Quantum Optics

In the past decade many important advances have taken place in the field of quantum optics, with numerous potential applications. This textbook provides an up-to-date account of the basic principles of the subject, and is ideal for graduate courses.

Focusing on applications of quantum optics, the textbook covers recent developments such as engineering of quantum states, quantum optics on a chip, nano-mechanical mirrors, quantum entanglement, quantum metrology, spin squeezing, control of decoherence, and many other key topics. Readers are guided through the principles of quantum optics and their uses in a wide variety of areas including quantum information science and quantum mechanics.

The textbook features over 150 end-of-chapter exercises with solutions available for instructors at www.cambridge.org/9781107006409. It is invaluable to both graduate students and researchers in physics and photonics, quantum information science, and quantum communications.

Girish S. Agarwal is Noble Foundation Chair and Regents Professor at Oklahoma State University. A recognized leader in the field of theoretical quantum optics, he is a Fellow of the Royal Society and has won several awards, including the Max-Born Prize from the Optical Society of America and the Humboldt Research Award.
Quantum Optics

GIRISH S. AGARWAL

Oklahoma State University
Dedicated to the memory of my wife Sneh who had hoped that the book would be completed and to my daughters Anjali and Mranjali who have been a constant source of inspiration.
Contents

Preface

1 Quantized electromagnetic field and coherent state representations
  1.1 Quantization of the electromagnetic field 1
  1.2 State space for the electromagnetic field – Fock space and Fock states 5
  1.3 Quadratures of the field 6
  1.4 Coherent states 7
  1.5 Mixed states of the radiation field 12
  1.6 Diagonal coherent state representation for electromagnetic fields – $P$-representation 15
  1.7 The Wigner function for the electromagnetic field 18
  1.8 Bosonic systems with finite mass – coherent states and phase-space representations 22
Exercises 24
References 26

2 Nonclassicality of radiation fields
  2.1 The Mandel $Q_M$ parameter 28
  2.2 Phase-dependent measure of nonclassicality – squeezing parameter $S$ 29
  2.3 Single-mode squeezed states – squeezed vacuum 31
  2.4 Squeezed coherent state 37
  2.5 Other measures of nonclassicality 39
  2.6 Mixed nonclassical states – degradation in squeezing 43
Exercises 45
References 47

3 Two-mode squeezed states and quantum entanglement
  3.1 The two-mode squeezed states 49
  3.2 Nonclassicality of the two-mode squeezed vacuum 50
  3.3 Quantum phase-space distributions and quadrature distributions 52
  3.4 Cauchy–Schwarz inequalities for nonclassicality in two-mode states 54
  3.5 Conditional measurements on the two-mode squeezed vacuum 55
  3.6 Quantum entanglement in the two-mode squeezed vacuum 56
  3.7 Peres–Horodecki separability criterion for continuous variable systems 56
  3.8 Generation of two-mode nonclassical and entangled states – optical parametric down-conversion 58
3.9 Parametric amplification of signals 60
3.10 Type-II optical parametric down-conversion – production of entangled photons 61
3.11 Four-photon entanglement using optical parametric down-conversion 63
3.12 Two-mode mixed nonclassical states 65
3.13 Entanglement in two-mode mixed Gaussian states 66
3.14 Application of entanglement to the teleportation of a quantum state 67
3.15 Nonclassical fields in optical fibers 69
Exercises 72
References 74

4 Non-Gaussian nonclassical states 76
4.1 Schrödinger cat state and the cat paradox 76
4.2 Photon-added and -subtracted states 82
4.3 Single-photon-added coherent and thermal states 84
4.4 Squeezing and sub-Poissonian properties of single-photon-added states 86
4.5 Experimental realization of photon-added nonclassical non-Gaussian states 88
4.6 Single-photon-subtracted states 89
4.7 Single-photon-subtracted two-mode states with vortex structure 93
4.8 Pair-coherent states 97
Exercises 101
References 102

5 Optical interferometry with single photons and nonclassical light 103
5.1 Transformation of quantized light fields at beam splitters 103
5.2 Beam splitter transformation equivalent to evolution under a Hamiltonian 105
5.3 Transformation of states by the beam splitter 105
5.4 Transformation of photon number states by a beam splitter 106
5.5 Single photons at beam splitters 107
5.6 Pairs of single photons at beam splitters 108
5.7 Generalization of the Hong–Ou–Mandel interference to $N$ photons from both ports of the beam splitter 109
5.8 Transformation of a two-mode squeezed state by a 50-50 beam splitter 109
5.9 Generation of two-mode entangled states by the interference of coherent fields and single photons 110
5.10 Beam splitter as an attenuator 111
5.11 Transformation of quantized light fields by phase shifters 112
5.12 The Mach–Zehnder interferometer 113
5.13 Wheeler’s delayed choice gedanken experiment 117
5.14 Interaction-free measurements 117
5.15 Two-photon Mach–Zehnder interferometer 119
5.16 Multiphoton interference and engineering of quantum states 121
5.17 Mach–Zehnder interferometer with two-mode squeezed vacuum as input 123
5.18 Balanced homodyne interferometers for measuring the squeezing of light 125
5.19 Manipulation of quantum states by homodyning and feed-forward 126
5.20 Quantum state tomography 128
5.21 Sensitivity of an optical interferometer 129
5.22 Heisenberg limited sensitivity of interferometers based on parametric amplifiers or four-wave mixers 131
5.23 The quantum statistics of fields at the output ports 133
Exercises 134
References 136

6 Polarization and orbital angular momentum of quantum fields 138
6.1 Characterization of the polarization properties of quantized fields 138
6.2 Polarization of quantized fields – Stokes operators 139
6.3 Action of polarizing devices on quantized fields 141
6.4 Description of unpolarized light beyond Stokes parameters 143
6.5 Stokes operator tomography 144
6.6 Orbital angular momentum of fields – HG and LG modes 146
6.7 Orbital Stokes operators and the Poincaré sphere 149
6.8 Mixed states of orbital angular momentum 151
6.9 Entangled states of the orbital angular momentum 152
6.10 Transformation of entanglement between polarization and orbital angular momentum q-plates 154
Exercises 155
References 156

7 Absorption, emission, and scattering of radiation 158
7.1 The interaction of radiation and matter in the electric dipole approximation 158
7.2 Rates for the absorption and emission of radiation 159
7.3 Single-mode limit – Einstein’s $\beta$ coefficient and the absorption coefficient $\alpha(\omega)$ 165
7.4 Scattering of radiation 166
7.5 Quantum interferences in scattering 169
7.6 Radiative decay of states – Weisskopf–Wigner theory 170
7.7 Control of spontaneous emission through the design of the electromagnetic vacuum 174
Exercises 177
References 178
## Partial coherence in multimode quantum fields

8.1 Correlation functions for electromagnetic fields 179
8.2 Young’s interferometer and spatial coherence of the field 181
8.3 Photon–photon correlations – intensity interferometry 184
8.4 Higher-order correlation functions of the field 187
8.5 Interferometry in the spectral domain 188
8.6 Squeezing spectrum and spectral homodyne measurement 191
8.7 Coherence effects in two-photon absorption 192
8.8 Two-photon imaging – ghost imaging using $G^{(2)}$ 194

Exercises 197
References 198

## Open quantum systems

9.1 Master equation description of open systems 200
9.2 Dissipative dynamics of harmonic oscillators 204
9.3 Dissipative dynamics of a two-level system 206
9.4 Dissipative dynamics of a multilevel system 208
9.5 Time correlation functions for multilevel systems 210
9.6 Quantum Langevin equations 212
9.7 Exactly soluble models for the dissipative dynamics of the oscillator 213
9.8 Exact dissipative dynamics of a two-level system under dephasing 215

Exercises 218
References 219

## Amplification and attenuation of quantum fields

10.1 Quantum theory of optical amplification 220
10.2 Loss of nonclassicality in the amplification process 223
10.3 Amplification of single-photon states 229
10.4 Amplification of entangled fields 230
10.5 Realising a phase-insensitive amplifier from a phase-sensitive amplifier 232
10.6 Degradation of nonclassicality and entanglement due to the absorption of quantum fields 233
10.7 Loss of coherence on interaction with the environment 235

Exercises 238
References 240

## Quantum coherence, interference, and squeezing in two-level systems

11.1 Two-level approximation: atomic dynamics in a monochromatic field 242
11.2 Application of atomic coherence – Ramsey interferometry 247
11.3 Atomic coherent states 249
11.4 Minimum uncertainty states for two-level systems – spin squeezing 252
11.5 Atomoc squeezed states by nonlinear unitary transformations 254
11.6 Atomic squeezed states produced by supersensitivity of Ramsey interferometers 256
### Contents

11.7 Phase-space representation for a collection of two-level systems 258
11.8 Phase-space description of EPR correlations of spin systems 262
Exercises 264
References 265

12 Cavity quantum electrodynamics 267
12.1 Exact solution of the Jaynes–Cummings model: dressed states 268
12.2 Collapse and revival phenomena in JCM 271
12.3 Dispersive limit of the JCM 273
12.4 Dissipative processes in cavity QED – the master equation 275
12.5 Spectroscopy of the ladder of dressed states 277
12.6 Multi-atom effects in cavity QED 284
12.7 Effective dipole–dipole interaction in a dispersive cavity from Lamb shift of the vacuum 288
12.8 Atomic cat states using multi-atom dispersive JCM 290
12.9 Application of atomic cat states in Heisenberg limited measurements 293
12.10 Engineering anti-Jaynes–Cummings interaction 296
12.11 QED in coupled cavity arrays – single-photon switch 298
Exercises 300
References 301

13 Absorption, emission, and scattering from two-level atoms 304
13.1 Effects of relaxation: optical Bloch equations 304
13.2 Absorption and amplification of radiation by a strongly pumped two-level system 309
13.3 Resonance fluorescence from a coherently driven two-level atom 314
13.4 Quantum dynamics of the two-level atom and spectrum of fluorescence 317
Exercises 325
References 327

14 Quantum interference and entanglement in radiating systems 328
14.1 Young’s interference with microscopic slits – atoms as slits 328
14.2 Spatial bunching and antibunching of photons 330
14.3 Interference in radiation from two incoherently excited atoms 333
14.4 Atom–photon entanglement 337
14.5 Atom–atom entanglement via detection of spontaneously emitted photons 338
14.6 Multi-atom entanglement 341
14.7 Quantum entanglement in Dicke states and superradiance 343
14.8 Multi-path quantum interference as the source of Dicke superradiance 345
14.9 Entanglement of photons produced in an atomic cascade 348
Exercises 351
References 352
## Contents

### 15 Near field radiative effects
- 15.1 Near field radiative effects – coupling between dipoles 354
- 15.2 Radiative coupling between dipoles and dynamics 358
- 15.3 Vacuum-induced deterministic entanglement 360
- 15.4 Two-photon resonance induced by near field radiative effects 362
- 15.5 The dipole blockade 365

Exercises 368

References 368

### 16 Decoherence and disentanglement in two-level systems
- 16.1 Decoherence due to the interaction of a two-level system with the environment 370
- 16.2 Disentanglement in two-level systems 371
- 16.3 Decoherence-free subspace 373
- 16.4 Protection of decoherence due to dephasing via dynamical decoupling 374
- 16.5 Control of the spectral density of environment for protection against decoherence 378
- 16.6 Modulation produced protection against disentanglement in cavity QED 380

Exercises 382

References 383

### 17 Coherent control of the optical properties
- 17.1 A simple model for coherent control 385
- 17.2 Dark states and coherent population trapping 394
- 17.3 EIT in single-atom fluorescence 397
- 17.4 Control of two-photon absorption 400
- 17.5 Vacuum-induced coherence and interference 404

Exercises 409

References 410

### 18 Dispersion management and ultraslow light
- 18.1 Group velocity and propagation in a dispersive medium 413
- 18.2 Electromagnetically induced waveguides 417
- 18.3 Storage and retrieval of optical pulses 418
- 18.4 Adiabatons and storage and retrieval of pulses 423
- 18.5 Non-EIT mechanisms for ultraslow light 426

Exercises 429

References 430

### 19 Single photons and nonclassical light in integrated structures
- 19.1 Quantum optics in a coupled array of waveguides 432
- 19.2 The Hong–Ou–Mandel interference in a system of two coupled waveguides 434
19.3 Single-photon transport and coherent Bloch oscillations in a coupled array
19.4 The Anderson localization of quantum fields in coupled waveguide arrays
19.5 Discrete quantum walks via waveguide couplers on a chip
Exercises
References

20 Quantum optical effects in nano-mechanical systems
20.1 The radiation pressure on the nano-mechanical mirror
20.2 Basic quantum Langevin equations for the coupled system of cavity and NMO
20.3 Steady-state solution of quantum Langevin equations in the mean field limit and bistability
20.4 Quantum fluctuations in optomechanical systems
20.5 Sideband cooling of the nano-mechanical mirror
20.6 Normal-mode splitting
20.7 Squeezing of a nano-mechanical oscillator
20.8 Electromagnetically induced transparency (EIT) in the mechanical effects of light
20.9 Quantized states of the nano-mechanical mirror coupled to the cavity
Exercises
References

Index
The development of new sources of radiation that produce nonclassical and entangled light has changed the landscape of quantum optics. The production, characterization, and detection of single photons is important not only in understanding fundamental issues but also in the transfer of quantum information. Entangled light and matter sources as well as ones possessing squeezing are used for precision interferometry and for implementing quantum communication protocols. Furthermore, quantum optics is making inroads in a number of interdisciplinary areas, such as quantum information science and nano systems.

These new developments require a book which covers both the basic principles and the many emerging applications. We therefore emphasize fundamental concepts and illustrate many of the ideas with typical applications. We make every possible attempt to indicate the experimental work if an idea has already been tested. Other applications are left as exercises which contain enough guidance so that the reader can easily work them out. Important references are given, although the bibliography is hardly complete. Thus students and postdocs can use the material in the book to do independent research. We have presented the material in a self-contained manner. The book can be used for a two-semester course in quantum optics after the students have covered quantum mechanics and classical electrodynamics at a level taught in the first year of graduate courses. Some advanced topics in the book, such as exact non-Markovian dynamics of open systems, quantum walks, and nano-mechanical mirrors, can be used for seminars in quantum optics.

The material in the book is broadly divided into two parts. The first part deals with many old and emerging aspects of the quantized radiation fields, such as the engineering and characterization of quantum states and the generation of entanglement. The working of an interferometer using one or a few photons is extensively treated. A chapter is devoted to quantum optics with fields carrying orbital angular momentum. Many applications of entangled fields are given. A thorough discussion of quantum noise in amplification and attenuation is also given. The second part deals with the interaction of radiation with matter. Coherent, squeezed, and cat states of atoms are treated. Dissipative processes are treated from a microscopic approach. This part includes a discussion of electromagnetically induced transparency and a host of applications. Special emphasis is placed on quantum interference and entanglement. It is shown how measurement can produce entanglement. Furthermore, the deterministic production of entanglement is discussed. Many relatively newer aspects of cavity QED, such as single photon switches, photon blockade, and anti-Jaynes–Cummings interaction are presented. The book concludes with a look at developments such as quantum optics on a chip, quantum optical effects arising from radiation pressure and mechanical motion, quantum walks, control of decoherence, and disentanglement.
The focus in the book is on emerging areas in quantum optics and therefore important topics like the quantum theory of lasers, micromasers, optical multi-stability, self-induced transparency, etc. have been left out as these are well covered in earlier textbooks like those of Mandel and Wolf (Optical Coherence and Quantum Optics) and Scully and Zubairy (Fundamentals of Quantum Optics). Other topics like polarization are treated from the new perspective of quantum fluctuations in Stokes parameters. Clearly it is impossible to present in a single volume all that has happened since the discovery of the laser and the pioneering quantum optical works in the early 1960s. We have emphasized aspects that we consider essential for the newer directions in which quantum optics is moving.

This book is the outcome of teaching courses in Quantum Optics at the University of Hyderabad and the Oklahoma State University and extensive lecturing at the International Center for Theoretical Physics, Trieste, and at many scientific schools in India and elsewhere. Research carried out over several decades and my earlier writings, as well as interaction with students, postdocs, and collaborators, has shaped the book. A part of the chapters dealing with states of the radiation field was evolved while I spent half a year in 1992–93 at the Max-Planck Institute for Quantum Optics, Garching, Germany. My collaborator Subhash Chaturvedi contributed by refining some of this material.

The book would never have been completed except for the tireless efforts of my student Sumei Huang who extensively worked on it. I am grateful to her. I also acknowledge considerable assistance from my student Kenan Qu.

I thank my collaborators Jay Banerji, Subhash Chaturvedi, Tarak Dey, Jacques Perk, Gautam Vemuri, and Joachim von Zanthier for reading several chapters and for providing useful input. I thank Ravi Puri, Surya Tewari, Subhasish Dutta Gupta, and a large number of students whose works have been used for the writing of the book. I thank Mustansir Barma for the hospitality at TIFR, Mumbai where I continued to work on my book.

Over the years I have learnt a lot through interactions with a large number of physicists. These interactions have had a deep influence in the writing of the book and I thank especially Bob Boyd, Jinx Cooper, Joe Eberly, Marlan Scully, Herman Haken, Sudhansu Jha, Peter Knight, Emil Wolf, the late Len Mandel, and Herbert Walther.

I am grateful to the Oklahoma State University, especially Dean Peter Sherwood, and the Noble Foundation for supporting my work and for providing ideal facilities.

I thank Dr. John Fowler who was instrumental in getting the book project going and Dr. Simon Capelin for his deep interest in the project. Finally, I thank the very supportive staff at Cambridge University Press, and in particular Ms. Lindsay Barnes, who always came up with a solution to my problems.