The Capital Asset Pricing Model in the 21st Century
Analytical, Empirical, and Behavioral Perspectives

The Capital Asset Pricing Model (CAPM) and the mean-variance (M-V) rule, which are based on classic expected utility theory (EUT), have been heavily criticized theoretically and empirically. The advent of behavioral economics, prospect theory, and other psychology-minded approaches in finance challenges the rational investor model from which CAPM and M-V derive. Haim Levy argues that the tension between the classic financial models and behavioral economics approaches is more apparent than real. This book aims to relax the tension between the two paradigms. Specifically, Professor Levy shows that although behavioral economics contradicts aspects of EUT, CAPM and M-V are intact in both EUT and Cumulative Prospect Theory (CPT) frameworks. There is, furthermore, no evidence to reject CAPM empirically when ex-ante parameters are employed. Professionals may thus comfortably teach and use CAPM and behavioral economics or CPT as coexisting paradigms.

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Modern finance is relatively new. Before the breakthrough “Portfolio Selection” article was published by Markowitz in 1952, research in finance was basically nonquantitative and the use of quantitative models in teaching and in research was rare. A glance at finance textbooks that were used in teaching before 1952 and textbooks that are currently used suffices to reveal the revolution induced in the finance profession by the publication of this 1952 Mean-Variance (M-V) article. The next revolutionary papers in portfolio selection and equilibrium pricing were published by Sharpe, Lintner, and Black in 1964, 1965, and 1972, respectively. These three papers use Markowitz’s M-V model as a springboard in developing equilibrium prices of risky assets in the capital market and in identifying beta rather than sigma as the risk measure of an individual asset in a portfolio context. The model developed by Sharpe and Lintner, known as the Capital Asset Pricing Model (CAPM), is used in virtually all research studies that deal with risk and return and occupies a substantial portion of textbooks on investments and corporate finance.

The other pillars of modern finance are the papers published by Modigliani and Miller in 1958, which focus on the optimal capital structure, and the two breakthrough papers published by Black and Scholes and by Merton on option pricing in 1973. No wonder Markowitz, Sharpe, Scholes, Merton, Modigliani, and Miller have all been awarded the Nobel Prize in Economics for their revolutionary contributions (the other researchers mentioned were not alive in relevant years when the prizes were awarded). Because this book focuses on portfolio selection and the CAPM, we mainly discuss and analyze
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the contributions of Markowitz, Sharpe, Lintner, and Black to the financial literature.

The publication of the Prospect Theory (PT) article by Kahneman and Tversky in 1979, for which Kahneman won the Nobel Prize in Economics in 2002, has shaken the foundations of the Expected Utility Theory (EUT); and, as the M-V framework and the CAPM have been developed within the EUT framework, PT indirectly has also shaken the foundations of these two models.

PT’s criticism of EUT is based on experimental findings. Additional criticism of the CAPM is based on empirical findings, showing that beta has very little or even no explanatory power at all. Leading this criticism is the 1992 empirical study of Fama and French, revealing that the coefficient of the CAPM’s beta is statistically insignificant; hence, in contradiction to the CAPM, beta does not explain the cross section of stock returns. Therefore, this finding allegedly casts doubt on the validity of beta as a measure of risk.

Thus, we have the M-V and the CAPM, which are widely used in teaching, in research, and by practitioners on the one hand, and PT’s experimental findings and empirical studies that criticize these two models on the other hand. Because PT has been known since 1979 and the empirical studies that criticize the M-V and the CAPM models have also been known for decades, one must wonder why academics as well as professional investors keep adhering to the M-V and the CAPM and why virtually all curriculums in finance still heavily rely on these two models. We devote this book to this question. We show that PT and M-V and the CAPM can coexist, even though PT and EUT cannot. We also show that although the CAPM is rejected with ex-post parameters, it cannot be rejected with ex-ante parameters.

We hope that after reading this book, professors of finance can comfortably teach the M-V and the CAPM, as well as the behavioral PT model, as we show that there is no contradiction between these two frameworks. Also, this book provides a somewhat different interpretation of the CAPM’s empirical studies, which, in a nutshell, asserts that the M-V and the CAPM cannot be rejected with the ex-ante parameters. Similarly, professional investors and consulting firms can continue relying on the M-V and the CAPM models, although some modifications may be needed.
In this book, we present all the material needed to achieve the integration of the M-V, CAPM, and Cumulative PT (CPT). For example, EUT and stochastic dominance rules are discussed, as we employ both to show that the M-V and the CAPM do not contradict CPT. Of course, we could refer the reader to this material in other books or articles but, to facilitate the reading of this book, we prefer to have all the relevant material contained in one place. The same principle is valid regarding PT and CPT material needed to prove that the behavioral model and the classical portfolio models can coexist. Finally, although we rely on the CPT, which is the modified version of PT, realizing the growing role of behavioral finance in recent years, we also devote a chapter to the original PT.

This book is mainly written for professors of finance and professional investors who use the M-V framework and the CAPM and who are also certainly aware of the criticisms of these two models. We hope that this book will resolve some conflicts and increase their confidence in the employed models. The book can be used in advanced courses in economics and finance and in Ph.D. classes in these two areas.

The book could not achieve its present form and level without the help of many people. I would like to thank Turan Bali, Rob Brown, Harry Markowitz, Richard Roll, William Sharpe, Jim Yoder, and an anonymous reader for their many helpful comments. It is a pleasure for me to thank Moshe (Shiki) Levy and Michal Orkan, who read the whole manuscript and provided me with many detailed comments.

Finally, I would like to thank Scott Parris and Adam Levine at Cambridge University Press and Peggy Rote at Aptara, Inc., for their great assistance in making writing and producing this book a pleasure.