Do the movements of animals, including humans, follow patterns that can be described quantitatively by simple laws of motion? If so, then why? These questions have attracted the attention of scientists in many disciplines and stimulated debates on ecological matters to queries such as, “how can there be free will if one follows a law of motion?”

This is the first book on this rapidly evolving subject, which introduces random searches and foraging in a way that can be understood by readers with no previous background on the subject. It reviews theory as well as experiments, addresses open problems and perspectives, and discusses applications ranging from the colonization of Madagascar by Austronesians to the diffusion of genetically modified crops.

The book will interest physicists working in the field of anomalous diffusion and movement ecology as well as ecologists already familiar with the concepts and methods of statistical physics.

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THE PHYSICS OF FORAGING

An Introduction to Random Searches and Biological Encounters

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Contents

Preface

Part I Introduction: Movement

1 Empirical motivation for studying movement
  1.1 How do organisms really move, and why? 3
  1.2 Biological encounters as a reaction-diffusion process 4
  1.3 Impact and scientific importance 7
  1.4 Follow the data 9
  1.5 Beyond model comparison 12

2 Statistical physics of biological motion
  2.1 Optimal foraging theory 14
  2.2 Microscopic versus macroscopic levels of description 15
  2.3 Disorder and incomplete information 16
  2.4 Scaling and universality 17
  2.5 The extraordinary success of limiting models 20

3 Random walks and Lévy flights
  3.1 Central limit theorems 23
  3.2 Normal diffusion and Brownian motion 24
  3.3 Anomalous diffusion 27
  3.4 Lévy flights and Lévy walks 34

4 The wandering albatross
  4.1 Do good theories always come from good data? 42
  4.2 Lévy flights of the wandering albatross 42
  4.3 Power laws and Pareto’s principle 46
  4.4 Scientific progress as a random walk 48
Contents

Part II  Experimental findings

5  Early studies 53
   5.1  Fickian transport 53
   5.2  Directional persistence 54
   5.3  A new idea: Lévy flights and walks 55

6  Evidence of anomalous diffusion 58
   6.1  Arthropods and mollusks 58
   6.2  Marine and aquatic animals 60
   6.3  Mammals 61
   6.4  Micro-organisms 62
   6.5  Birds 63

7  Human dispersal 64
   7.1  Hunter-gatherers and archaeological evidence 64
   7.2  Lévy flights of dollar bills 65
   7.3  GPS tracking of humans 65
   7.4  Fishermen as foragers 66
   7.5  Austronesians in Madagascar 67

8  How strong is the evidence? 71
   8.1  Measurement and data analysis 71
   8.2  Special issues related to power laws 72
   8.3  Anomalous diffusion: Not if, but when and why 72

Part III  Theory of foraging

9  Optimizing encounter rates 77
   9.1  A general theory of searchers and targets 77
   9.2  A limiting but general model of optimal foraging 79
   9.3  Random walk propagators and encounter rates 80

10 Lévy flight foraging 85
   10.1  The Lévy flight foraging hypothesis 85
   10.2  Analytical and numerical results 87
   10.3  Discrete versus continuous media 91
   10.4  Energy and entropy 96

11 Other search models 100
   11.1  Correlated random walks with a single scale 100
   11.2  Intermittent searches with two scales 103
   11.3  A unified approach 106
Contents ix

Part IV Finale: A broader context 111
12 Superdiffusive random searches 111
12.1 Submarine warfare and operations research 111
12.2 Enzymatic searches on DNA 112
12.3 Robot foraging 113
12.4 Eye microsaccades 113
12.5 Learning, memory, and databases 114
12.6 Genetically modified crops and disease vectors 115

13 Adaptational versus emergent superdiffusion 116
13.1 Are Lévy walks really adaptive? 117
13.2 Self-organization and emergence 118
13.3 Deterministic induction of Lévy behavior 119
13.4 Why the answer is crucial 121

14 Perspectives and open problems 123
14.1 The flavor of foraging research 123
14.2 Biological mechanisms underlying superdiffusion 124
14.3 Determinism, randomness, and free will 127
14.4 Globally optimum random searches 129
14.5 Final remarks 130

Appendices 131
A Data analysis 131
A.1 A criterion for inferring superdiffusion 131
A.2 Log-log plots and surrounding controversies 133
A.3 Maximum likelihood estimation 134

B Lévy walkers inside absorbing boundaries 136

References 140
Index 161
Preface

As the FBI helps a 14-year-old victim who escaped from a dangerous polygamist self-proclaimed prophet, it is faced with the question of how to search 2200 square miles of mountain desert.

“How rough is the terrain? Because the rougher the terrain, the more likely she was forced into a Lévy flight type movement. I can create a viable search pattern,” says Charlie Eppes, the mathematical genius.

“It’s like when you lose your keys,” explains Amita Ramanujan, his girlfriend and former doctoral student. “You don’t methodically search every inch of your house from front to back. You look like crazy in one area, and then jump to the next most likely area and look there.”

The preceding dialogue, from the American television series *Numb3rs*, shows how far the theory of Lévy flight foraging has penetrated mainstream science. Although the term *foraging* has a biological connotation, in fact, biological foraging is a special case of random searches. Michael Shlesinger, for instance, has pointed out the relevance of random searches to operations research in World War II, involving the hunt for enemy submarines.

There are intriguing aspects of the random search problem that are peculiar to biological foraging. Why should the movements of freely moving animals follow any natural law at all? This is a fascinating question, and we find it remarkable that animals – and even humans – that possess a degree of “free will” actually move in a manner that can be described quantitatively by physical principles.

Questions such as how they move (and why) have attracted the attention of physicists for a number of other reasons as well. The trajectories of individual organisms as they forage and search closely resemble certain kinds of random walks studied by theoretical physicists. Encounter interactions of moving organisms thus have a close parallel with reaction-diffusion processes seen in nonequilibrium statistical mechanics. These mathematical similarities and the abundance of experimental...
data have allowed physicists to make contributions toward the study of foraging. Hence the choice of title, *The Physics of Foraging*.

This book is an introduction to the interdisciplinary study of how organisms move in order to encounter and interact. Our goal is to bring together relevant theoretical concepts, along with a review of the experimental findings, to allow the current literature to be readable and understandable. The focus is on the statistical properties of the trajectories of single organisms. Such quantities are statistically coercive, making them useful in constraining the range of possible behaviors or in predicting the conditions under which the adoption of a given random walk strategy might confer some relative advantage, and so forth.

The statistical physics approach to the problem of foraging and random searches in many ways complements the more traditional treatment given in other disciplines. Collaboration between biologists and ecologists, on one hand, and physicists, on the other, began in earnest about two decades ago. By the early 1990s, there was enough empirical evidence to suggest that organisms do not always diffuse like particles performing Brownian motion. In collaboration with Sergey Buldyrev and Shlomo Havlin, we proposed in 1999 what has become known as the Lévy flight foraging hypothesis to try to account for the observed phenomena. Since then, a number of works coauthored by physicists have further impacted the interdisciplinary subfield of theoretical movement ecology. At present, the field is still evolving rapidly; hence we also discuss in the book a number of open problems and perspectives, both from theoretical and empirical points of view.

We are in great debt to many people with whom, along the years, we have discussed many aspects of random searches, Lévy flights, and related topics. We thank our key collaborators in the two first *Nature* papers that congealed our interest in the topic: V. Afanasyev, S. V. Buldyrev, S. Havlin, E. J. Murphy, and the late P. A. Prince. We thank Ivars Peterson for the appealing article on the first *Nature* paper that helped focus attention on statistical physics approaches to understanding the more enigmatic aspects of foraging. We also thank Mark Buchanan for drawing considerable attention to the topic in a more recent feature article in *Nature*. We have had discussions with many other people to whom we are grateful: J. S. Agnaldo, F. Bartumeus, M. W. Beims, D. Boyer, C. Carvalho, J. Catalan, S. Cavalcante, M. D. Coutinho-Filho, J. C. Cressoni, A. Davis, A. M. Edwards, C. L. Faustino, M. L. Felisberto, N. M. Freeman, U. L. Fulco, M. Gitterman, L. Giuggioli, I. M. Gléria, H. D. Jennings, V. M. Kenkre, J. Klafter, A. Y. Kasakov, L. S. Lucena, M. L. Lyra, A. Marshak, J. L. Mateos, R. Metzler, O. Miramontes, C. M. Nascimento, A. M. Nemirovsky, R. W. Nowak, E. J. Nunes-Pereira, F. S. Passos, R. A. Phillips, M. C. Santos, M. F. Shlesinger, L. R. da Silva, M. A. A. da Silva, I. M. Sokolov, C. Tsallis, T. M. Viswanathan, and N. W. Watkins. Moreover, we are in great debt for the support and encouragement provided by our families. Last,
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