AN INTRODUCTION TO FLUID MECHANICS

This is a modern and elegant introduction to engineering fluid mechanics enriched with numerous examples, exercises, and applications. The goal of this textbook is to introduce the reader to the analysis of flows using the laws of physics and the language of mathematics. The approach is rigorous, but mindful of the student. Emphasis is on building engagement, competency, and problem-solving confidence that extends beyond a first fluids course.

This text delves deeply into the mathematical analysis of flows, because knowledge of the patterns fluids form and why they are formed and the stresses fluids generate and why they are generated is essential to designing and optimizing modern systems and devices. Inventions such as helicopters and lab-on-a-chip reactors would never have been designed without the insight brought by mathematical models.

Faith A. Morrison is Professor of Chemical Engineering at Michigan Technological University, where she has taught for 22 years. Morrison's expertise is in polymer rheology, in particular focusing on materials with structure, including high-molecular-weight polymers, block copolymers, hydrogels, and composites. She is the Past President of the Society of Rheology and Editor of the *Rheology Bulletin*. Morrison is the author of *Understanding Rheology* (2001).

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Faith A. Morrison

Department of Chemical Engineering Michigan Technological University



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Cover photo: The Naruto Whirlpools, Japan, as seen from a tourist cruise boat. Photo taken by Hellbuny.

> This book is dedicated to my mother Frances P. Morrison, my father Philip W. Morrison, and my elder brother Professor Philip W. Morrison, Jr.

Contents

Preface

PART I PREPARING TO STUDY FLOW

1 W	Why Study Fluid Mechanics?		
1.1	Gettin	3	
1.2	Quick	8	
	1.2.1	MEB with no friction, no work: Macroscopic	
		Bernoulli equation	15
	1.2.2	MEB with shaft work	26
	1.2.3	MEB with friction	34
1.3	Connecting mathematics to fluid mechanics		49
	1.3.1	Calculus of continuous functions	50
		1.3.1.1 Derivatives	50
		1.3.1.2 Integrals	54
	1.3.2		58
		1.3.2.1 Coordinate systems	61
		1.3.2.2 Tensors	67
		1.3.2.3 Differential operations	70
		1.3.2.4 Curvilinear coordinates	74
	1.3.3		84
	1.3.4	Practical advice	91
1.4	Proble	ems	93
2 Ho	ow Fluid	s Behave	106
2.1	Viscos	sity	106
2.2	Drag		113
2.3	Bound	118	
2.4	Lamin	127	
2.5	Aerod	137	
2.6	Supers	143	
2.7	Surfac	145	
2.8	Flows with curved streamlines		
2.9	Magnetohydrodynamics		

viii

2.10 Particulate flow

Summary

Contents

2.11

2.12	Problems	15
PAR [.]	T II THE PHYSICS OF FLOW	
3 M	odeling Fluids	16
3.1	Motion of rigid bodies	10
3.2	Motion of deformable media	
	3.2.1 The continuum model	17
	3.2.1.1 Field variables	17
	3.2.1.2 The continuum hypothesis	18
	3.2.1.3 Fluid particles	18
	3.2.2 Control-volume approach	18
	3.2.2.1 Momentum balance on a control volur	ne 19
	3.2.2.2 The convective term	19
	3.2.3 Problem solving with control volumes	20
	3.2.3.1 Microscopic control-volume problem	20
	3.2.3.2 Macroscopic control-volume problem	21
3.3	Summary	21
3.4	Problems	2
4 M	olecular Fluid Stresses	22
4.1	Forces on a control volume	22
4.2	Stationary fluids: Hydrostatics	23
	4.2.1 Gases	23
	4.2.2 Liquids	24
	4.2.3 Pascal's principle	20
	4.2.4 Static fluid devices	27
	4.2.4.1 Manometers	27
	4.2.4.2 Hydraulic lifts	27 28
4.3	Fluids in motion	
	4.3.1 Total molecular stress	28
	4.3.1.1 Stress tensor	28
	4.3.1.2 Stress sign convention	29
	4.3.2 Isotropic and anisotropic stress	30
4.4	Free-surface stress effects	32
4.5	Problems	33
5 St	ress-Velocity Relationships	34
5.1	Simple shear flow	34
	5.1.1 Velocity field	35
	5.1.2 Stress field	35
	5.1.3 Viscosity	30
5.2	Newtonian fluids	
	5.2.1 The constitutive equation	30
	5.2.2 Using the constitutive equation	37

154

157

Conte	nts		
5.3	Non-N	Jewtonian fluids	393
	5.3.1	Non-Newtonian viscosity	394
	5.3.2	-	397
		Inelastic constitutive equations	402
		Viscoelastic constitutive equations	414
5.4		•	418
5.5	Proble	•	418
PAR	TIII FI	LOW FIELD CALCULATIONS	
6 N	icroscop	pic Balance Equations	429
6.1	Derivi	ng the microscopic balance equations	430
	6.1.1	Gauss-Ostrogradskii divergence theorem	432
	6.1.2	Mass balance	433
	6.1.3	Momentum balance	438
		6.1.3.1 General fluids	438
		6.1.3.2 Newtonian fluids	441
	6.1.4	Energy balance	442
6.2	Using	microscopic-balance equations	445
	6.2.1	Solution methodology	446
		6.2.1.1 The equations	447
		6.2.1.2 Applying the equations	452
	6.2.2		464
	6.2.3	Engineering quantities from velocity and stress	
		fields	472
		6.2.3.1 Total force on a wall	472
		6.2.3.2 Torque	478
		6.2.3.3 Flow rate and average velocity	481
		6.2.3.4 Velocity and stress extrema	483
6.3	Summ	•	485
6.4	Proble	•	486
7 Ir	iternal Fl	ows	494
7.1	Circula	ar pipes	494
	7.1.1	Laminar flow in pipes	497
	7.1.2	Turbulent flow in pipes	511
		7.1.2.1 Momentum balance in turbulent flow	517
		7.1.2.2 Dimensional analysis	518
		7.1.2.3 Data correlations	529
7.2	Noncia	rcular conduits	540
	7.2.1	Laminar flow in noncircular ducts	541
		7.2.1.1 Poisson equation	541
		7.2.1.2 Poiseuille number and hydraulic diameter	554
	7.2.2	Turbulent flow in noncircular ducts	570
7.3		complex internal flows	572
	7.3.1	Unsteady-state solutions	573
	7.3.2	Quasi-steady-state solutions	577
	-	 V 	

Contents

	7.3.3	Geometrically complex flows (including lubrication		
		approximation, converging flows, and entry flows)	580	
7.4	Proble	ms	585	
8 External Flows				
8.1	Flow around a sphere			
	8.1.1	Creeping flow around a sphere	604	
	8.1.2	Noncreeping flow around a sphere	622	
		8.1.2.1 Dimensional analysis of noncreeping flow	628	
		8.1.2.2 Flow patterns	647	
		8.1.2.3 Potential flow	650	
8.2 Boundary layers		lary layers	673	
	8.2.1	Laminar boundary layers	678	
	8.2.2	Turbulent boundary layers	696	
	8.2.3	Flow past blunt objects	705	
8.3	3 More complex external flows		718	
	8.3.1	Vorticity	718	
	8.3.2	Dimensional analysis redux	726	
8.4	Problems 7			

PART IV ADVANCED FLOW CALCULATIONS

9 Ma	Macroscopic Balance Equations			
9.1	Derivi	741		
	9.1.1	Macroscopic mass-balance equation	742	
	9.1.2	Macroscopic momentum-balance equation	745	
	9.1.3	Energy balance	750	
		9.1.3.1 Closed systems	751	
		9.1.3.2 Open systems	753	
		9.1.3.3 Mechanical energy balance	759	
9.2	Using	766		
	9.2.1	Pressure-measurement devices	769	
	9.2.2	Flow-rate-measurement devices	772	
	9.2.3	Valves and fittings	779	
	9.2.4	Pumps	800	
		9.2.4.1 Pump sizing	801	
		9.2.4.2 Net positive suction head	814	
	9.2.5	Open-channel flow	823	
9.3	Proble	ms	830	
10 H	low Flui	ds Behave (Redux)	838	
10.1	Viscos	sity, drag, and boundary layers	838	
10.2	Numer	Numerical solution methods		
	10.2.1	Strategy	840	
	10.2.2	Software packages	842	
	10.2.3	Accuracy	843	
10.3	Lamin	845		

xi	Contents		
	10.3.1 Statistical modeling of turbulence10.3.2 Flow instability	846 851	
	10.4 Lift, circulation	853	
	10.5 Flows with curved streamlines	855	
		867	
	r · · · · · · · · · · · · · · · · · · ·		
	10.7 Summary	874	
	10.8 Problems	874	
	PART V APPENDICES		
	Appendix A: Glossary		
	Appendix B: Mathematics	892	
	B.1 Differential operations on vectors and tensors	892	
	B.2 Differential operations in rectangular and curvilinear		
	coordinates	898	
	Bibliography		
	Index	917	

Preface

This book forms the basis of a one-semester introductory course in fluid mechanics for engineers and scientists. Students working with this text are expected to have a background in multivariable calculus, linear algebra, and differential equations; review of these topics as applied to fluid mechanics is provided in Chapter 1. Problem solving is taught by example throughout the text. We include numerous solved examples and end-of-chapter problems, and a complete solution manual is available for instructors.

Fluid mechanics can be a difficult subject. Nonlinear physics governs flow, and thus we often resort to a variety of simplifications to obtain solutions. Different simplifications are used under different conditions, making fluid mechanics intimidating, at least to a beginner. *An Introduction to Fluid Mechanics* presents the topic through a discovery process, as described in this preface, that mimics engineering practice. The process used seeks solutions by answering the following questions:

- 1. What is the problem?
- 2. What do we need to know, and do, to address the problem?
- 3. What is the solution to the problem?
- 4. What other problems/opportunities may be addressed now that we have solved this problem?

This organizational choice builds critical thinking skills by emphasizing the thought processes that lead to model development. The book is divided into four parts that answer these four questions for the study of fluid mechanics.

1. What is the problem? [Part I: Preparing to Study Flow] Chapter 1: Why Study Fluid Mechanics Chapter 2: How Fluids Behave

The problem addressed in this book is how to bring readers to an understanding of flow behavior and to mastery of flow-modeling calculations. To accomplish this objective, students must come to the task with skills in mathematics and simple flow calculations. In Chapter 1 we introduce the problem, cover needed background calculations (i.e., the macroscopic mass balance and the mechanical engineering balance), and review mathematics that is prerequisite to the study of fluid mechanics (i.e., calculus and differential equations). In Chapter 2, we showcase the diversity and complexity of fluid behaviors—showing readers that the mechanical energy balance is insufficient to explain flow patterns and making

xiv Preface

the case that effort spent learning fluid mechanics is worth it. The presentation in Chapter 2 is at the survey level and spans from the introduction of viscosity to discussions of magnetohydrodynamics and vorticity. Overall, the text follows a path inspired by the spiral learning curve [Bruner, 1966], with the topics of Chapter 2 revisited at the end of the book (*Chapter 10: How Fluids Behave* (*Redux*)). That final chapter demonstrates how the intervening presentation leads to the ability to solve complex flow problems.

2. What do we need to know, and do, to address the problem? [Part II: The Physics of Flow] Chapter 3: Modeling Fluids Chapter 4: Molecular Fluid Stresses Chapter 5: Stress-Velocity Relationships

Having clarified our objectives in Part I, we seek methods to address the objectives in Part II. The continuum and the control volume are introduced in Chapter 3, and the stress components, fluid statics, and surface tension are presented in Chapter 4. To apply momentum conservation to a continuum, we need the stress constitutive equations, developed in Chapter 5 (Newtonian and non-Newtonian). These three chapters introduce the complete continuum model.

It can be a challenge to maintain student focus when covering background material, and we address this issue in a unique way: we provide a storyline. At the end of Chapter 3 we introduce two flow calculations and follow them longitudinally throughout Part II. These two problems (flow down an incline plane and flow in a 90-degree bend) are addressed in a just-in-time format, beginning before readers know enough fluid mechanics to be able to solve them. The solution develops gradually, incorporating new model pieces as they are covered. The repeated appearance of the two highlighted problems focuses readers on new developments, demonstrating the utility of the most recent step. Both highlighted problems are completed in Chapter 5, and Part II closes with the continuum model in place.

3. What is the solution to the problem? [Part III: Flow Field Calculations] Chapter 6: Microscopic Balance Equations

Chapter 7: Internal Flows

Chapter 8: External Flows

Model in hand, we turn to flows of interest. In Chapter 6 we develop the microscopic momentum balance (i.e., the Navier-Stokes equation), which represents an adaptation of the methods of Part II to the general case. We introduce the expressions for flow rates, fluid forces on walls, and fluid torques and show how to use these. In Chapter 7 a range of internal flows is discussed (pipes and ducts); in Chapter 8 external flows and boundary-layer flows are presented in detail (drag and lift).

The reader's path through Chapters 7 and 8 follows once again a storyline of a pair of highlighted flow problems. Chapter 7 begins with the quest to determine the extent of a home flood. Although not transparently related to the continuum model, the home flood problem is readily associated with pipe flow and motivates

Preface

xv

the examination of pressure drop/flow rate relationships, laminar and turbulent flow, and other internal-flow topics. We repeat this structure in Chapter 8, asking about a skydiver, which raises the question of flow past an obstacle in general, leading to discussion of drag, lift, and boundary layers.

Throughout Part III we employ dimensional analysis when the models we develop are too difficult to solve. Dimensional analysis is presented as a natural step in a problem-solving methodology that begins with addressing simplified versions of a real problem (because those are the problems we can solve and they give us insight), progresses to solving mathematically complex models, and turns ultimately to obtaining practical data correlations.

4. What other problems/opportunities may be addressed now that we have solved this problem? [Part IV: Advanced Flow Calculations] Chapter 9: Macroscopic Balance Equations Chapter 10: How Fluids Behave (Redux)

The final two chapters of *An Introduction to Fluid Mechanics* guide readers through advanced modeling calculations on a variety of flows. In Chapter 9 the macroscopic balances, including the mechanical energy balance and the macroscopic momentum balance, are derived and applied. Although simple uses of the mechanical energy balance are covered in Chapter 1, in Chapter 9 the applications are more involved, including pump sizing and open-channel flow. Applying the macroscopic momentum balance is generally considered to be a difficult topic; we systemize macroscopic momentum solutions, making them more accessible. In Chapter 10, the learning spiral returns us to the more complex flows introduced in Chapter 2, and we apply the now-familiar continuum model to begin to understand these flows. Chapter 10 discusses numerical solutions, statistical aspects of turbulence, lift, circulation, vorticity, and supersonic flow.

The text includes reference materials provided to aid the student. The appendices contain a glossary of terms and mathematical tables. There is additional mathematical assistance available on the Internet in the Web Appendix. Finally, key equations are presented on the inside covers as an aid to problem solving.

REFERENCE

Bruner, Jerome S., *The Process of Education* (Harvard University Press: Cambridge, MA, 1966).

Acknowledgments

My path to choosing this presentation method for my fluids class began in 1998 when I was first asked to teach fluid mechanics. I looked at the texts available, and, given the goals of both my course and my students, I had difficulty choosing a text. Although I did not find a book that satisfied my needs, I did find notes from a colleague, Professor Davis W. Hubbard, that got me started in the right direction. Professor Hubbard passed away in 1994, before this text was conceived, but his contribution to pedagogy lives on through his influence on this book.

Preface

xvi

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