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Gregory Falkovich
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Fluid Mechanics

A Short Course for Physicists

The multi-disciplinary field of fluid mechanics is one of the most actively developing fields of physics, mathematics and engineering. In this book, the fundamental ideas of fluid mechanics are presented from a physics perspective.

Using examples taken from everyday life, from hydraulic jumps in a kitchen sink to Kelvin–Helmholtz instabilities in clouds, the book provides readers with a better understanding of the world around them. It teaches the art of making fluid-mechanical estimates and shows how the ideas and methods developed to study the mechanics of fluids are used to analyze other systems with many degrees of freedom in statistical physics and field theory.

Aimed at undergraduate and graduate students, the book assumes no prior knowledge of the subject and only a basic understanding of vector calculus and analysis. It contains 32 exercises of varying difficulty, from simple estimates to elaborate calculations, with detailed solutions to help readers understand fluid mechanics. A link to a website accompanying the book, hosted by the author, can be found at www.cambridge.org/9781107005754.

GREGORY FALKOVICH is a Professor in the Department of Physics of Complex Systems, Weizmann Institute of Science, Israel. He has researched in plasma, condensed matter, fluid mechanics, statistical and mathematical physics, and cloud physics and meteorology, and has won several awards for his work.

Fluid Mechanics

A Short Course for Physicists

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Preface

Why study fluid mechanics? The primary reason is not even technical, it is cultural: a physicist is defined as one who looks around and understands at least part of the material world. One of the goals of this book is to let you understand how the wind blows and the water flows so that flying or swimming you may appreciate what is actually going on. The secondary reason is to do with applications: whether you are to engage with astrophysics or biophysics theory or build an apparatus for condensed matter research, you need the ability to make correct fluid-mechanics estimates; some of the art of doing this will be taught in the book. Yet another reason is conceptual: mechanics is the basis of the whole of physics in terms of intuition and mathematical methods. Concepts introduced in the mechanics of particles were subsequently applied to optics, electromagnetism, quantum mechanics, etc.; here you will see the ideas and methods developed for the mechanics of fluids, which are used to analyze other systems with many degrees of freedom in statistical physics and quantum field theory. And last but not least: at present, fluid mechanics is one of the most actively developing fields of physics, mathematics and engineering, so you may wish to participate in this exciting development.

Even for physicists who are not using fluid mechanics in their work, taking a one-semester course on the subject would be well worth the effort. This is one such course. It presumes no prior acquaintance with the subject and requires only basic knowledge of vector calculus and analysis. On the other hand, applied mathematicians and engineers working on fluid mechanics may find in this book several new insights presented from a physicist's perspective. In choosing from the enormous wealth of material produced by the last four centuries of ever-accelerating research, preference was given to the ideas and concepts that teach lessons whose importance transcends the confines of one specific subject as they prove useful time and again across the whole spectrum of modern physics. To much delight, it turned out to be possible to weave the subjects into a single coherent narrative so that the book is a novel rather than a collection of short stories.

We approach every subject as physicists: start from qualitative considerations (dimensional reasoning, symmetries and conservation laws), then use back-of-the-envelope estimates and crown it with concise yet consistent derivations. Fluid mechanics is an essentially experimental science, as is any branch of physics. Experimental data guide us at each step, which is often far from trivial: for example, energy is not conserved even in the frictionless limit and other symmetries can be unexpectedly broken, which makes a profound impact on estimates and derivations.

Lecturers and students using the book for a course will find out that its 12 sections comfortably fit into 12 lectures plus, if needed, problem-solving sessions. Sections 2.3 and 3.1 each contain one extra subsection that can be treated in a problem-solving session (specifically, Sections 2.3.5 and 3.1.2, but the choice may be different). For second-year students, one can use a shorter version, excluding Sections 3.2–3.4 and the two small-font parts in Sections 2.2.1 and 2.3.4. The lectures are supposed to be self-contained so that no references are included in the text. The epilogue and endnotes provide guidance for further reading; the references are collected in the reference list at the end. Those using the book for self-study will find out that in about two intense weeks one is able to master the basic elements of fluid physics. Those reading for amusement can disregard the endnotes, skip all the derivations and half of the resulting formulae and still be able to learn a lot about fluids and a bit about the world around us, helped by numerous pictures.

In many years of teaching this course at the Weizmann Institute, I have benefitted from the generations of brilliant students who taught me never to stop looking for simpler explanations and deeper links between branches of physics. I also learnt from V. Arnold, E. Balkovsky, E. Bodenschatz, G. Boffetta, A. Celani, M. Chertkov, B. Chirikov, G. Eyink, U. Frisch, K. Gawedzki, V. Geshkenbein, L. Kadanoff, K. Khanin, D. Khmelnitskii, I. Kolokolov, G. Kotkin, R. Kraichnan, E. Kuznetsov, A. Larkin, V. Lebedev, B. Lugovtsov, S. Lukaschuk, V. L'vov, K. Moffatt, A. Newell, A. Polyakov, I. Procaccia, A. Pumir, A. Rubenchik, D. Ryutov, V. Serbo, A. Shafarenko, M. Shats, B. Shraiman, E. Siggia, Ya. Sinai, M. Spektor, K. Sreenivasan, V. Steinberg, K. Turitsyn, S. Turitsyn, G. Vekshtein, M. Vergassola, P. Wiegmann, V. Zakharov, A. Zamolodchikov, Ya. Zeldovich. Special thanks to Itzhak Fouxon and Marija Vucelja, who were instructors in problem-solving sessions and wrote draft solutions for some of the exercises. Errors, both of omission and of commission, are my responsibility alone. This book is dedicated to my family.

Prologue

> The water's language was a wondrous one, some narrative
> on a recurrent subject ...
> *A. Tarkovsky, translated by A. Shafarenko*

There are two protagonists in this story: inertia and friction. One meets them first in the mechanics of particles and solids where their interplay is not very complicated: inertia tries to keep the motion while friction tries to stop it. Going from a finite to an infinite number of degrees of freedom is always a game-changer. We will see in this book how an infinitesimal viscous friction makes fluid motion infinitely more complicated than inertia alone ever could. Without friction, most incompressible flows would stay potential, i.e. essentially trivial. At solid surfaces, friction produces vorticity, which is carried away by inertia and changes the flow in the bulk. Instabilities then bring about turbulence, and statistics emerges from dynamics. Vorticity penetrating the bulk makes life interesting in ideal fluids though in a way different from superfluids and superconductors.

On the other hand, compressibility makes even potential flows non-trivial as it allows inertia to develop a finite-time singularity (shock), which friction manages to stop. It is only in a wave motion that inertia is able to have an interesting life in the absence of friction, when it is instead partnered with medium anisotropy or inhomogeneity, which cause the dispersion of waves. The soliton is a happy child of that partnership. Yet even there, a modulational instability can bring a finite-time singularity in the form of self-focusing or collapse. At the end, I discuss how inertia, friction and dispersion may act together.

On a formal level, inertia of a continuous medium is described by a non-linear term in the equation of motion. Friction and dispersion are described by linear terms, which, however, have the highest spatial derivatives so that the limit of zero friction and zero dispersion is singular. Friction is not only singular but also a symmetry-breaking perturbation, which leads to an anomaly when the effect of symmetry breaking remains finite even in the limit of vanishing viscosity.

The first chapter introduces basic notions and describes stationary flows, inviscid and viscous. Time starts to run in the second chapter, which discusses instabilities, turbulence and sound. The third chapter is devoted to dispersive waves, it progresses from linear to non-linear waves, solitons, collapses and wave turbulence. The epilogue gives a guide to further reading and briefly describes present-day activities in fluid mechanics. Detailed solutions of the exercises are given.