Introduction

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A stimulating factor in discussions of infinity is that the concept arises in many different contexts: mathematics, physics, metaphysics, theology, psychology, and even the arts. The founder of modern set theory, Georg Cantor, was well aware of these distinctions, and he collapses them into three domains.

> The actual infinite arises in three contexts: *first* when it is realized in the most complete form, in a fully independent other-worldly being, *in Deo*, where I call it the Absolute Infinite or simply Absolute; *second* when it occurs in the contingent, created world; *third* when the mind grasps it *in abstracto* as a mathematical magnitude, number, or order type. I wish to make a sharp contrast between the Absolute and what I call the Transfinite, that is, the actual infinities of the last two sorts, which are clearly limited, subject to further increase, and thus related to the finite.[1]

Mathematical infinities occur as, for instance, the number of points on a continuous line, the size of the endless natural number sequence 1, 2, 3, . . . , or the class of all sets.

In physics, we encounter infinities when we wonder if there might be infinitely many stars, if the universe might last forever, or if matter might be infinitely divisible.

In metaphysical discussions of the Absolute, we can ask whether an ultimate entity must be infinite, whether lesser things can be infinite as well, and how the infinite relates to our seemingly finite lives.

The metaphysical questions carry over to the theological realm, and with an added emotional intensity. Theologians might, for instance, speculate about how a finite, created mind experiences an infinite God's love.

In the psychological domain, some might argue that it's impossible to talk coherently about infinity at all, whereas others report meditative mental perceptions of the infinite.

[1] Georg Cantor. 1980. *Gesammelte Abhandlungen,* p. 378. Berlin: Springer.. This translation is taken from my book, *Infinity and the Mind*, p. 9. Princeton: Princeton University Press, 2004. Robert John Russell also mentions this quote in his chapter in the present volume, "God and Infinity: Theological Insights from Cantor's Mathematics."

And, finally, in the arts, we seek to find representations of our looming intimations of the infinite, perhaps in paintings, or in music, poetry, or prose.

In the following remarks, I'll say a bit about the different kinds of infinity, with special attention to the contents of the essays gathered in this volume. I'll divide my remarks into four sections: (1) mathematical infinities, (2) physical infinities, (3) metaphysical and theological infinities, and (4) psychological and artistic infinities.

Mathematical Infinities

Enrico Bombieri's genial, discursive chapter, "The Mathematical Infinity," gives us a historical survey of many areas in which infinity has cropped up in mathematics, running from the Pythagoreans to the P and NP problem in computer science. Wolfgang Achtner's chapter, "Infinity as a Transformative Concept in Science and Theology," describes how the evolution of mathematical and physical notions of infinity has advanced in concert with our theological notions of infinity. I'll say more about Achtner's chapter in the section on metaphysical and theological infinities.

For now, I'll describe a high point of the history of mathematical infinity in my own words. Set theory, or the mathematical theory of infinity, was in large part created by Georg Cantor in the late 1800s. Cantor distinguishes between a specific set and the abstract notion of its size. In Cantor's theory, there's no contradiction or incoherence in having, say, two times a transfinite cardinal be the same transfinite cardinal. And, unlike finite sets, an infinite set can have the same cardinality as a proper subset of itself. Cantor calls these infinite number sizes "transfinite cardinals."

Cantor's celebrated theorem of 1873 shows that there are transfinite cardinals of strictly different sizes. Using a so-called diagonal argument, Cantor proved that the size of the set of whole numbers is strictly less than the size of the set of all points on a line. More generally, he showed that the cardinality of any set must be less than the cardinality of its power set, that is, the set that contains all the given set's possible subsets. Along with a principle known as the axiom of choice, the proof method of Cantor's theorem can be used to ensure an endless sequence of ever-larger transfinite cardinals.

The transfinite cardinals include aleph-null (the size of the set of whole numbers), aleph-one (the next larger infinity), and the continuum (the size of the set of points on a line). These three numbers are also written as $\aleph_0$, $\aleph_1$, and c. By definition, $\aleph_0$ is less than $\aleph_1$, and by Cantor's theorem, $\aleph_1$ is less than or equal to c. And we can continue on past $\aleph_1$ to such numbers as $\aleph_2$ and $\aleph_{\aleph_0}$.

The continuum problem is the question of which of the alephs is equal to the continuum cardinality c. Cantor conjectured that $c = \aleph_1$; this is known as Cantor's continuum hypothesis, or CH for short. The continuum hypothesis can also be thought of as stating that any set of points on the line must either be countable (of size less than or equal to $\aleph_0$) or have a size as large as the entire space (be of size c).

In the early 1900s a formalized version of Cantor's theory of infinite sets arose. This theory is known as ZF, which stands for "Zermelo-Fraenkel set theory." Informally speaking, the theory is often taken to include the axiom of choice.

The continuum hypothesis is known to be undecidable on the basis of the axioms in ZF. In 1940, the logician Kurt Gödel was able to show that ZF can't disprove CH, and in 1963, the mathematician Paul Cohen showed that ZF can't prove CH. Set theorists continue to explore ways to extend the ZF axioms in a reasonable way so as to resolve CH. In the early 1970s, Kurt Gödel suggested that CH may be false and that the true size of c could be the larger infinity $\aleph_2$. In 2001, the mathematician W. Hugh Woodin seemingly espoused this view as well.[2]

In one of his two chapters for the present volume, "The Realm of the Infinite," W. Hugh Woodin mounts farther than ever into the pinnacles of the infinite. Woodin and like-minded set theorists feel that Cantor's continuum problem may in fact be solvable, and that the answer is likely to have some relation to large cardinal axioms, which posit higher and higher levels of infinity.

Near the start of his chapter, Woodin makes the point that asserting the consistency of set theory is equivalent to asserting that certain destructive types of proofs will never be found to exist in the physical world. Thus, in this sense there is a direct, albeit subtle, connection between set theory and the physical world. Woodin feels that this connection lends some validity to the belief that the universe of set theory is real.

Some skeptics maintain that Cantor's continuum problem is, in fact, a meaningless question, akin to asking about, say, the fictional Frodo Baggins's precise height. In "The Realm of the Infinite," Woodin deploys a number of refined arguments for the reality and the objectivity of the continuum problem.

In particular, he wants to undermine what he calls the "generic-multiverse position," which suggests that, although we have many diverse models of set theory, there is really no one true model wherein something like the continuum hypothesis is definitely true or false. In the multiversal kind of view, CH is true in some models, false in others, and that's the end of it.

The perennial hope among set theorists is that if we can attain still higher conceptions of infinity, these insights may end up by shedding light on even such relatively low-level questions as the continuum problem. These novel kinds of infinity are collectively known as large cardinals. It may be that by extending set theory with some new axioms about large cardinals, we could narrow in on a more complete and satisfying theory.

Let me remark in passing that there is some similarity between a contemplative monk's quest for God and a set theorist's years-long and highly focused study of large cardinals.

Woodin formulates his analysis in terms of such highly advanced modern notions as his Ω Conjecture, the Inner Model Program, and his Set Theorist's Cosmological Principle. This is twenty-first-century mathematics, and a delight to behold, even if the details will lie beyond many of us.

Woodin is arguing for a maximally rich universe of set theory, in which new levels of surprise and creativity can be found at arbitrarily large levels. In his words, "It is a fairly common (informal) claim that the quest for truth about the universe of sets is analogous to the quest for truth about the physical universe. However, I am claiming an important distinction. While physicists would rejoice in the discovery that the conception of the physical universe reduces to the conception of some simple fragment or model, the set theorist rejects this possibility. By the very nature of its conception, the set of all truths

[2] W. Hugh Woodin. 2001. The continuum hypothesis, part I and part II. *Notices of the American Mathematical Society* (June/July and August).

of the transfinite universe (the universe of sets) cannot be reduced to the set of truths of some explicit fragment of the universe of sets."

Infinity isn't the only concept that lies on the interface between mathematics and philosophy. In his other startling contribution to this volume, "A Potential Subtlety Concerning the Distinction between Determinism and Nondeterminism," Woodin uses his brand of intellectual legerdemain to argue that there really is no coherent distinction between free will and determinism. The proofs are rigorous and subtle, drawing in deep results from recursion theory and nonstandard model theory.

Woodin's line of thought is similar to the following argument, which is drawn from computer science. Philosophers often suppose that there are only two options. *Either* we are deterministic and all of our decisions are predictable far in advance *or* our behavior is utterly capricious and essentially random. But there is a third way. A human being's behavior may indeed be generated by something like a mathematical algorithm, but it may be that the workings of the algorithm do not admit for any kind of shortcuts or speedups. That is, human behavior can be deterministic without being predictable.

Those not well versed in mathematical set theory sometimes imagine there to be a strong likelihood that our formal science of the infinite may contain an inconsistency. If, for instance, ZF set theory were to be inconsistent, then at some point we'd learn that the theory breaks down and begins "proving" things like $0=1$. In this case, the theory would be useless.

Most professional set theorists develop a kind of sixth sense, whereby they feel themselves to be proving things about a Platonic world of actually infinite objects. One of my thesis advisors, Gaisi Takeuti, used to say, "Why would you believe in electrons or in a tiny village in Russia that you've never seen – yet deny the reality of $\aleph_1$ or of the set of all real numbers?" To a mathematician who's "looking" at the class of all sets every day, it seems quite evident that set theory is consistent – in much the same way that a physicist is sure that the laws of physics are consistent. A theory has a concrete model if and only if it is consistent.

In his chapter, "Concept Calculus: Much Better Than," Harvey Friedman, who has often worked in the field of proof theory, takes a novel approach to questions about the consistency or inconsistency of the standard theory or ZF set theory. How do we win the confidence of someone who hesitates to believe in a world of actually infinite sets? Friedman's new idea is that we might possibly model set theory in terms of the ordinary informal concepts of "better than" and "much better than." The chapter is an interesting tour de force, quite technical and demanding in its details. Friedman's intended program is to find further deep connections between logic and common sense, and this is to be commended.

Cantor was well aware that some people are in some sense blind to the possibility of infinity. As Cantor puts it: "The fear of infinity is a form of short-sightedness that destroys the possibility of seeing the actual infinite, even though it in its highest form has created and sustains us, and in its secondary transfinite forms occurs all around us and even inhabits our minds."[3]

[3] I quote and translate this remark in my book, *Infinity and the Mind*, p. 43. The original quote appears in Georg Cantor, *Gesammelte Abhandlungen*, p. 374.

In his chapter, "Warning Signs of a Possible Collapse of Contemporary Mathematics," Edward Nelson speaks of "the strong emotions of loathing and oppression that the contemplation of an actual infinity arouses in me. It is the antithesis of life, of newness, of becoming – it is finished." Nelson proceeds to argue that the notion of infinity is somehow inconsistent. He takes an extreme finitist tack and presents a case that even the existence of very large finite sets is questionable. But, in the end, there's a certain circularity to any a priori arguments for or against the possibility of infinity.

Physical Infinities

The science of physical infinities is much less developed than the science of mathematical infinities. The main reason is simply that the status of physical infinities is quite undecided. In physics, one might look for infinities in space, time, divisibility, or dimensionality, and I'll discuss this in this section.

It is worth noting, however, that we are still conspicuously lacking in any physical application for the transfinite numbers of set theory. Along these lines, Georg Cantor hoped he could find an application for transfinite set theory in the realm of physics – at one time he proposed that ordinary matter might be made of aleph-null particles and that aether (which we might now term electromagnetic fields) might be made of aleph-one particles. Cantor conjectured that the matter and fields might be decomposable into meaningful pieces based on his notions of the accumulation points of infinite series. It would be a great day for set theorists if anything along the lines of such theories ever reached fruition.

Some of the Greeks speculated that space had to be infinite because the notion of an edge to space is incoherent. But, as Carlo Rovelli mentions in "Some Considerations on Infinity in Physics," if we view the space of our three-dimensional surface as curved into the hypersurface of a hypersphere, we're able to have a space that is both finite and unbounded.

Is our universe really shaped like that? In "Infinity and the Nostalgia of the Stars," Marco Bersanelli discusses how recent measurements of the cosmic microwave background indicate that the overall curvature of our space is very close to being that of a flat, Euclidean space, although the possibility still remains that our space might after all be a large hypersphere, or even a negatively curved space. Bersanelli couches the result in an amusing way:

> It is as if Eratosthenes in his famous measurement of the radius of the Earth in 250 BC was not able to measure any curvature: then his conclusion would have been that the Earth might be flat and infinite, or that its radius is greater than a given size compatible with the accuracy of his observation.
>
> (Bernaselli, Chapter 9)

Bersanelli makes another point that is not so well known. Even if we knew our space were to be precisely flat, we could not inevitably conclude that it was as infinite as an endless plane, for we might allow for the possibility that space might be multiply connected – like the surface of a torus. It is possible to find finitely large and multiply connected spaces that are, in fact, flat or negatively curved.

But suppose our physical universe were indeed infinite in its spatial extent. What then? In his chapter, "Warning Signs of a Possible Collapse of Contemporary Mathematics," Edward Nelson speaks of his discomfort with the concept of an actually infinite universe, and he remarks that in such a universe, every possible variation of our planet Earth would appear infinitely many times. Anthony Aguirre mentions this possibility as well in "Cosmological Intimations of Infinity." In "Infinity and the Nostalgia of the Stars," Marco Bersanelli makes the point that spatial infinity may be boring: "It seems that spatial infinity, in order to be perceived as a fascinating concept, has to maintain some kind of genuine element of variety and surprise."

It is worth remarking that repetition is not inevitable in an infinite universe. Put differently, the mere fact that a collection is infinite does not entail that it's exhaustive. As a very simple counterexample, consider an infinite set of integers that has only *one* odd member, the number 3. Someone who starts at 3 and looks for another odd number is going to be disappointed. As a slightly more sophisticated counterexample, think of a nonperiodic tessellation of a plane, for instance, by Penrose tiles. Although the same few tiles reoccur infinitely often, there is no one particular pattern that can be repeated to obtain the whole. It is possible for each location in an infinite universe to have its own unique qualities.

This being said, if the physical universe really were to be infinite, then there really might be other people exactly like us out there somewhere. Simply working through the number crunching needed to formalize such an argument gives us a little taste of how big infinity really is. Is it discouraging to imagine a copy of oneself on another world? Perhaps not – perhaps it's liberating. Even if you do something wrong here, maybe one of your copies will get it right!

In his chapter, "Infinities in Cosmology," Michael Heller also mentions the question of repetition in an infinite space. As Heller remarks, the physicist Max Tegmark has observed that, in some senses, the notion of a spatially infinite universe is close to the notion of a multiverse of many mutually inaccessible spacetimes.[4] Like me, Heller is unwilling to grant that spatial infinity entails endless duplication. As he puts it, ". . . in the set of real numbers, each number is an individual entity that is never repeated in the entire uncountable infinite set of reals. The 'individualization principle,' in this case, consists in both peculiar properties of a given real number and the ordering properties of the whole set of reals. If such a principle works with respect to such apparently simple entities as real numbers, should we not expect that something analogous could be at work at much higher levels of complexity?"

And what of temporal infinities? In the light of the Big Bang theory, cosmologists think of our universe as having a finitely long past; whether it might have an endless future is an open question.

Under the infinite-future view we might suppose that the space of our universe will continue much as it is now, with the galaxies drifting farther and farther apart, the stars burning to dust, and the remaining particles possibly decaying into radiation. In the finite-future view, we suppose that at some definite future time a cosmic catastrophe will destroy our universe: it might be that our space collapses to a point, or perhaps

[4] Max Tegmark. 2003. Parallel universes. *Scientific American* (May). Also available online at http://space.mit.edu/home/tegmark/crazy.html.

it might be that a parallel sheet of space moves through ours, annihilating everything. In any of the catastrophic finite-future scenarios, we can still wonder if the end of our universe might be followed by the birth of a new one, in which case the future might, in some sense, be infinite after all.

In his stimulating chapter, “Cosmological Intimations of Infinity,” Anthony Aguirre takes up some deep considerations relating to the possible infinitude of space and time. To begin with, he points out that, even if we adopt the Big Bang scenario under which the universe in some sense sprang into being at some finitely removed past time, it is possible for a Big Bang universe to be infinitely large. Even more heartening for those infinitistically inclined, Aguirre remarks that, even though it *appears* as if a Big Bang occurred, it may also be that our past time line is, in fact, infinite.

Along these lines, in their popular exposition, *The Endless Universe*,[5] physicists Paul Steinhardt and Neil Turok envision, as hinted at earlier, two sheets of hyperspace passing through each other and, at a stroke, filling each other with energy and light.

Aguirre prefers the popular inflationary universe scenario, under which our universe, whether finite or infinite, has at some point expanded very much more rapidly than the speed of light. Aguirre points out that, mathematically speaking, one of these inflationary universes can be infinite instead of finite – it takes only a touch of mathematical trickery to make the bubbles infinitely large, at least as seen from the inside. Even more intriguing, we can have an infinite number of inflationary “pocket universes” coexisting, and these pocket universes may themselves be infinite. One might think of them as bubbles spontaneously forming in a pot of boiling water.

A variation on the theme of multiple pocket universes is the notion of the multiverse, wherein, as mentioned previously, many versions of our universe may all exist, nestled together in some quantum mechanical, infinite-dimensional Hilbert space. As Marco Bersanelli remarks in “Infinity and the Nostalgia of the Stars,” the notion of an exhaustive multiverse lacks a certain aesthetic appeal. In “Infinities in Cosmology,” Michael Heller points out that the multiverse model in some sense neuters the interesting philosophical question regarding why our world is the particular way it is. Heller adds that the multiverse idea, although it seems to make superfluous the notion of God as Great Designer, gives added force to the image of God as Creator.

Continuing in the vein of finding subtler kinds of physical infinities, Anthony Aguirre’s “Cosmological Intimations of Infinity” closes with an argument that if our universe had only finitely many states, then our lives would be dominated by random large-scale thermodynamic fluctuations. From this, Aguirre concludes, “. . . the reasoning may indeed be telling us something profound: that the very coherence of our experience means that the universe has infinite possibilities.”

In his chapter, “Infinities in Cosmology,” Michael Heller distinguishes between two kinds of infinities in cosmology. On the one hand, he talks about “infinitely distant” regions such as one might find beyond the reaches of an endless space or an endless time. On the other hand, he talks about the “infinitely divergent” regions called singularities, where “the standard structure of spacetime breaks down: when one approaches such regions, some physical magnitudes tend to infinity.”

[5] Paul J. Steinhardt and Neil Turok. 2008. *Endless Universe: Beyond the Big Bang.* New York: Broadway Books.

Heller describes how mathematically oriented cosmologists have come to terms with the infinitely distant regions by using a trick from differential geometry. They apply specialized representations of distance that allow them to draw an infinite spacetime as a tidy diamond shape known as a Penrose diagram, In these diagrams, the infinities lie along the corners and edges of the diamond, in much the same style that the artist M. C. Escher uses in some of his infinitely regressing images.

The singularities of spacetime are more difficult to deal with. Heller strikingly describes one approach in terms of geodesics, which are spacetime paths that objects might naturally follow, and in terms of apophatic theology, which is the technique of describing God by using negations, that is, by describing the things that *cannot* be said about God. As he puts it, "We collect information from inside a given spacetime (by following the behavior of geodesics in it) to learn something about the way its structure breaks down. The *apophatic* character of our knowledge is mitigated by tracing vestiges of what we do not know in the domain open for our investigation."

Heller makes the interesting point that both kinds of cosmological infinity can have global effects, in that their presence in a given universe can affect all of its spacetime. Here again he makes an interesting connection to theology. "Both 'infinitely distant' and 'infinitely divergent' transcend the regular parts of spacetime and at the same time are, as nonlocal elements of the model, somehow present everywhere in the model. Analogously, God transcends the world and at the same time is present within it."

What of the possibility of infinities in the small? Might matter or space itself be infinitely subdivisible? If this were the case, then each object would, in principle, contain a potentially infinite collection of particles. Of course, that perennial spoilsport quantum mechanics bids to rule this out, but perhaps there's a way around the barrier.

In "Some Considerations on Infinity in Physics," Carlo Rovelli discusses the fact that several modern theories of physics propose that space itself may be quantized. Perhaps quantum mechanics does pose an unbreachable lower bound on size – often this is identified with the so-called Planck length. Or it may be that there's some underlying deeper structure to the universe that also resists endless subdivision. In any of these cases, it seems that neither matter nor space can be viewed as infinitely divisible.

Even so, as I hinted at earlier, there remains a possibility that there may still be some kind of physics that operates below the quantum level. I recently came across a passage in Michio Kaku's book, *Parallel Worlds*, in which he discusses a 1984 theory of "string duality" ascribed to Keiji Kikkawa and Masami Yamasaki. The string duality theory also allows for interesting physics below the Planck length. The Planck length becomes something like an interface between two worlds, one that is, so to say, "inside" the Planck length, and another world that is "outside." As Kaku puts it:

> This means that the physics within the Planck length is identical to the physics outside the Planck length. At the Planck length, spacetime may become lumpy and foamy, but the physics inside the Planck length and the physics at very large distances can be smooth and are in fact identical.[6]

I'll leave the last word on the topic of physical infinities to Carlo Rovelli. In "Some Considerations on Infinity in Physics," he recognizes that all of our current speculations

[6] Michio Kaku. 2006. *Parallel Worlds: A Journey through Creation, Higher Dimensions and the Future of the Cosmos,* p. 237. New York: Anchor.

are inconclusive. As he puts it, "I think that what is truly infinite may just be the abyss of our ignorance."

Metaphysical and Theological Infinities

Although Plato thought of the Absolute as finite, all theologians and metaphysicians from Plotinus on have supposed the Absolute to be infinite. What is meant by "the Absolute" depends, of course, on the philosopher in question; it might be taken to mean God, an overarching universal mind, or simply the class of all possible thoughts.

As mentioned earlier, Wolfgang Achtner's chapter, "Infinity as a Transformative Concept in Science and Theology," gives us a very rich and interesting historical overview, which dovetails nicely with the survey in Enrico Bombieri's "The Mathematical Infinity." Working very much in the spirit of our volume, Achtner looks for the ways in which mathematical, physical, and theological attitudes toward infinity have advanced hand in hand. Achtner sees four steps in this advance.

1. The passage from what the Greeks called the *peiron* (limited, clearly defined, having a simple form) to the *apeiron* (unlimited, indescribable, chaotic). An early example of something *apeiron* was the irrational number length of the diagonal of a square. For the early thinkers, being infinite was a privation, a lack of structure, and it seemed natural to deny that God or the One would have such an unpleasant property.

2. Aristotle's realization that the *apeiron* could be represented in a logical form by using the notion of potential infinity. Rather than throwing up our hands in horror because an irrational number like pi or the square root of two can't be represented as a simple ratio of two whole numbers, we've learned to write our irrational numbers as endless sequences of decimal digits. If we view these sequences as approximation schemata, we are viewing them as potential infinities. As mentioned previously, the mystic philosopher Plotinus was one of the first to view being infinite as a positive attribute, and it seems fair to say that he sometimes thought of God as a potential infinity, a goal toward which a human soul might strive.

3. Gregory of Nyssa, Nicholas of Cusa, and, much later, Georg Cantor all came to think that a truly divine being might have an actually infinite nature, rather than being a potentially infinite process of endless growth. I'll say more about Gregory later. It's important to note that, before Cantor, *apeiron* notions of logical incoherence were still mixed in with the concept of the actual infinite. Although Galileo paved the way, it was Cantor's great achievement to demonstrate that we could, in fact, discuss the infinite in scientific as well as in theological terms.

4. It was Cantor as well who saw that there exists an Absolute Infinity that lies beyond the mathematical transfinites that set theorists discuss. As Achtner mentions, when we reach out to this realm, it's easy to fall into paradoxes. If the Absolute Infinite is the set of all transfinite numbers, then we need to be careful not to call the Absolute Infinite a transfinite number itself, for then it takes on the contradictory quality of being a number that is less than itself.

In closing, Achtner remarks that "the step from finiteness (*peiron*) to potential infinity and the transfinite is associated with the liberation from a purely sensual encounter of the world in favor of a rational relation. The step from the transfinite to absolute infinity is the liberation from merely rational quantitative thinking to the intuitive insight of the unity and infinity of the all-encompassing infinity of God." However, in defense of mathematics, I would add that Gödel-Bernays class-set theory or Gaisi Takeuti's more esoteric nodal transfinite type theory make it clear that mathematics, too, can speak about the Absolute.[7]

Graham Oppy's chapter, "God and Infinity: Directions for Future Research," outlines some possible connections between theology and transfinite set theory, or, as he puts it, the study of the "ultimate source of everything" and the mathematics of "limits and bounds."

One of the salient questions here is whether we might speak of God as infinite, and most theologians would answer this in the affirmative. If, for instance, God is to be omniscient, then it seems as if God might well know infinitely many propositions. But once we learn about Cantor's transfinite numbers, we find ourselves on a slippery slope. It would seem odd to say that there's some respect in which God is as big as aleph-one, but not as big as aleph-two. Hence, as Oppy points out, we're more likely to end up saying that God is "Absolutely Infinite," in Cantor's phrase.

In line with what I said earlier, and despite what Oppy remarks in one of his footnotes, most set theorists would be comfortable with identifying the Absolute Infinite with the proper class of all transfinite ordinals, a class that is variously called On or Ω. This identification is, to repeat a point, an example of how transfinite set theory is a kind of mathematical metaphysics, that is, an exact science of the Absolute.

In "God and Infinity: Theological Insights from Cantor's Mathematics," Robert John Russell further pursues the connections between theology and Cantorian set theory, bringing up two points relating to Cantor's formulation of the ordinal numbers and to the reflection principle used in the foundations of set theory. These points seem rather central to the aims of this book, and I'll summarize them in some detail.

Russell's first point has to do with the fact that Cantor distinguishes between cardinal and ordinal numbers. Cardinality has to do with the size of a number, whereas ordinality has to do with the linear order pattern in which the number elements are arranged. In the finite realm, these notions are equivalent, but in the transfinite realm we can have two numbers of the same cardinality that differ in their ordinality: for instance, the ordinals ω and $\omega + \omega$ have the same cardinality, but they represent different linear order patterns and are different ordinal numbers. A cardinal is simply an ordinal that doesn't have the same size as any previous ordinals. For Russell, what is significant in this context is that set theorists formally represent an ordinal α as the set of all ordinals less than α. Thus, aleph-null is the set of all finite ordinals, aleph-one is the set of all finite or countable ordinals, and so on.

Russell now applies this idea to a certain dilemma faced by theologians, which he describes as follows: "1. The Infinite is the negation of the finite. Yet if it is nothing more than this negation, the Infinite too is finite. 2. To avoid being merely finite through this negation, the Infinite transcends the negation by uniting itself with the finite without destroying their difference."

[7] Rudy Rucker. 1977. The one/many problem in the foundations of set theory. In *Logic Colloquium '76*, R. O. Gandy and J. M. E. Hyland (eds.), pp. 567–93. Amsterdam: North-Holland.