Advanced Topics in Applied Mathematics

This book is ideal for engineering, physical science, and applied mathematics students and professionals who want to enhance their mathematical knowledge. *Advanced Topics in Applied Mathematics* covers four essential applied mathematics topics: Green's functions, integral equations, Fourier transforms, and Laplace transforms. Also included is a useful discussion of topics such as the Wiener-Hopf method, finite Hilbert transforms, Cagniard–De Hoop method, and the proper orthogonal decomposition. This book reflects Sudhakar Nair's long classroom experience and includes numerous examples of differential and integral equations from engineering and physics to illustrate the solution procedures. The text includes exercise sets at the end of each chapter and a solutions manual, which is available for instructors.

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For Engineering and the Physical Sciences

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CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town,
Singapore, São Paulo, Delhi, Tokyo, Mexico City

Cambridge University Press
32 Avenue of the Americas, New York, NY 10013-2473, USA

www.cambridge.org
Information on this title: www.cambridge.org/9781107006201

First published 2011

Printed in the United States of America

A catalog record for this publication is available from the British Library.

Library of Congress Cataloging in Publication data
Nair, Sudhakar, 1944– author.
Advanced Topics in Applied Mathematics: for Engineering and the Physical
Sciences/Sudhakar Nair.
p. cm
Includes index.
ISBN 978-1-107-00620-1 (hardback)
1. Differential equations. 2. Engineering mathematics. 3. Mathematical physics.
I. Title.
TA347.D45N35 2011
620.001′51–dc22 2010052380

ISBN 978-1-107-00620-1 Hardback

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Preface

This text is aimed at graduate students in engineering, physics, and applied mathematics. I have included four essential topics: Green's functions, integral equations, Fourier transforms, and Laplace transforms. As background material for understanding these topics, a course in complex variables with contour integration and analytic continuation and a second course in differential equations are assumed. One may point out that these topics are not all that advanced – the expected advanced-level knowledge of complex variables and a familiarity with the classical partial differential equations of physics may be used as a justification for the term "advanced." Most graduate students in engineering satisfy these prerequisites. Another aspect of this book that makes it "advanced" is the expected maturity of the students to handle the fast pace of the course. The fours topics covered in this book can be used for a one-semester course, as is done at the Illinois Institute of Technology (IIT). As an application-oriented course, I have included techniques with a number of examples at the expense of rigor. Materials for further reading are included to help students further their understanding in special areas of individual interest. With the advent of multiphysics computational software, the study of classical methods is in general on a decline, and this book is an attempt to optimize the time allotted in the curricula for applied mathematics.

I have included a selection of exercises at the end of each chapter for instructors to choose as weekly assignments. A solutions manual for these exercises is available on request. The problems are numbered in such a way as to simplify the assignment process, instead of clustering a number of similar problems under one number.

Classical books on integral transforms by Sneddon and on mathematical methods by Morse and Feshbach and by Courant and Hilbert form the foundation for this book. I have included sections on the Boundary Element Method and Proper Orthogonal Decomposition under integral equations – topics of interest to the current research community. The Cagniard–De Hoop method for inverting combined Fourier-Laplace transforms is well known to researchers in the area of elastic waves, and I feel it deserves exposure to applied mathematicians in general. Discrete Fourier transform leading to the fast Fourier algorithm and the Z-transform are included.

I am grateful to my numerous students who have read my notes and corrected me over the years. My thanks also go to my colleagues, who helped to proofread the manuscript, Kevin Cassel, Dietmar Rempfer, Warren Edelstein, Fred Hickernell, Jeff Duan, and Greg Fasshauer, who have been persistent in instilling applied mathematics to believers and nonbelievers at IIT, and, especially, for training the students who take my course. I am also indebted to my late colleague, Professor L. N. Tao, who shared the applied mathematics teaching with me for more than twenty-five years.

The editorial assistance provided by Peter Gordon and Sara Black is appreciated.

The Mathematica™ package from Wolfram Research was used to generate the number function plots.

My wife, Celeste, has provided constant encouragement throughout the preparation of the manuscript, and I am always thankful to her.