energies. The most extensively studied examples are applications to the physics of the quark–gluon plasma, a new strongly coupled state of matter at temperatures above the QCD deconfinement temperature. The quark–gluon plasma has been observed experimentally and continues to be under experimental study, in particular at the RHIC accelerator in Brookhaven and at the LHC at CERN, Geneva. In this context, gauge/gravity duality has contributed the celebrated result for η/s, the ratio of shear viscosity to entropy density, of Kovtun, Son and Starinets, which agrees well with experimental observations. This result provides an example of *universality* in gauge/gravity duality, which means that gravity theories with different structure, dimensionality and field content all give the same result for η/s. On the field theory side, this implies that the precise form of the microscopic degrees of freedom is irrelevant for the dynamics.

More recently, gauge/gravity duality has also been applied to strongly coupled systems in condensed matter physics. In this context, the concept of universality is also of central importance, and is realised for instance near quantum phase transitions. These are phase transitions at zero temperature generated by quantum fluctuations.

Given the central importance of the new research area of gauge/gravity duality, this book aims to introduce a wide audience, including beginning graduate students and researchers from neighbouring areas, to its central ideas and concepts. The book is structured in three parts. The first part covers the prerequisites for explaining the duality. In the second part, the duality is established. The third part is devoted to applications.

To explain the subtle relations provided by gauge/gravity duality, in part I we first present the many ingredients which the duality relates. This involves elements of gauge theory, such as the large N expansion, conformal symmetry and supersymmetry on the one hand, and the geometry and gravity of Anti-de Sitter spaces on the other. Moreover, since the duality is firmly rooted within string theory, we also present an overview of relevant string theory topics.

Part II of the book is devoted to establishing the duality. We explain in detail the motivation for the AdS/CFT correspondence. We state the associated conjecture and give a number of examples of the compelling evidence supporting the conjecture. A particularly important approach is based on the use of integrability. Moreover, the correspondence is generalised to non-conformal examples, and we introduce holographic renormalisation group (RG) flows. We also discuss generalisations to finite temperature, which are obtained by considering a black hole in Anti-de Sitter space.

In part III, applications of gauge/gravity duality are presented. As examples, we consider holographic hydrodynamics, as relevant in particular to applications to the quark–gluon plasma. We also consider applications to theories similar to low-energy QCD. Finally, we present applications to systems of relevance in condensed matter physics, such as quantum phase transitions, superfluids and superconductors as well as Fermi surfaces. We conclude with a discussion of holographic entanglement entropy.

Let us give a more detailed guide to these three parts. Part I contains four chapters reviewing the relevant aspects of quantum field theory, general relativity, symmetries such as conformal and supersymmetry, and string theory, respectively. Part I is intended primarily for graduate students. However, experienced readers may use it as a glossary of concepts used in parts II and III. Moreover, researchers interested in applications of gauge/gravity duality may find it useful to read chapter 4, which contains a short summary of string theory and supergravity as relevant for understanding the string theory origin of gauge/gravity duality.

In part II, the AdS/CFT correspondence is stated and non-trivial tests as well as extensions of the AdS/CFT correspondence are presented. The key chapter is chapter 5 in which the AdS/CFT correspondence is motivated within string theory, considering in particular the near-horizon limit of D3-branes. Moreover, the field-operator map is established and the important concept of holographic renormalisation is introduced. Also, an explanation of how to realise Wilson loops in AdS/CFT is given. Chapter 6 contains non-trivial tests of the AdS/CFT correspondence, such as the calculation of correlation functions and of the conformal anomaly. In chapter 7, aspects of integrability and scattering amplitudes are introduced, providing further tests, as well as further elucidating string theory aspects of the correspondence. In chapter 8, further examples of the AdS/CFT correspondence are presented, such as AdS/CFT for branes at singularities and for M2-branes. Moreover, as a first step towards generalising the correspondence, we consider examples of the duality in which conformal symmetry is broken. In chapter 9 we discuss holographic renormalisation group (RG) flows. We consider simple cases of flows linking a UV to an IR fixed point, as well as explicit realisations of RG flows within IIB supergravity. In chapter 10 we describe models with additional branes in supergravity. In particular, we consider flavour branes, which provide descriptions of particles with similar properties to quarks and electrons within gauge/gravity duality. In chapter 11, we formulate the correspondence at finite temperature in Lorentzian signature and explain how to obtain a causal structure which allows us to introduce retarded Green's functions.

Readers interested primarily in applications may omit chapters 7, 8 and the second half of chapter 9 (RG flows within IIB supergravity) at first reading. Readers interested primarily in foundations are encouraged to concentrate on all chapters of part II, including chapter 11 on finite temperature. Within part II, there are a few sections denoted by an asterisk *. These provide material at a more advanced level and are not a prerequisite for reading the subsequent sections and chapters.

Part III, devoted to applications, is organised as follows. In chapter 12 we introduce the linear response formalism and hydrodynamics and explain how both are implemented in gauge/gravity duality. This provides the tools for calculating transport coefficients. As an important example, we consider the shear viscosity over entropy ratio. We also discuss the important concept of quasinormal modes and their relation to the pole structure of Green's functions. In chapters 13 and 14 we introduce aspects of applications of gauge/gravity duality to theories related to QCD. Chapter 13 is devoted in particular to confinement, chiral symmetry breaking and light mesons. Chapter 14 deals with applications to QCD-like theories at finite temperature and density. In chapter 15, we introduce applications of gauge/gravity duality to systems of relevance in condensed matter physics. We review the concept of quantum phase transitions, calculate conductivities, introduce holographic superconductors, review the electron star and hyperscaling models and give an introduction to the gauge/gravity duality approach to entanglement entropy.

There are three appendices, on Grassmann numbers (appendix A), on Lie algebras, superalgebras and their representations (appendix B), and an appendix summarising our conventions (appendix C). Appendix B contains important information on group theory which is essential for establishing the field-operator map for the AdS/CFT correspondence.

We have chosen to list the relevant references at the end of each chapter. Each of these reference lists is preceded by a 'Further reading' section, which briefly describes the references used in preparing the text. Moreover, an outlook on further relevant literature is given.

There are exercises given in the text which are intended to help the reader become acquainted with the standard tools and methods of gauge/gravity duality.

Gauge/gravity duality is a fast growing area of research with a wealth of different aspects. This implies that certain topics had to be selected for inclusion in this book. Our main guiding principle is to provide a textbook style introduction to the subject. This implies that there is an extensive introduction, and examples of generalisations and applications. Our choice of generalisations and applications is influenced by our own research experience and interests. We hope that after studying this book, readers will be able to read and understand original research papers on many other exciting aspects of gauge/gravity duality, and to become involved with research in this fascinating area themselves.

Johanna Erdmenger and Martin Ammon

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