ESSENTIALS OF HAMILTONIAN DYNAMICS

Classical dynamics is one of the cornerstones of advanced education in physics and applied mathematics, with applications across engineering, chemistry, and biology.

In this book, the author uses a concise and pedagogical style to cover all the topics necessary for a graduate-level course in dynamics based on Hamiltonian methods. Readers are introduced to the impressive advances in the field during the second half of the twentieth-century, including KAM theory and deterministic chaos. Essential to these developments are some exciting ideas from modern mathematics, which are introduced carefully and selectively. Core concepts and techniques are discussed, together with numerous concrete examples to illustrate key principles. A special feature of the book is the use of computer software to investigate complex dynamical systems, both analytically and numerically.

This text is ideal for graduate students and advanced undergraduates who are already familiar with the Newtonian and Lagrangian treatments of classical mechanics. The book is well suited to a one-semester course, but is easily adapted to a more concentrated format of one-quarter or a trimester. A solutions manual and introduction to Mathematica® are available online at www.cambridge.org/Lowenstein.

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To Marcia

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Preface

This is a textbook on classical Hamiltonian dynamics designed primarily for students commencing graduate studies in physics. The aim is to cover all essential topics in a relatively concise format, without sacrificing the intellectual coherence of the subject, or the conceptual precision which is the *sine qua non* of advanced education in physics.

Encouraged by my colleagues at New York University, I have taken it as a pedagogical challenge to create a textbook suitable for a twenty-first-century course of duration no more than one semester (at NYU, the material is covered in about two-thirds of a semester). To do so, I have chosen to limit the scope of the book in certain important ways. It is assumed that the student has already had a course in which Newtonian mechanics, in both $\mathbf{F} = m\mathbf{a}$ and Lagrangian versions, has been systematically developed and applied to a standard array of soluble examples: the harmonic oscillator, the simple pendulum, the Kepler problem, small oscillations (normal modes), and rigid-body motion. In the present book, the Hamiltonian formulation in phase space is introduced at the outset and applied directly to the same familiar systems.

Topics usually found in more encyclopedic textbooks, but omitted from the present treatment, include dissipative systems, nonholonomic constraints, special and general theories of relativity, continuum mechanics, and classical field theory. A further choice I have made is to limit the use of advanced differential geometry. Although I consider the concept of a differential manifold to be absolutely essential for a clear understanding of such concepts as configuration space, phase space, degrees of freedom, and generalized coordinates, I have found that the use of differential forms is neither necessary nor desirable for this book.

One of the advantages which a twenty-first-century student enjoys in comparison with students of earlier generations is a ready access to computers and to software designed to assist in what would otherwise be long, tedious exercises in algebra and analysis. In writing this book, I have assumed that the reader has already acquired some skill and familiarity with elementary scientific programming, and is willing and able to apply such capabilities in solving some instructive numerical exercises in dynamics. I have found *Mathematica*® (a product of Wolfram Research, Inc.) to be well suited to the small-scale calculations which I have used to provide concrete examples, with illustrative graphs and figures, throughout the book. Obviously the choice of software for the numerical exercises is not unique, and different instructors will have their own preferences as to how to implement the numerical component of the course. For those who are interested, I have included in the appendix some samples of the *Mathematica* programming which I have used in the text.

Let us now briefly summarize the content of the book. The opening chapter contains a rapid review of classical mechanics, from Newton's laws through the Lagrangian and Hamiltonian formulations, with a number of instructive examples. This is followed, in Chapter 2, by an introduction to the core concepts of the Hamiltonian formalism, including phase-space geometry, Hamilton's equations of motion, Poisson brackets, and canonical transformations. Here we emphasize an algebraic approach that parallels in certain ways the canonical commutation algebra of quantum mechanics.

In Chapter 3 we turn to a systematic treatment of an extremely important class of dynamical systems, namely those which are integrable, in the sense that there is a complete set of conserved quantities. In the simplest of the integrable systems discussed in Chapter 3, the method of separation of variables leads directly to a reformulation in terms of special coordinates (the action-angle variables) for which each degree of freedom is described as uniform motion on a circle. More generally, the powerful Liouville–Arnol'd theorem provides not only important topological information about the phase space, but also a constructive method for finding an appropriate set of action-angle variables in models that are not separable in their original system of coordinates.

While integrable systems are very rare among Hamiltonian systems (rigorously, the probability of finding one in a random search of function space is zero), it is often the case that a realistic system can be well modeled in a portion of its phase space by an integrable one, with small corrections that can be estimated within the Hamiltonian formalism. This is the realm of canonical perturbation theory, the topic of Chapter 4.

In canonical perturbation theory, one constructs a formal solution of the equations of motion, correct to finite order in a small perturbation parameter. Although in each order the model is in some sense integrable, the perturbative solutions of the equations of motion do not always converge to exact solutions in the limit of infinite order. Nonetheless, thanks to the remarkable theorem of Kolmogorov, Arnol'd, and Moser (KAM), we now know how to set up the perturbation series, namely

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as a finely tuned renormalization process, so that, for a sufficiently small perturbation parameter, most initial conditions do indeed lead to a definite limit. For the remaining initial conditions, convergence may break down and seemingly random ("chaotic") orbits are possible. The KAM theory and the fascinating interplay between order and chaos in Hamiltonian dynamical systems will be explored in Chapter 5. Examples will be drawn from one of the most important current research areas in Hamiltonian dynamics, namely the motions, both stable and chaotic, of the planets and smaller bodies in the Solar System.

In the final chapter of the book, the full power of the concepts and methods developed in the preceding chapters is brought to bear on a particularly fascinating dynamical system, the elastic three-dimensional pendulum. This system, known as a swing-spring, provides an excellent model for certain excitations of the carbon dioxide molecule, a quantum-mechanical system for which experimentally verifiable information can be gleaned from quantization of the classical model. What makes the swing-spring particularly interesting in this regard is the presence of nontrivial monodromy, which complicates the task of classifying the quantized energy levels, but also is associated with an observable phenomenon, namely a kind of intermittent pendulum-like swinging. We will explore some of these features in some detail in Chapter 6.

John H. Lowenstein