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Edited by Michael Heller and W. Hugh Woodin  
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## Infinity

### *New Research Frontiers*

This interdisciplinary study of infinity explores the concept through the prism of mathematics and then offers more expansive investigations in areas beyond mathematical boundaries to reflect the broader, deeper implications of infinity for human intellectual thought. More than a dozen world-renowned researchers in the fields of mathematics, physics, cosmology, philosophy, and theology offer a rich intellectual exchange among various current viewpoints, rather than a static picture of accepted views on infinity.

The book starts with a historical examination of the transformation of infinity from a philosophical and theological study to one dominated by mathematics. It then offers technical discussions on the understanding of mathematical infinity. Following this, the book considers the perspectives of physics and cosmology: Can infinity be found in the real universe? Finally, the book returns to questions of philosophical and theological aspects of infinity.

Rev. Dr. Michael Heller is the Founder and Director of the Copernicus Center for Interdisciplinary Studies, Cracow. He is a Professor of Philosophy at the Pontifical University of John Paul II, Cracow, Poland; an Adjunct Member of the Vatican Observatory; and Ordinary Member of the Pontifical Academy of Sciences. Heller is the author of more than thirty books and nearly two hundred scientific papers.

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Edited by

**Michael Heller**

*Pontifical University of John Paul II*

**W. Hugh Woodin**

*University of California, Berkeley*



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The infinite! No other question has ever moved so profoundly the spirit of man; no other idea has so fruitfully stimulated his intellect; yet no other concept stands in greater need of clarification than that of the infinite. . . .

David Hilbert (1862–1943)

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## Preface

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*Infinity: New Research Frontiers* was developed to explore new research domains involving concepts of infinity in a technically rigorous, as well as interdisciplinary, context. It is the culmination of a creative research initiative that began with the international conference “New Frontiers in Research on Infinity,” held August 18 to 20, 2006, in San Marino. The conference was co-organized by the Center for Theology and the Natural Sciences (CTNS), Berkeley, California,<sup>1</sup> and the John Templeton Foundation (JTF), Philadelphia, Pennsylvania,<sup>2</sup> and it was funded by a generous grant to CTNS from JTF with assistance from Euresis, Milan,<sup>3</sup> and the Republic of San Marino.<sup>4</sup> The invitation-only conference, whose theme was the concept and meaning of infinity in the multifaceted contexts of mathematics, physics, cosmology, philosophy, and theology, laid the intellectual groundwork for this volume.

Esteemed researchers in these diverse fields were invited to contribute to this book with the purpose of pursuing one of the “biggest questions” facing humankind: the notion of infinity. As the great German mathematician David Hilbert stated, as quoted in the epigraph at the beginning of this volume, “No other question has ever moved so profoundly the spirit of man.” Infinity has usually been regarded as a “limiting concept,” that is, a concept to which we arrive by extrapolating from what we know and what is limited and finite. However, some thinkers, following the example of Descartes, claim that infinity is a “primordial concept” and that all other concepts are derived from it. Thus, in this book, we ask the basic question: Is our world of everyday experience embedded in something transcending it?

Within, we offer our intellectual adventures with the concept of infinity, showing its multiform, interdisciplinary nature. The perspective adopted in the book is to show the tension between different viewpoints and opinions, presenting an academic “dispute in action” rather than a static panorama of accepted views. Previously residing in the realm of philosophical and theological speculations, considerations about infinity are

<sup>1</sup> “Promoting the creative mutual interaction between contemporary *theology and the natural sciences*”: <http://www.ctns.org/>.

<sup>2</sup> “Supporting science, investing in the big questions”: <http://www.templeton.org/>.

<sup>3</sup> The Association for the Promotion of the Development of Culture and Scientific Work: <http://www.euresis.org/>.

<sup>4</sup> See <http://www.ctns.org/infinity/> for information about the symposium on which this book is based.

currently dominated by mathematics, and it is mathematics that influences our thinking about infinity in other domains.

Thus, first viewed through the prism of mathematics, infinity is then investigated more expansively in areas beyond mathematical boundaries to reflect the broader, deeper implications of the concept for human intellectual thought and explorations of our lives in the vastness of the universe: infinity in mathematics, in physics and cosmology, and in philosophy and theology.

Following the comprehensive introduction, which takes us through some of the aspects of infinity from the vantage points of various human pursuits, Part I presents a chapter showing the historical process of the expansion of investigations of infinity into other areas of human endeavor – infinity as a transformative concept in science and theology. Mathematics is powerfully represented in the book in a series of five mathematical chapters (divided into two parts) by prominent mathematicians, who present modern views of infinity. Part II contains two somewhat less technical discussions, and Part III contains three advanced studies contributing to our understanding of mathematical infinity. These include how infinity fits into current research in the foundations of mathematics and range from a prediction of imminent collapse to a prediction of complete vindication in the conception of the infinite.

Can infinity be found in the real universe? The question is far from trivial, and it is considered by prominent scientists in physics and cosmology in the four chapters presented in Part IV. Given the profundity of the subject matter, the editors devoted the remaining four chapters to exploring the inevitable historical, philosophical, and theological aspects of the vast notion of infinity in Part V. These reflections are based on Western metaphysics and the Judeo-Christian tradition and have no agenda other than to explore developments in these areas of human concern.

The integration of the somewhat disparate disciplines represented in this volume has resulted in an exploration of the concept and meaning of infinity that would not otherwise have been possible. Given the recent surge in activity among researchers trying to expand the mathematical and conceptual foundations of our understanding of the universe, we believe this book will appeal to a broad, educated readership. In addition, a volume of this sort makes it possible to present in one place a rather high level of technical information while rounding out the discussion of a profound conceptual and foundational question in a multidisciplinary format that would be inappropriate in a technical mathematics research volume. Thus, we hope this book will be of benefit to those without advanced mathematics and scientific backgrounds who are deeply interested in a topic that is typically explored only by specialists and who may find new inspiration in contemplating a boundless universe within these pages, which provide fascinating reading for all those who are not afraid of intellectual challenge.

\*\*\*

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The editors wish to thank the co-organizers of the 2006 conference, “New Frontiers in Research on Infinity,” on which this volume is based: the Center for Theology and the Natural Sciences (CTNS), Berkeley, California, and the John Templeton Foundation (JTF), Philadelphia, Pennsylvania. In addition, we wish to thank JTF, Euresis, and the Republic of San Marino for co-sponsoring the conference.

While much of the conceptual structure of, and resources for, the conference and this book came initially from JTF senior staff and consultants, Robert John Russell, Director of CTNS, and his staff worked closely with JTF to develop and host the conference. Dr. Russell is also a contributor to this volume and, in addition, he worked with the co-editors during the writing and review process.

We also wish to thank Charles L. Harper, Jr., currently Chancellor for International Distance Learning and Senior Vice President, Global Programs, of the American University System, as well as President of Vision-Five.com Consulting, who served as one of the original project developers and the conference convener in his former role as Senior Vice President and Chief Strategist of JTF, working with Robert Russell of CTNS.

Hyung S. Choi, currently Director of Mathematical and Physical Sciences at JTF, assumed an integral role in developing the academic program for the symposium in conjunction with Charles Harper and Robert Russell.

Pamela M. Contractor, President and Director of Ellipsis Enterprises, Inc., working in conjunction with JTF and the volume editors, served as developmental editor of this book, along with Matthew P. Bond, Assistant Editor and Manager of Client Services at Ellipsis.

Finally, the editors acknowledge Lauren Cowles, Senior Editor, Mathematics and Computer Science at Cambridge University Press–New York, for supporting and overseeing this book project.