

Quantum Optics

In the past decade many important advances have taken place in the field of quantum optics, with numerous potential applications. This textbook provides an up-to-date account of the basic principles of the subject, and is ideal for graduate courses.

Focusing on applications of quantum optics, the textbook covers recent developments such as engineering of quantum states, quantum optics on a chip, nano-mechanical mirrors, quantum entanglement, quantum metrology, spin squeezing, control of decoherence, and many other key topics. Readers are guided through the principles of quantum optics and their uses in a wide variety of areas including quantum information science and quantum mechanics.

The textbook features over 150 end-of-chapter exercises with solutions available for instructors at www.cambridge.org/9781107006409. It is invaluable to both graduate students and researchers in physics and photonics, quantum information science, and quantum communications.

Girish S. Agarwal is Noble Foundation Chair and Regents Professor at Oklahoma State University. A recognized leader in the field of theoretical quantum optics, he is a Fellow of the Royal Society and has won several awards, including the Max-Born Prize from the Optical Society of America and the Humboldt Research Award.

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GIRISH S. AGARWAL

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**Dedicated to the memory of my wife Sneha who had hoped that the book would be
completed and to my daughters Anjali and Mranjali
who have been a constant source of inspiration.**

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Preface

The development of new sources of radiation that produce nonclassical and entangled light has changed the landscape of quantum optics. The production, characterization, and detection of single photons is important not only in understanding fundamental issues but also in the transfer of quantum information. Entangled light and matter sources as well as ones possessing squeezing are used for precision interferometry and for implementing quantum communication protocols. Furthermore, quantum optics is making inroads in a number of interdisciplinary areas, such as quantum information science and nano systems.

These new developments require a book which covers both the basic principles and the many emerging applications. We therefore emphasize fundamental concepts and illustrate many of the ideas with typical applications. We make every possible attempt to indicate the experimental work if an idea has already been tested. Other applications are left as exercises which contain enough guidance so that the reader can easily work them out. Important references are given, although the bibliography is hardly complete. Thus students and postdocs can use the material in the book to do independent research. We have presented the material in a self-contained manner. The book can be used for a two-semester course in quantum optics after the students have covered quantum mechanics and classical electrodynamics at a level taught in the first year of graduate courses. Some advanced topics in the book, such as exact non-Markovian dynamics of open systems, quantum walks, and nano-mechanical mirrors, can be used for seminars in quantum optics.

The material in the book is broadly divided into two parts. The first part deals with many old and emerging aspects of the quantized radiation fields, such as the engineering and characterization of quantum states and the generation of entanglement. The working of an interferometer using one or a few photons is extensively treated. A chapter is devoted to quantum optics with fields carrying orbital angular momentum. Many applications of entangled fields are given. A thorough discussion of quantum noise in amplification and attenuation is also given. The second part deals with the interaction of radiation with matter. Coherent, squeezed, and cat states of atoms are treated. Dissipative processes are treated from a microscopic approach. This part includes a discussion of electromagnetically induced transparency and a host of applications. Special emphasis is placed on quantum interference and entanglement. It is shown how measurement can produce entanglement. Furthermore, the deterministic production of entanglement is discussed. Many relatively newer aspects of cavity QED, such as single photon switches, photon blockade, and anti-Jaynes–Cummings interaction are presented. The book concludes with a look at developments such as quantum optics on a chip, quantum optical effects arising from radiation pressure and mechanical motion, quantum walks, control of decoherence, and disentanglement.

The focus in the book is on emerging areas in quantum optics and therefore important topics like the quantum theory of lasers, micromasers, optical multi-stability, self-induced transparency, etc. have been left out as these are well covered in earlier textbooks like those of Mandel and Wolf (*Optical Coherence and Quantum Optics*) and Scully and Zubairy (*Fundamentals of Quantum Optics*). Other topics like polarization are treated from the new perspective of quantum fluctuations in Stokes parameters. Clearly it is impossible to present in a single volume all that has happened since the discovery of the laser and the pioneering quantum optical works in the early 1960s. We have emphasized aspects that we consider essential for the newer directions in which quantum optics is moving.

This book is the outcome of teaching courses in Quantum Optics at the University of Hyderabad and the Oklahoma State University and extensive lecturing at the International Center for Theoretical Physics, Trieste, and at many scientific schools in India and elsewhere. Research carried out over several decades and my earlier writings, as well as interaction with students, postdocs, and collaborators, has shaped the book. A part of the chapters dealing with states of the radiation field was evolved while I spent half a year in 1992–93 at the Max-Planck Institute for Quantum Optics, Garching, Germany. My collaborator Subhash Chaturvedi contributed by refining some of this material.

The book would never have been completed except for the tireless efforts of my student Sumei Huang who extensively worked on it. I am grateful to her. I also acknowledge considerable assistance from my student Kenan Qu.

I thank my collaborators Jay Banerji, Subhash Chaturvedi, Tarak Dey, Jacques Perk, Gautam Vemuri, and Joachim von Zanthier for reading several chapters and for providing useful input. I thank Ravi Puri, Surya Tewari, Subhasish Dutta Gupta, and a large number of students whose works have been used for the writing of the book. I thank Mustansir Barma for the hospitality at TIFR, Mumbai where I continued to work on my book.

Over the years I have learnt a lot through interactions with a large number of physicists. These interactions have had a deep influence in the writing of the book and I thank especially Bob Boyd, Jinx Cooper, Joe Eberly, Marlan Scully, Herman Haken, Sudhansu Jha, Peter Knight, Emil Wolf, the late Len Mandel, and Herbert Walther.

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