

Numerical Methods in Finance with C++

Driven by concrete computational problems in quantitative finance, this book provides aspiring quant developers with the numerical techniques and programming skills they need.

The authors start from scratch, so the reader does not need any previous experience of C++. Beginning with straightforward option pricing on binomial trees, the book gradually progresses towards more advanced topics, including non-linear solvers, Monte Carlo techniques for path-dependent derivative securities, finite difference methods for partial differential equations, and American option pricing by solving a linear complementarity problem. Further material, including solutions to all exercises and C++ code, is available online.

The book is ideal preparation for work as an entry-level quant programmer, and it gives readers the confidence to progress to more advanced skill sets involving C++ design patterns as applied in finance.

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[DMFM] Discrete Models of Financial Markets, Marek Capiński, Ekkehard Kopp

[PF] Probability for Finance,

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[SCF] Stochastic Calculus for Finance,

Marek Capiński, Ekkehard Kopp, Janusz Traple

[BSM] The Black–Scholes Model,

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[PTRM] Portfolio Theory and Risk Management,

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[NMFC] Numerical Methods in Finance with C++,

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[SIR] Stochastic Interest Rates,

Daragh McInerney, Tomasz Zastawniak

[CR] Credit Risk,

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[FE] Financial Econometrics,

Marek Capiński, Jian Zhang

[SCAF] Stochastic Control Applied to Finance, Szymon Peszat, Tomasz Zastawniak

Series editors Marek Capiński, *AGH University of Science and Technology, Kraków*; Ekkehard Kopp, *University of Hull*; Tomasz Zastawniak, *University of York*



Numerical Methods in Finance with C++

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To Monika and Nina





Contents

	Prefe	ace	page ix
1	Bino	1	
	1.1	Program shell	2
	1.2	Entering data	4
	1.3	Functions	7
	1.4	Separate compilation	10
	1.5	CRR pricer	13
	1.6	Pointers	18
	1.7	Function pointers	21
	1.8	Taking stock	26
2	Bino	omial pricer revisited	28
	2.1	Our first class	28
	2.2	Inheritance	36
	2.3	Virtual functions	43
	2.4	Summing up	49
3	American options		53
	3.1	Multiple inheritance	54
	3.2	Virtual inheritance	59
	3.3	Class templates	64
4	Non-linear solvers		75
	4.1	Implied volatility	75
	4.2	Bisection method	76
	4.3	Newton-Raphson method	77
	4.4	Function pointers	77
	4.5	Virtual functions	80
	4.6	Function templates	83
	4.7	Computing implied volatility	87
	4.8	Remarks on templates	91
5	Monte Carlo methods		93
	5.1	Path-dependent options	93
	5.2	Valuation	95
	5.3	Pricing error	100
	5.4	Greek parameters	103

vii



viii	Contents		
	5.5	Variance reduction	106
	5.6	Path-dependent basket options	114
6	Finite difference methods		125
	6.1	Parabolic partial differential equations	125
	6.2	Explicit method	132
	6.3	Implicit schemes	140
	6.4	Changing coordinates	149
	6.5	American options	154
	6.6	Proofs	162
	Inde:	x	165



Preface

This volume of the 'Mastering Mathematical Finance' series is all about numerical methods combined with C++ programming skills, driven by concrete computational problems in quantitative finance. It begins with straightforward option pricing on binomial trees, and gradually progresses towards more advanced topics, including non-linear solvers, Monte Carlo techniques for path-dependent derivative securities, finite difference methods for partial differential equations, and American option pricing by solving a linear complementarity problem.

Familiarity with C++ is not a prerequisite. The exposition starts from scratch in that respect. Nonetheless, the learning curve is steep, and some, if only limited, experience of computer programming, in any language, might be helpful. In terms of quantitative finance background, working knowledge of the binomial and Black–Scholes models of the market is sufficient. Prior knowledge of numerical methods is not necessary, though it would give some advantage.

This book takes an accelerated route through C++, picking and choosing whatever programming language tools are required to tackle the job in hand. It is not a substitute for a systematic C++ manual. We recommend that the reader should frequently consult such a manual (or Internet resources) for reference about the finer points of the various C++ constructs.

The emphasis is on solving and implementing numerical problems of increasing complexity that arise in finance. Of equal importance is code design that reflects the structure of such problems, facilitates future extensions and encourages collaboration between programmers. This provides motivation to master both the numerical techniques and programming language at the same time.

One of the aims of this course is to prepare the reader for work as an entry-level quant programmer. It can also be used as a springboard to embark on more advanced texts, for example, the brilliant book by Mark Joshi.¹

The material has been tried and tested at the University of York, as part of an MSc in Mathematical Finance, both campus-based and by online

¹ M. Joshi, C++ Design Patterns and Derivatives Pricing, Cambridge University Press 2004.



x Preface

distance learning, and it has also been taught to final-year undergraduates. We are indebted to our students for their feedback and enthusiasm, constructive criticism and creative ideas, which resulted in many improvements both in the code and the text.

The accompanying C++ code is available on the linked website

www.cambridge.org/9781107003712

which also contains solutions to all exercises. Errata and some bonus material will be posted on this website in due course.