Dynamics: Theory and Application of Kane's Method

This book is ideal for teaching students in engineering or physics the skills necessary to analyze motions of complex mechanical systems such as spacecraft, robotic manipulators, and articulated scientific instruments. Kane's method, which emerged recently, reduces the labor needed to derive equations of motion and leads to equations that are simpler and more readily solved by computer, in comparison to earlier, classical approaches. Moreover, the method is highly systematic and thus easy to teach. This book is a revision of *Dynamics: Theory and Applications* by T. R. Kane and D. A. Levinson and presents the method for forming equations of motion by constructing generalized active forces and generalized inertia forces. Important additional topics include approaches for dealing with finite rotation, an updated treatment of constraint forces and constraint torques, an extension of Kane's method to deal with a broader class of nonholonomic constraint equations, and other recent advances.

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This work contains material taken from *Dynamics: Theory and Applications* by Thomas R. Kane and David A. Levinson. It also contains material drawn from *Spacecraft Dynamics* by Thomas R. Kane, Peter W. Likins, and David A. Levinson. These gentlemen have generously allowed Carlos M. Roithmayr and Dewey H. Hodges, the authors of this derivative work, to sample freely from the rich content of their two books. Original material and minor revisions contributed by the authors are distributed throughout the work.

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PREFACE

The authors of the earlier version of this book succeeded in accomplishing the goals stated in their preface. Since it was written, *Dynamics: Theory and Applications* has served as a textbook for teaching graduate students a method of formulating dynamical equations of motion for mechanical systems. The method has proved especially useful for dealing with the complex multibody mechanical systems that in the twentieth and twenty-first centuries have challenged engineers in industry, government, and universities: the *Galileo* spacecraft sent to Jupiter, the International Space Station, and the robotic manipulator arms aiding astronauts on the Space Shuttle and International Space Station are but a few examples. Kane's method is systematic and easily taught, in a way that enables the student to be conversant with colleagues trained to apply traditional approaches found in the classical literature.

Although the fundamental aspects of the method have not changed during the past three decades, advances and refinements have been made in a number of areas. In certain cases the newer developments facilitate exposition of the topic at hand and lend themselves well to integration with material in the original textbook. The primary purpose of this text, then, is to make the benefits of this progress available for current courses in dynamics.

The preface to the earlier version (which immediately follows this Preface) includes a discussion of the organization of the original book and supporting rationale. Here, we give an overview of the modest alterations made to the earlier structure.

The initial chapter now begins with three brief sections that put the student into position to give a mathematical description of the orientation of a rigid body with respect to a reference frame, when the rigid body has been subjected to successive rotations. Inclusion of these sections provides a formal presentation of topics that typically were covered in classroom discussion. The final section of the first chapter is concerned with differentiation of a scalar function of vectors, which subsequently comes into play in Chapter 6. The original second chapter is divided in two; Chapter 2 deals solely with kinematics, and Chapter 3 is devoted to constraints. The separation focuses attention on the subject of constraints, where there are important distinctions to be made between Kane's method and the classical approaches. The treatment of motion constraints has been broadened. Satisfaction of a constraint entails application of certain forces and torques that are the center of attention in Chapter 6. The practice of expressing constraint equations in terms of vectors, as illustrated in Chapter 3, makes it possible to identify, by inspection, the direction of each constraint force and the point at which it must be applied, as well as the direction of the torque of each constraint force couple, together with the body on which the couple acts. Constraint forces and constraint torques can be identified in this manner if they are of interest in a particular analysis. If, however, they are immaterial, they need not enter the picture at all; indeed, this is a central feature of Kane's method. Thus, Chapter 6 concludes with a discussion of noncontributing forces in two sections that have been relocated from Chapter 5. Extraction of information from equations of motion, formerly covered in the final chapter, is now taken up in Chapter 9. The checking function, introduced in an additional section, can be constructed even when an energy integral does not exist, and is used for the same purpose; namely, to test the results of numerical integrations of equations of motion. The section dealing with momentum integrals has been revised to demonstrate that they can be regarded as nonholonomic constraint equations. Finally, the orientation of a rigid body in a reference frame, addressed at a basic level at the beginning of Chapter 1, receives advanced treatment in Chapter 10. With the exception of two sections dealing with Wiener-Milenković parameters, the material in this chapter is drawn largely from the book *Spacecraft Dynamics*. An understanding of this chapter is especially helpful to the dynamicist who is tackling a problem involving a rigid body (for example, an aircraft or a spacecraft) that is not mechanically attached to the reference frame in question. Nevertheless, the preceding chapters can be mastered without referring to the last one.

A small number of problems have been revised to be consistent with the revisions made to the text. Likewise, problems have been created to cover the newly added material. The significance of a star in connection with a problem, and the importance of solving all unstarred problems, remains unchanged. As before, and for the same pedagogical reasons, results are supplied for all problems.

It is our sincere hope that this updated book will serve as the basis for continued graduate instruction in dynamics so that Kane's method can be applied to the challenging problems that face us now and in the future. We are indebted to the authors of the earlier version for instructing us in their classrooms, and for their generosity in allowing us to make use of their material here.

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PREFACE TO *DYNAMICS: THEORY AND APPLICATIONS*

Dissatisfaction with available graduate-level textbooks on the subject of dynamics has been widespread throughout the engineering and physics communities for some years among teachers, students, and employers of university graduates; furthermore, this dissatisfaction is growing at the present time. A major reason for this is that engineering graduates entering industry with advanced degrees, when asked to solve dynamics problems arising in fields such as multibody spacecraft attitude control, robotics, and design of complex mechanical devices, find that their education in dynamics, based on the textbooks currently in print, has not equipped them adequately to perform the tasks confronting them. Similarly, physics graduates often discover that, in their education, so much emphasis was placed on preparation for the study of quantum mechanics, and the subject of rigid body dynamics was slighted to such an extent, that they are handicapped, both in industry and in academic research, by their inability to design certain types of experimental equipment, such as a particle detector that is to be mounted on a planetary satellite. In this connection, the ability to analyze the effects of detector scanning motions on the attitude motion of the satellite is just as important as knowledge of the physics of the detection process itself. Moreover, the graduates in question often are totally unaware of the deficiencies in their dynamics education. How did this state of affairs come into being, and is there a remedy?

For the most part, traditional dynamics texts deal with the exposition of eighteenth-century methods and their application to physically simple systems, such as the spinning top with a fixed point, the double pendulum, and so forth. The reason for this is that, prior to the advent of computers, one was justified in demanding no more of students than the ability to formulate equations of motion for such simple systems, for one could not hope to extract useful information from the equations governing the motions of more complex systems. Indeed, considerable ingenuity and a rather extensive knowledge of mathematics were required to analyze even simple systems. Not surprisingly, therefore, ever more attention came to be focused on analytical intricacies of the *mathematics* of dynamics, while the process of formulating equations of motion came to be regarded as a rather routine matter. Now that computers enable one to extract highly valuable information from large sets of complicated equations of motion, all this has changed. In fact, the inability to *formulate* equations of motion effectively can be as great a hindrance at present as the inability to *solve* equations was formerly. It follows that the subject of formulation of equations of motion demands careful reconsideration. Or, to say it another way, a major goal of a modern dynamics course must be to produce students who are proficient in the use of the best available methodology for formulating equations of motion. How can this goal be attained?

In the 1970s, when extensive dynamical studies of multibody spacecraft, robotic devices, and complex scientific equipment were first undertaken, it became apparent that straightforward use of classical methods, such as those of Newton, Lagrange, and Hamilton, could entail the expenditure of very large, and at times even prohibitive, amounts of analysts' labor, and could lead to equations of motion so unwieldy as to render computer solutions unacceptably slow for technical and/or economic reasons. Now, while it may be impossible to overcome this difficulty entirely, which is to say that it is unlikely that a way will be found to reduce formulating equations of motion for complex systems to a truly simple task, there does exist a method that is superior to the classical ones in that its use leads to major savings in labor, as well as to simpler equations. Moreover, being highly systematic, this method is easy to teach. Focusing attention on motions, rather than on configurations, it affords the analyst maximum physical insight. Not involving variations, such as those encountered in connection with virtual work, it can be presented at a relatively elementary mathematical level. Furthermore, it enables one to deal directly with nonholonomic systems without having to introduce and subsequently eliminate Lagrange multipliers. It follows that the resolution of the dilemma before us is to instruct students in the use of this method (which is often referred to as Kane's method). This book is intended as the basis for such instruction.

Textbooks can differ from each other not only in content but also in organization, and the sequence in which topics are presented can have a significant effect on the relative ease of teaching and learning the subject. The rationale underlying the organization of the present book is the following. We view dynamics as a deductive discipline, knowledge of which enables one to describe in quantitative and qualitative terms how mechanical systems move when acted upon by given forces, or to determine what forces must be applied to a system in order to cause it to move in a specified manner. The solution of a dynamics problem is carried out in two major steps, the first being the formulation of equations of motion, and the second the extraction of information from these equations. Since the second step cannot be taken fruitfully until the first has been completed, it is imperative that the distinction between the two be kept clearly in mind. In this book, the extraction of information from equations of motion is deferred formally to the last chapter, while the preceding chapters deal with the material one needs to master in order to be able to arrive at valid equations of motion.

Diverse concepts come into play in the process of constructing equations of motion. Here again it is important to separate ideas from each other distinctly. Major attention must be devoted to kinematics, mass distribution considerations, and force concepts. Accordingly, we treat each of these topics in its own right. First, however, since differentiation of vectors plays a key role in dynamics, we devote the initial chapter of the book to this topic. Here we stress the fact that differentiation of a vector with respect to a scalar variable requires specification of a reference frame, in which connection we dispense with the use of limits because such use tends to confuse rather than clarify matters; but we draw directly on students' knowledge of scalar calculus. Thereafter, we devote one chapter each to the topics of kinematics, mass distribution, and generalized forces, before discussing energy functions, in Chapter 5, and the formulation of equations of motion, in Chapter 6. Finally, the extraction of information from equations of motion is considered in Chapter 7. The material in these seven chapters has formed the basis for a one-year course for first-year graduate students at Stanford University for more than 20 years.

Dynamics is a discipline that cannot be mastered without extensive practice. Accordingly, the book contains 14 sets of problems intended to be solved by users of the book. To learn the material presented in the text, the reader should solve *all* of the *unstarred* problems, each of which covers some material not covered by any other. In their totality, the unstarred problems provide complete coverage of the theory set forth in the book. By solving also the starred problems, which are not necessarily more difficult than the unstarred ones, one can gain additional insights. Results are given for *all* problems, so that the correcting of problem solutions needs to be undertaken only when a student is unable to reach a given result. It is important, however, that both students and instructors expend whatever effort is required to make certain that students know what the *point* of each problem is, not only how to solve it. Classroom discussion of selected problems is most helpful in this regard.

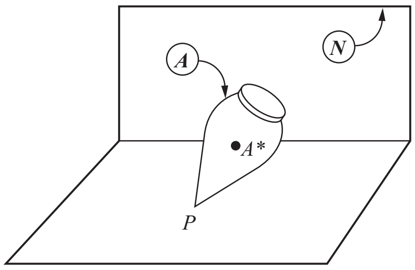

Figure i

Finally, a few words about notation will be helpful. Suppose that one is dealing with a simple system, such as the top A, shown in Fig. *i*, the top terminating in a point P that is fixed in a Newtonian reference frame N. The notation needed here certainly can be simple. For instance, one can let $\boldsymbol{\omega}$ denote the angular velocity of A in N, and let $\mathbf{v}$ stand for the velocity in N of point $A^\star$, the mass center of A. Indeed, notations more elaborate than these can be regarded as objectionable because they burden the analyst with unnecessary writing. But suppose that one must undertake the analysis of motions of a complex system, such as the Galileo spacecraft, modeled as consisting of eight rigid bodies $A, B, \ldots, H$, coupled to each other as indicated in Fig. *ii*. Here, unless one employs notations more elaborate than $\boldsymbol{\omega}$ and $\mathbf{v}$, one cannot distinguish from each other such quantities as, say, the angular velocity of A in a Newtonian reference frame N, the angular velocity of B in N, and the angular velocity of B in A, all of which may enter the analysis. Or, if $A^\star$ and $B^\star$ are points of interest fixed on A and B, perhaps the respective mass centers, one needs a notation that permits one to distinguish from each other, say, the velocity of $A^\star$ in N, the velocity of $B^\star$ in N, and the velocity of $B^\star$ in A. Therefore, we establish, and use consistently throughout this book, a few notational practices that work well in such situations. In particular, when a vector denoting an angular velocity or an angular acceleration of a rigid body in a certain reference frame has two superscripts, the right superscript stands for the rigid body, whereas the left superscript refers to the reference frame. Incidentally, we use the terms "reference frame" and "rigid body" interchangeably. That is, every rigid body can serve as a reference frame, and every reference frame can be regarded as a massless rigid body. Thus, for example, the three angular velocities mentioned in connection with the system depicted in Fig. *ii*, namely, the angular velocity of A in N, the angular velocity of B in N, and the angular velocity of B in A, are denoted by ${}^N\boldsymbol{\omega}^A$, ${}^N\boldsymbol{\omega}^B$, and ${}^A\boldsymbol{\omega}^B$, respectively. Similarly, the right superscript on a vector denoting a velocity or acceleration of a point in a reference frame is the name of the point, whereas the left superscript identifies the reference frame. Thus, for example, the aforementioned velocity of $A^\star$ in N is written ${}^N\mathbf{v}^{A^\star}$, and ${}^A\mathbf{v}^{B^\star}$ represents the velocity of $B^\star$ in A. Similar conventions are established in connection with angular momenta, kinetic energies, and so forth.

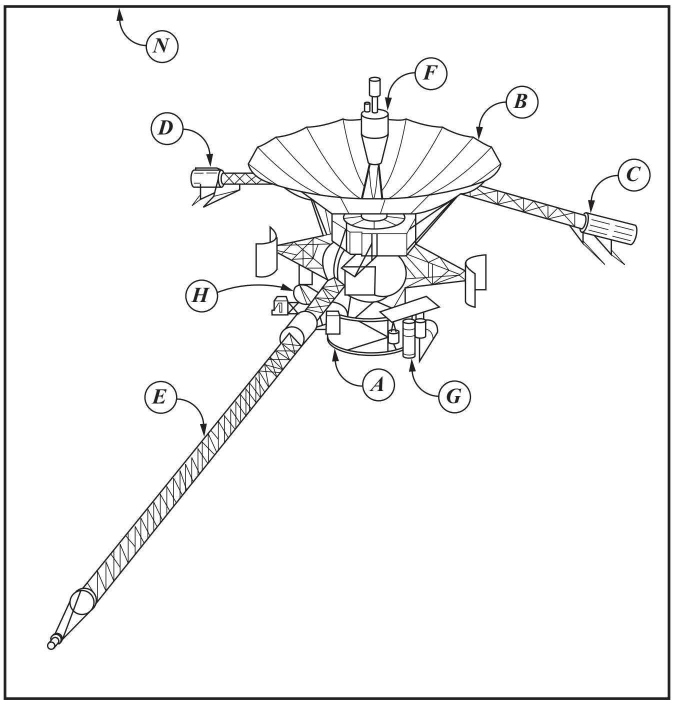

Figure ii

While there are distinct differences between our approach to dynamics, on the one hand, and traditional approaches, on the other hand, there is no fundamental conflict between the new and the old. On the contrary, the material in this book is entirely compatible with the classical literature. Thus, it is the purpose of this book not only to equip students with the skills they need to deal effectively with present-day dynamics problems, but also to bring them into position to interact smoothly with those trained more conventionally.

Thomas R. Kane
David A. Levinson

TO THE READER

Each of the ten chapters of this book is divided into sections. A section is identified by two numbers separated by a decimal point, the first number referring to the chapter in which the section appears, and the second identifying the section within the chapter. Thus, the identifier 3.6 refers to the sixth section of the third chapter. A section identifier appears at the *top of each page*.

Equations are numbered serially within sections. For example, the equations in Secs. 3.6 and 3.9 are numbered (1)–(31) and (1)–(50), respectively. References to an equation may be made both within the section in which the equation appears and in other sections. In the first case, the equation number is cited as a single number; in the second case, the section number is included as part of a three-number designation. Thus, within Sec. 3.6, Eq. (2) of Sec. 3.6 is referred to as Eq. (2); in Sec. 3.9, the same equation is referred to as Eq. (3.6.2). To locate an equation cited in this manner, one may make use of the section identifiers appearing at the tops of pages.

Figures appearing in the chapters are numbered so as to identify the sections in which the figures appear. For example, the two figures in Sec. 5.7 are designated Fig. 5.7.1 and Fig. 5.7.2. To avoid confusing these figures with those in the problem sets and in Appendix III, the figure number is preceded by the letter P in the case of problem set figures, and by the letter A in the case of Appendix III figures. The double number following the letter P refers to the problem statement in which the figure is introduced. For example, Fig. P13.3 is introduced in Problem 13.3. Similarly, Table 4.4.1 is the designation for a table in Sec. 4.4, and Table P13.19(*b*) is associated with Problem 13.19.