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A Student's Guide to Geophysical Equations

The advent of accessible student computing packages has meant that geophysics students can now easily manipulate datasets and gain first-hand modeling experience – essential in developing an intuitive understanding of the physics of the Earth. Yet to gain a more in-depth understanding of the physical theory, and to be able to develop new models and solutions, it is necessary to be able to derive the relevant equations from first principles.

This compact, handy book fills a gap left by most modern geophysics textbooks, which generally do not have space to derive all of the important formulae, showing the intermediate steps. This guide presents full derivations for the classical equations of gravitation, gravity, tides, Earth rotation, heat, geomagnetism, and foundational seismology, illustrated with simple schematic diagrams. It supports students through the successive steps and explains the logical sequence of a derivation – facilitating self-study and helping students to tackle homework exercises and prepare for exams.

WILLIAM LOWRIE was born in Hawick, Scotland, and attended the University of Edinburgh, where he graduated in 1960 with first-class honors in physics. He achieved a masters degree in geophysics at the University of Toronto and, in 1967, a doctorate at the University of Pittsburgh. After two years in the research laboratory of Gulf Oil Company he became a researcher at the Lamont-Doherty Geological Observatory of Columbia University. In 1974 he was elected professor of geophysics at the ETH Zürich (Swiss Federal Institute of Technology in Zurich), Switzerland, where he taught and researched until retirement in 2004. His research in rock magnetism and paleomagnetism consisted of deducing the Earth's magnetic field in the geological past from the magnetizations of dated rocks. The results were applied to the solution of geologic-tectonic problems, and to analysis of the polarity history of the geomagnetic field. Professor Lowrie has authored 135 scientific articles and a second edition of his acclaimed 1997 textbook *Fundamentals of Geophysics* was published in 2007. He has been President of the European Union of Geosciences (1987–9) and Section President and Council member of the American Geophysical Union (2000–2). He is a Fellow of the American Geophysical Union and a Member of the Academia Europaea.

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Preface

This work was written as a supplementary text to help students understand the mathematical steps in deriving important equations in classical geophysics. It is not intended to be a primary textbook, nor is it intended to be an introduction to modern research in any of the topics it covers. It originated in a set of handouts, a kind of "do-it-yourself" manual, that accompanied a course I taught on theoretical geophysics. The lecture aids were necessary for two reasons. First, my lectures were given in German and there were no comprehensive up-to-date texts in the language; the recommended texts were in English, so the students frequently needed clarification. Secondly, it was often necessary to explain classical theory in more detail than one finds in a multi-topic advanced textbook. To keep such a book as succinct as possible, the intermediate steps in the mathematical derivation of a formula must often be omitted. Sometimes the unassisted student cannot fill in the missing steps without individual tutorial assistance, which is usually in short supply at most universities, especially at large institutions. To help my students in these situations, the "do-it-yourself" text that accompanied my lectures explained missing details in the derivations. This is the background against which I prepared the present guide to geophysical equations, in the hope that it might be helpful to other students at this level of study.

The classes that I taught to senior grades were largely related to potential theory and primarily covered topics other than seismology, since this was the domain of my colleagues and better taught by a true seismologist than by a paleomagnetist! Theoretical seismology is a large topic that merits its own treatment at an advanced level, and there are several textbooks of classical and modern vintage that deal with this. However, a short chapter on the relationship of stress, strain, and the propagation of seismic waves is included here as an introduction to the topic.

Computer technology is an essential ingredient of progress in modern geophysics, but a well-trained aspiring geophysicist must be able to do more than

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apply advanced software packages. A fundamental mathematical understanding is needed in order to formulate a geophysical problem, and numerical computational skills are needed to solve it. The techniques that enabled scientists to understand much about the Earth in the pre-computer era also underlie much of modern methodology. For this reason, a university training in geophysics still requires the student to work through basic theory. This guide is intended as a companion in that process.

Historically, most geophysicists came from the field of physics, for which geophysics was an applied science. They generally had a sound training in mathematics. The modern geophysics student is more likely to have begun studies in an Earth science discipline, the mathematical background might be heavily oriented to the use of tailor-made packaged software, and some students may be less able to handle advanced mathematical topics without help or tutoring. To fill these needs, the opening chapter of this book provides a summary of the mathematical background for topics handled in subsequent chapters.

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