Part I

Money

Cambridge University Press
978-1-107-00349-1 - Modeling Monetary Economies, Third Edition
Bruce Champ, Scott Freeman and Joseph Haslag
Excerpt
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Chapter 1

A Simple Model of Money: Building a Model of Money

IN THIS BOOK, we will try to learn about monetary economies through the construction of a series of model economies that replicate essential features of actual monetary economics. All such models are simplifications of the complex economic reality in which we live. They may be useful, however, if they are able to illustrate key elements of the behavior of people who choose to hold money and to predict the reactions of important economic variables such as output, prices, government revenue, and public welfare to changes in policies that involve money. We start our analysis with the simplest conceivable model of money. We will learn what we can from this simple model and then ask how the model fails to adequately represent reality. Throughout the book, we try to correct the model's oversights by adding, one by one, the features it lacks.

To arrive at the simplest possible model of money, we must ask ourselves which features are essential to monetary economics. The demand for money is distinct from the demand for the goods studied elsewhere in economics. People want goods for the utility received from their consumption. In contrast, people do not want money in order to consume it; they want money because money helps them get the things they want to consume. In this way, money is a medium of exchange—something acquired to make it easier to trade for the goods whose consumption is desired.

A model of this distinction in the demand for money therefore requires two special features. First, there must be some "friction" to trade that inhibits people from directly acquiring the goods they desire in the absence of money. If people could costlessly trade what they have for what they want, there would be no role for money.

Second, someone must be willing to hold money from one period to the next. This is necessary because money is an asset held over some period of time, however short, before it is spent. Therefore, we will look for models in which there is always someone who will live into the next period.

Two possible frameworks meet this second requirement. People (or households) could live infinite lives or could live finite lives in generations that overlap (so that some, but not all, people will live into the next period). For many of the topics we study, life span does not matter. We identify where it does matter in Appendix B of Chapter 16, where infinitely lived households are studied in detail.

With the exception of that appendix, we concentrate on the second framework—the overlapping generations model. This model, introduced by Paul Samuelson (1958), has been applied to the study of a large number of topics in monetary theory and macroeconomic theory. Among its desirable features are the following:

- Overlapping generations models are highly tractable. Although they can be used to analyze quite complex issues, they are relatively easy to use. Many of their predictions may be described on a simple two-dimensional graph.
- Overlapping generations models provide an elegantly parsimonious framework in which to introduce the existence of money. Money in overlapping generations models dramatically facilitates exchange between people who otherwise would be unable to trade.
- Overlapping generations models are dynamic. They demonstrate how behavior in the present can be affected by anticipated future events. They stand in marked contrast to static models, which assume that only current events affect behavior.

We begin this chapter with a very simple version of an overlapping generations model. As we proceed through the book, we introduce extensions to this basic model. These extensions allow us to analyze a variety of interesting issues.

Other model economies share the same three characteristics we identified previously. Our aim is not to be all encompassing and cover all of these alternatives. Rather, our approach is more topic driven. After building the basic framework, the extensions we introduce are tied to questions. By focusing on the overlapping generations model, we are able to utilize its flexibility. Over time, other model economies with the same three characteristics will likely exhibit the same flexibility, and coverage of the same broad set of topics will be made available.

To foreshadow one such avenue, we recognize recent work by Narayana Kocherlakota (1999), who has identified a market mechanism that is a perfect substitute for the trading mechanisms in which money is valued. In the overlapping generations economy, money is the means for executing intergenerational transfers. Mutually beneficial trades are conducted despite the friction between generations. In constast, without money, the old generation has nothing the young generation wants. Money embodies both features by overcoming the intergenerational friction and being durable enough to carry from one period to the next. Kocherlakota demonstrates that perfect memory is equivalent to money. In other words, with perfect social record keeping, young people will trade with old people, knowing that the record of the young's trade will overcome the intergenerational friction. When old, a person will turn to the accounting device and trade with young people. Perfect record keeping provides the same mutually beneficial trade as money. We end the

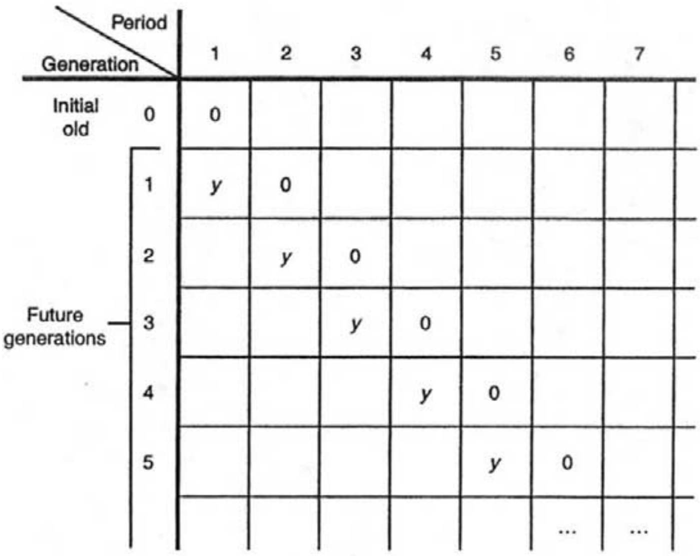

Figure 1.1. The pattern of endowments. In each period t, generation t is born. Each individual lives for two periods. Individuals are endowed with y units of the consumption good when young and 0 units when old. In any given period, one generation of young people and one generation of old people are alive. The name of this model, the overlapping generations model, follows from this generational structure.

chapter by formally presenting the notion that money is memory. For now, let us turn to the development of the basic overlapping generations model.

The Environment

In the basic overlapping generations model, individuals live for two periods. We call people in the first period of life "young" and those in the second period of life "old."

The economy begins in period 1. In each period $t \geq 1$, N_t individuals are born. Note that we index time with a subscript. For example, N_2 is our notation for the number of individuals born in period 2. The individuals born in periods 1, 2, 3, and so forth are called the "future generations" of the economy. In addition, in period 1, there are N_0 members of the initial old.

Hence, in each period t, there are N_t young individuals and N_{t-1} old individuals alive in the economy. For example, in period 1, there are N_0 initial old individuals and N_1 young individuals who were born at the beginning of period 1.

For simplicity, there is only one good in this economy. The good cannot be stored from one period to the next. In this basic setup, each individual receives an endowment of the consumption good in the first period of life. The amount of this endowment is denoted as y. Each individual receives no endowment in the second period of life. This pattern of endowments is illustrated in Figure 1.1.

We can also interpret the endowment as an endowment of labor—the ability to work. By using this labor endowment (by working), the individual is able to obtain a real income of y units of the consumption good.

Preferences

Individuals consume the economy's sole commodity and obtain satisfaction—or, in the economist's jargon, utility—from having done so.

Future Generations

Members of future generations in an overlapping generations model consume both when young and when old. An individual member's utility therefore depends on the combination of personal consumption when young and when old. We make the following assumptions about an individual's preferences regarding consumption:

1. For a given amount of consumption in one of the periods, an individual's utility increases with the consumption obtained in the other period.
2. Individuals like to consume some of this good in both periods of life. An individual prefers the consumption of positive amounts of the good in both periods of life over the consumption of any quantity of the good in only one period of life.
3. To receive another unit of consumption tomorrow, an individual is willing to give up more consumption today if the good is currently abundant than if it is scarce relative to consumption tomorrow.

With these assumptions, we are assuming that individuals are capable of ranking combinations (or bundles) of the consumption good over time in order of preference. We denote the amount of the good that is consumed in the first period of life by an individual born in period t with the notation $c_{1,t}$. Similarly, $c_{2,t+1}$ denotes the amount the same individual consumes in the second period of life. It is important to note that $c_{2,t+1}$ is consumption that actually occurs in period $t+1$, when the person born at time t is old. When the time period is not crucial to the discussion, we denote first- and second-period consumption as $c_1$and c_2.

Suppose we offer an individual the following consumption choices:

- Bundle A, which consists of 3 units of the consumption good when a person is young and 6 units of the consumption good when a person is old. We denote this bundle as $c_1 = 3$ and $c_2 = 6$.
- Bundle B, which consists of 5 units of the consumption good when a person is young and 4 units of the consumption good when a person is old ($c_1 = 5$ and $c_2 = 4$).

By assuming that an individual can rank these bundles, we are saying that he or she can state a preference for bundle A over bundle B or for bundle B over bundle

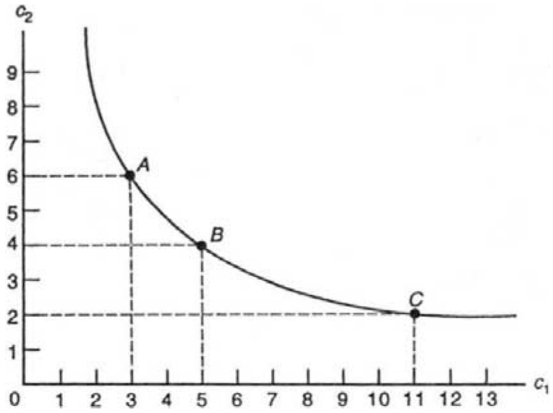

Figure 1.2. An indifference curve. Individual preferences are represented by indifference curves. The figure portrays an indifference curve for a typical individual. Along any particular indifference curve, utility is constant. Here, the individual is indifferent between points A, B, and C.

A or equal happiness with either bundle. The individual can rank any number of bundles of the consumption good that we might offer in this manner.

It will be extremely useful to portray an individual's preferences graphically. We do this with indifference curves. An indifference curve connects all consumption bundles that yield equal utility to the individual. In other words, if offered any two bundles on a given indifference curve, the individual would say, "I do not care which I receive; they are equally satisfying to me." In the preceding example, if the individual were indifferent to bundles A and B, then those two bundles would lie on the same indifference curve. Figure 1.2 displays a typical indifference curve.

On this indifference curve, we show the two points A and B from our earlier example. We also illustrate a third point, C, representing the bundle $c_1 = 11$ and $c_2 = 2$. Because C lies on the same indifference curve as points A and B, point C yields the same level of utility as points A and B for the individual. In fact, any point along the illustrated indifference curve represents a bundle that yields the same utility level.

Note some features of the indifference curve. The first is that the curve becomes flatter as we move from left to right. This is how indifference curves represent assumption 3. This property of indifference curves is called the "assumption of diminishing marginal rate of substitution." To illustrate this assumption, start at point A, where $c_1 = 3$ and $c_2 = 6$. Suppose we reduce the individual's second period consumption by 2 units. The indifference curve tells us that to keep the individual's utility constant, we must compensate him or her by providing 2 more units of first-period consumption. This places the individual at point B on the indifference curve. Now suppose we reduce second-period consumption by another 2 units. To remain indifferent, 6 more units of first-period consumption must

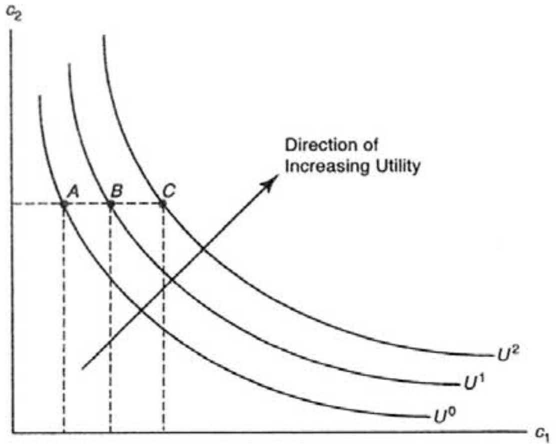

Figure 1.3. An indifference map. An indifference map consists of a collection of indifference curves. For a constant amount of consumption in one period, individuals prefer a greater amount of consumption in the other period. For this reason, individuals prefer point C to point B and point B to point A. Utility increases in the general direction of the arrow.

be given to the individual. In other words, we must compensate the individual with ever-increasing amounts of first-period consumption as we successively cut second-period consumption. This should make intuitive sense; individuals are more reluctant to give up something they do not have much of to begin with.

Consider food and clothing as an example. A person who has a large amount of clothing and very little food would be willing to give up a fairly large amount of clothing for another unit of food. Conversely, this person would be willing to give up only a small amount of food to obtain another unit of clothing.

We demonstrate this assumption of diminishing marginal rate of substitution by drawing an indifference curve that becomes flatter as we move downward and to the right along the curve.

We also assume that the indifference curves become infinitely steep as we approach the vertical axis and perfectly flat as we approach the horizontal axis. The curves never cross either axis. This might be justified by saying that consuming nothing in any one period would mean horrible starvation, to which consuming even a small amount is preferable. This is assumption 2.

It is also important to keep in mind that the indifference curves are dense in the (c_1, c_2) space. This means that if you pick a combination of first- and second-period consumption, there is an indifference curve running through that point. However, to avoid clutter, we normally show only a few of these indifference curves. A group of indifference curves shown on one graph is often called an "indifference map." Figure 1.3 illustrates an indifference map that obeys our assumptions.

Note that utility is increasing in the direction of the arrow. How do we know this? Compare points A, B, and C. Each of these bundles gives the individual the same amount of second-period consumption. However, moving from point A to B

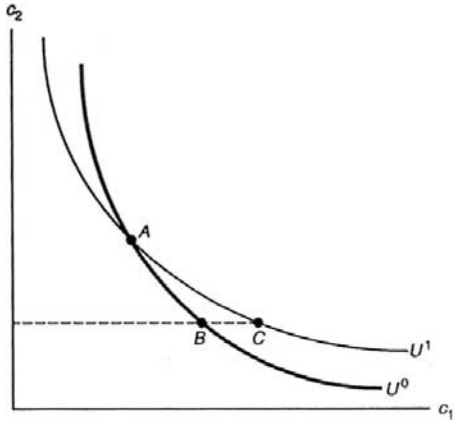

Figure 1.4. Indifference curves cannot cross. By our first assumption about preferences, the individual whose preferences are represented by these indifference curves prefers bundle C over bundle B because bundle C consists of more first-period consumption and the same amount of second-period consumption compared with bundle B. However, because the individual must be indifferent between all three bundles, A, B, and C, a contradiction arises. Our assumptions rule out the possibility of indifference curves that cross.

to C, the individual receives more and more first-period consumption. Hence, the individual will prefer point B to point A. Likewise, point C will be preferable to points A and B. This is assumption 1.

It is often useful to draw an analogy between an indifference map and a contour map that shows elevation. On an indifference map, the curves represent points of constant utility; on a contour map, the curves represent points of constant elevation. Extending the analogy, if we think of traversing the indifference map in a northeasterly direction, we would be going uphill. In other words, utility would be increasing. In fact, an indifference map, like a contour map, is merely a handy way to illustrate a three-dimensional concept on a two-dimensional drawing. The three dimensions here are first-period consumption, second-period consumption, and utility.

One other important concept is that our individual's rankings of preferences are transitive. If an individual prefers bundle B to bundle A and bundle C to bundle B, then that individual must also prefer bundle C to bundle A. Graphically, this implies that indifference curves cannot cross. To do so would violate this property of transitivity and assumption 1 (Figure 1.4). In portrays two indifference curves that cross at point A. We know that indifference curves represent bundles that give an individual the same level of utility. In other words, the individual whose preferences are represented by Figure 1.4 is indifferent between bundles A and B because they lie on the same indifference curve U^0. Similarly, the individual must be indifferent between bundles A and C on indifference curve U^1. We see, then, that the individual is indifferent between all three bundles. However, if we compare bundles B and C, we also observe that they consist of the same amount of second-period consumption but that C contains more first-period consumption than B. According to assumption 1, the individual must prefer C to B. But this contradicts our earlier statement about indifference among the three bundles. For this reason, indifference curves that cross violate our assumptions about preferences.

The Initial Old

The preferences of the initial old are much easier to describe than those of future generations. The initial old live and consume only in the initial period and thus simply want to maximize their consumption in that period.

The Economic Problem

The problem facing future generations of this economy is very simple. They want to acquire goods they do not have. Each has access to the nonstorable consumption good only when young but wants to consume in both periods of life. They must therefore find a way to acquire consumption in the second period of life and then decide how much they will consume in each period of life.

We examine, in turn, two solutions to this economic problem. The first, a centralized solution, proposes that an all-knowing, benevolent planner will allocate the economy's resources between consumption by the young and by the old. In the second, decentralized solution, we allow individuals to use money to trade for what they want. We then compare the two solutions and ask which is more likely to offer individuals the highest utility. The answer helps to provide a first illustration of the economic usefulness of money.

Feasible Allocations

Imagine for a moment that we are central planners with complete knowledge of and total control over the economy. Our job is to allocate the available goods among the young and old people alive in the economy at each point in time.

As central planners, under what constraint would we operate? Put simply, at any given time, we cannot allocate more goods than are available in the economy. Recall that only the young people are endowed with the consumption good at time t. There are N_t of these young people at time t. We have

$$(\textit{total amount of consumption good})_t = N_t y. \tag{1.1}$$

Suppose that every member of generation t is given that same lifetime allocation $(c_{1,t}, c_{2,t+1})$ of the consumption good (our society's view of equity). In this case, total consumption by the young people in period t is

$$(\textit{total young consumption})_t = N_t c_{1,t}. \tag{1.2}$$

Furthermore, total old consumption in period t is

$$(\textit{total old consumption})_t = N_{t-1}c_{2,t}. \tag{1.3}$$

Let us make sure the notation is clear. Recall that the old people in time t are those who were born at time $t-1$. There were N_{t-1} of these people born at time $t-1$. Furthermore, recall that $c_{2,t}$ denotes the second period (time t) consumption by someone who was born at time $t-1$. This implies that total consumption by the old at time t must be $N_{t-1}c_{2,t}$.

Total consumption by young and old is the sum of the amounts in Equations 1.2 and 1.3. We are now ready to state the constraint facing us as central planners: Total consumption by young and old cannot exceed the total amount of available goods (Equation 1.1). In other words,

$$N_t c_{1,t} + N_{t-1}c_{2,t} \leq N_t y. \tag{1.4}$$

For simplicity, we assume for now that the population is constant ($N_t = N$ for all t). In this case, we rewrite Equation 1.4 as

$$Nc_{1,t} + Nc_{2,t} \leq Ny.$$

Dividing through by N, we obtain the per capita form of the constraint facing us as central planners:

$$c_{1,t} + c_{2,t} \leq y. \tag{1.5}$$

For now, we are also concerned with a stationary allocation. A stationary allocation is one that gives the members of every generation the same lifetime consumption pattern. In other words, in a stationary allocation, $c_{1,t} = c_1$ and $c_{2,t} = c_2$ for every period $t = 1, 2, 3,$ and so on. However, it is important to realize that a stationary allocation does not necessarily imply that $c_1 = c_2$. With a stationary allocation, the per capita constraint becomes

$$c_1 + c_2 \leq y. \tag{1.6}$$

This represents a very simple linear equation in c_1 and c_2, which is illustrated in Figure 1.5.

The set of stationary, feasible, per capita allocations—the "feasible set"—is bounded by the triangle in the diagram. We refer to the triangular region as the feasible set. The thick diagonal line on the boundary of the feasible set is called the "feasible set line." The feasible set line represents Equation 1.6, evaluated at equality.

The Golden Rule Allocation

If we now superimpose a typical individual's indifference map on this diagram, we can identify the preferences of future generations among feasible stationary allocations. This is shown in Figure 1.6.