

## FOUNDATIONS OF QUANTUM GRAVITY

Exploring how the subtleties of quantum coherence can be consistently incorporated into Einstein's theory of gravitation, this book is ideal for researchers interested in the foundations of relativity and quantum physics.

The book examines those properties of coherent gravitating systems that are most closely connected to experimental observations. Examples of consistent co-gravitating quantum systems whose overall effects upon the geometry are independent of the coherence state of each constituent are provided, and the properties of the trapping regions of non-singular black objects, black holes, and a dynamic de Sitter cosmology are discussed analytically, numerically, and diagrammatically.

The extensive use of diagrams to summarize the results of the mathematics enables readers to bypass the need for a detailed understanding of the steps involved. Assuming some knowledge of quantum physics and relativity, the book provides textboxes featuring supplementary information for readers particularly interested in the philosophy and foundations of the physics.

JAMES LINDESAY is a Professor of Physics at Howard University, and was the founding Director of the Computational Physics Lab. He has been a visiting professor at Hampton University, Stanford University, and a visiting faculty scientist at MIT.

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## Preface

This book is the result of decades of efforts by the author to understand physics at its most basic foundations, and to construct models that can address some of the unanswered questions about gravity, quantum mechanics, and the spectrum of fundamental particles. It is felt that foundational explorations should examine the conceptual and philosophical basis of a discipline. As such, less emphasis was placed upon the mathematics of complex calculations, while more emphasis was placed upon the consistencies and critiques of basic premises, in the preparation of this book. As a scientist, there is often a tendency to be drawn towards mathematical and formal pursuits, simply because of the beauty and elegance of mathematics. Such approaches sometimes bypass difficult conceptual approaches and thought experiments, or develop speculative formulations just because of their elegance. This book attempts to establish a balance towards the exploration of *basic* concepts and puzzles.

The prior book on black holes by Lenny Susskind and myself serves as an introduction to quantum physics and relativity for static geometries, as well as information in horizon physics. However, some of my more recent explorations indicate that qualitative modifications in descriptions of the physics occur once dynamics has been incorporated. This then calls into question any intuitions from static geometries that have been used to imply that those very geometries cannot be static. This book incorporates dynamic, spatially coherent geometries, as well as expanding upon the well-established foundations of the prior work. It represents a modest attempt to address some of the questions that were posed to me by Lenny during the writing of our book, as well as an attempt to explore the degree of flexibility in micro-physics that can be consistent with gravitational macro-physics. For instance, Lenny once asked me if an asymptotic Schwarzschild observer is freely falling or fiducial, whether Hawking radiation will be observed by such an observer, and how my answer would be consistent with complementarity. He also had me consider the drastic change in the interior causal structure of a

Schwarzschild space-time that would result from a single, charged electron falling into the center, since the center should change from space-like to time-like as a result of the charge. How does the geometry get modified so drastically?

I also had several fundamental dilemmas of my own. Are the thermal properties of dynamic horizons different than those of static horizons? Can a quantized geometric interaction localize the phase information of a quantum system without breaking its coherence? When the Penrose diagram for the formation of a black hole is constructed, why should its formation modify the scales of a distant Minkowski observer? With regards to microscopic physics, is renormalizability a fundamental requirement for all viable physical theories? After all, some very useful models, like BCS superconductivity, are not analytic in the small coupling limit, and bound states are inherently non-perturbative in the coupling. This book is a compilation of my attempts to answer these and other questions for myself. It is my hope that the reader will find some nugget(s) of interest for further exploration or development out of the various results presented.

I would like to make a few acknowledgments of the many individuals and groups that have been supportive of my efforts, in particular since this is my first manuscript as a single author. As a teacher myself, I have come to even more fully appreciate the dedication that my mentors have exemplified in their support of my development as a scholar. My early general education, science, and math teachers Lee Roy Pitts, Craig Hall, Felton Denham, John Rice, Sharon Belden, Arnold Webb, John Henderson, and Calvin Glasgow, spent hours of evenings and weekends mentoring an immature student within whom they saw some potential, in various projects and activities of pre-college nurturing.

My mentors for undergraduate research at MIT, Francis Low, Ulrich Becker, and Harry Morrison, were truly phenomenal, introducing me to world-class scholarship worthy of attempts at emulation. Harry Morrison, in particular, gave continuing mentorship and support for which I could never express sufficient gratitude. I am likewise grateful for superb mentorship at Stanford by Stan Brodsky, Cliff Will, Roberto Peccei, and my advisor H. Pierre Noyes, who has continued his support throughout my academic career.

Several of my former students and present colleagues have directly contributed to this effort, including Alex Markevich, Ed Jones, Beth Brown, Paul Sheldon, Tehani Finch, Marcus Alfred, and Tepper Gill. In particular, my understanding of black holes and horizons would be rudimentary were it not for the expert tutelage of Lenny Susskind, whose brilliant insights present concepts that are contagious and inspiring. I would also like to acknowledge that my work on linear spinor fields was done partially while I was a Peace Corp Volunteer with the faculty at the University of Dar-es-Salaam, whose hospitality remains unforgettable, and with

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the support of my mother, Penelope Brown, and grandmother, Elnora Herod, during the intermediate periods of my service in the mid 1980s.

I would like to acknowledge my brother, Crayge Lindesay, for technical expertise in the development of several of the complex computer graphics. I also thank my friend Eileen Johnston, artist, for the design of the book cover. Lastly, I must express my appreciation for the support of friends and family throughout the writing of this book.



Notations and Conventions

Part I

Section

1.1.1 Minkowski metric  $((\eta_{\mu\nu})) = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

1.1.1  $\tau = \text{proper time} \quad dc\tau^2 = -\eta_{\mu\nu} dx^\mu dx^\nu$

1.1.3 Electrodynamic equations will use cgs-Gaussian units.

1.1.3 Covariant  $\mathbf{F} = ((F_{\mu\nu})) = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & B_z & -B_y \\ E_y & -B_z & 0 & B_x \\ E_z & B_y & -B_x & 0 \end{pmatrix}$ .

1.4.2 Compact conformal coordinates:  
 $Y_{\rightarrow} = [\tanh(\frac{ct_*+r_*}{scale}) - \tanh(\frac{ct_*-r_*}{scale})] / 2$

$Y_{\uparrow} = [\tanh(\frac{ct_*+r_*}{scale}) + \tanh(\frac{ct_*-r_*}{scale})] / 2$

2.1 Relevant fundamental constants:

Kinematic constants: Planck’s constant  $\hbar \simeq 6.58 \times 10^{-25}$  GeV-s and  
speed of light  $c \simeq 3 \times 10^8$  m/s.

Geometric constant: Newtons gravitational constant  
 $G_N \simeq 6.71 \times 10^{-39} \hbar c^5 / GeV^2$ .

Statistical scaling: Boltzmann’s constant  $k_B$  relates entropy to a  
temperature scale.

Planck mass  $M_P \equiv \sqrt{\frac{\hbar c}{G_N}} \simeq 1.2 \times 10^{19} \text{ GeV}/c^2$ .

Planck length  $L_P \equiv \sqrt{\frac{\hbar G_N}{c^3}} \simeq 1.6 \times 10^{-35} \text{ m}$ .

2.1.1 Momentum state normalization:

$\langle \mathbf{p}'; m | \mathbf{p}; m \rangle = \frac{\epsilon(p)}{p_{(s)}^0 c} \delta^3(\mathbf{p}' - \mathbf{p})$   
 $\hat{1} = \int \frac{p_{(s)}^0 c}{\epsilon(p)} d^3 p |\mathbf{p}; m\rangle \langle \mathbf{p}; m| = \int d^4 p |\vec{p}\rangle \langle \vec{p}| \delta(\sqrt{-\vec{p} \cdot \vec{p}} - mc).$

2.3.2 Reduced Compton wavelength of mass m:  $\lambda_m \equiv \frac{\hbar}{mc}$ .

8.1 Cosmological phenomenology:

Age of universe  $t_o \approx 13.7 \times 10^9$  years  
Redshift at last scattering (CMB):  $z_{LS} \approx 1100, k_B T_{LS} \approx 0.3 \text{ eV}$   
Dark energy density ratio to critical density  $\Omega_{\Lambda_o} \approx 0.73$

Part II

Locally freely falling coordinate components  $\xi^{\tilde{\mu}}$   
Curvilinear metric coordinate components  $x^\beta$